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# Peak-load Pricing with Different Types of Dispatchability*

*Klaus Eisenack*[†] *and Mathias Mier*[‡]

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**Abstract**

We extend the theory of peak-load pricing by considering that the production with different technologies can be adjusted within their capacity at different speeds. In the established analysis, all production decisions can be made after the random variables realize. In our setting, in contrast, some decisions are made before, others after. We consider fixed load and three types of capacities: medium-dispatchable capacity needs to be scheduled ahead of actual production, non-dispatchable capacity produces randomly, and highly-dispatchable capacity can instantly adjust. If capacities differ in their dispatchability, some standard results of peak-load pricing break down, e.g., not all types of capacity will be employed. Either a system with medium-dispatchables only, or a system dominated by non-dispatchables and supplemented by highly-dispatchables occurs, where non- and highly-dispatchables could be substitutes or complements. For the latter system capacity decisions cannot be decentralized by markets since costs recovery is not possible.

**Keywords:** peak-load pricing; dispatchability; costs recovery; market design; renewable energy; energy transition

**JEL Classification:** Q21, Q41, Q42, L94, L97, L98

## 1 Introduction

Many essential goods like energy, transport, or telecommunication services can not be easily stored, and face demand that is changing over time but inelastic in the short-term. Consequently, difficulties arise because production capacities cannot be increased or reduced instantaneously. While

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these difficulties have long been studied in the peak-load pricing literature, the literature has not considered that technologies differ in their flexibility to adjust production within their capacity. We thus present an extension of the standard peak-load pricing model (Crew et al., 1995).

In general, peak-load pricing does not address a minor challenge. Production capacities in those sectors typically face extraordinary capacity expenditures. If capacities are idle in off-peak times so that they cannot charge high prices, one question centers around how to re-finance these expenditures. This is, more general, a question about the optimal capacity mix and whether it can be decentralized with appropriate prices. The problem becomes even more challenging if some technologies have difficulties in rising their production (within capacity) although they are needed on short notice. This can be the case if there are unexpected fluctuations in the production of some technologies, e.g., due to technical failures or weather conditions. Then, for instance, energy utilities face the challenge to provide enough back-up capacity that can flexibly be re-scheduled to avoid outages or disruptions.

One prominent application of peak-load pricing is in electricity market design. In diurnal, weekly, and annual cycles, electricity load changes considerably between peak-load and off-peak periods. A large share of electricity demand is not elastic, and it is typically very costly to store electricity (battery storage is currently not a large-scale option (see, e.g., Luo et al., 2015), while pumped hydropower is, if admitted by the geographical conditions (see, e.g., Gimeno-Gutiérrez and Lacal-Arántegui, 2015; Sinn, 2017)). Moreover, there can be unforeseen disruptions of single power plants. Under such conditions, the standard peak-load pricing model concludes that it is optimal to have a mix of power plants, some with higher capacity costs but lower variable costs in the base-load, and other power plants with lower capacity costs but higher variable costs to be used in the peak-load hours. Capacity costs are mainly re-financed during peak-load hours. Today, however, increasing capacities of renewable generators like wind turbines and photovoltaic power stations are integrated in electricity systems worldwide (REN21, 2018). This poses new challenges. As their production partially depends on the weather conditions, their production is fluctuating randomly. It is an open question of whether current market designs then remain appropriate (Fabra et al., 2011; Henriot and Glachant, 2013). While random production can be principally captured by the standard peak-load pricing model, one crucial implication cannot: When renewables produce less than expected, remaining technologies differ in how flexibly they can adjust. For instance, while gas turbines can ramp-up fast, nuclear power plants typically need scheduling some days ahead. With a rising share of fluctuating renewables, implementing more flexibility options becomes important (Lannoye et al., 2012; Kubik et al., 2015), but differences in flexibility cannot be represented by the standard model.

The integration of renewables in the electricity system will serve as prime example in this paper. The general model, however, is also applicable to other cases, for instance in transport, agriculture, or lean production, where technologies can be distinguished along two dimensions: reliability (as in the standard model) and dispatchability (flexibility or ability to be re-scheduled in the short-term).

The basics of peak-load pricing have been developed by Bye (1926, 1929); Boiteux (1960); Steiner (1957). Brown and Johnson (1969) first implement demand uncertainty. Chao (1983) develops the fundamentals of peak-load pricing with supply uncertainty (see also Kleindorfer and

Fernando, 1993). Crew et al. (1995) provide an excellent survey. The development of the peak load pricing literature can roughly sorted into three settings. In *deterministic settings*, also if extended to multiple technologies and periods, the optimal price in the peak period is equal to the long-run marginal costs of the peak technology, and the zero-profit condition holds (Steiner, 1957; Williamson, 1966; Crew and Kleindorfer, 1971). If there is *demand uncertainty*, some results carry over from the multi-period deterministic setting, but the probability and costs of unserved load or rationing need to be considered. Assumptions about the latter drive part of the differences in the results (Brown and Johnson, 1969; Visscher, 1973; Crew and Kleindorfer, 1976; Carlton, 1977). Chao (1983) and Kleindorfer and Fernando (1993) account for *stochastic supply* as well. They model available capacity as continuum of stochastically independent generating units, each technology with identical probability distribution and availability factor. The model then determines the optimal capacity mix via the probability that the technologies produce, including the optimal probability of unserved load. All technologies that are not strictly dominated in both variable and fixed costs have a positive probability in the optimum. It is important to be aware of the assumed timing of short-term decisions in this literature. First, load is scheduled. Then, all random generating units produce. Finally, after the random variables realize, all remaining production decisions can be made. For instance, if fluctuating renewables are the main random technology, this model assumes that nuclear power plants can be re-dispatched instantaneously. The main innovation of our paper is to assume another timing of short-term decisions to account of differences in dispatchability. Then, interestingly, some standard results of peak-load pricing break down.

In our extension of the theoretical model, some technologies are flexible enough to adjust after the random variable realizes, while other technologies are not. We distinguish three technology types by reliability and *dispatchability*: non-, medium-, and highly-dispatchables. Production of non-dispatchables is random and cannot be adjusted at all. Medium-dispatchables need to be scheduled ahead of non-dispatchable production, but are assumed to be perfectly reliable. Highly-dispatchables can be scheduled after non-dispatchables' random production is known. As in the standard literature, we assume that long-term capacity decisions are made first, and in the short-term (for each period) inelastic load is scheduled before production decisions are made. We also follow the assumption that random generating units are stochastically independent. However, we further consider the polar case where all generating units are perfectly correlated, which had not been done so far in the literature.

Our paper proves the following implications from technologies being able to adjust their production at different speeds. The competition between technologies with different types of dispatchability is much fiercer than suggested by the standard peak-load pricing model. Depending on a specific relation of the costs, it is either optimal just to employ medium-dispatchables, or just a composite of non- and highly-dispatchables. If there are no medium-dispatchables, highly-dispatchables balance random non-dispatchables. The share of the latter increases if their costs or the costs of unserved load fall. Higher costs of highly-dispatchables can lead to more or less non-dispatchable capacity, depending on a specific condition. We also find that the probability of unserved load is independent from the costs of non-dispatchables. On the other hand, non-dispatchables are the price-setting technology (and not the highly-dispatchable peak technology). Furthermore, while medium-dispatchables can recover their fixed cost, highly-dispatchables are never able to do so. A zero-profit condition for non-dispatchables only holds in unlikely boundary cases. This poses crucial challenges for designing markets that decentralize optimal prices.

The following text is organized as follows. Section 2 describes the assumptions of the model, while Section 3 solves it. We continue with the comparative statics in Section 4, followed by the results on cost recovery (Section 5). Section 6 compares our results to the standard model and presents extensions, in particular for multiple periods. The implications of our results for the timely questions on integrating renewables to the electricity system are discussed in Section 7, followed by conclusions in Section 8. Most mathematical proofs are delegated to the Appendix.

## 2 Model

The analysis distinguishes three technology types: non-dispatchable ($N$), medium-dispatchable ($M$), and highly-dispatchable ($H$) capacity. These letters are used as subscripts to denote variables for the different capacity types. Non-dispatchables randomly produce within their capacity constraints. Their average production can only be increased in the long-term by expanding the capacity. Scheduling medium-dispatchable capacity requires a plan certain time ahead of actual production. Highly-dispatchable capacities can instantly adjust to random non-dispatchable production. Medium- and highly-dispatchable capacities are assumed to be perfectly reliable.

Production from type $j = N, M, H$ is $x_j$, load is denoted by $D$, and $x_0 = \max\left\{D - \sum_j x_j, 0\right\}$ is unserved load. From load, consumers obtain utility $U(D)$, possibly net of losses from unserved load. The function $U(D)$ is strictly increasing, strictly concave, and fulfills the Inada conditions. Medium- and highly-dispatchable production are restricted by capacity, $x_M \leq k_M$, $x_H \leq k_H$. Non-dispatchable production at a given point in time is a (continuously differentiable) random variable $\tilde{x}_N$ with density $f = f(\tilde{x}_N; k_N)$, which is defined as a continuum of generating units $z$, $\tilde{x}_N := \int_0^{k_N} \omega(z)\, dz$. Production of marginal generating units $\omega(z) \in [0, 1]$ are stochastically identically distributed random variables (see Chao, 1983). We assume that $\tilde{x}_N$ is boundedly integrable. We call $a = E[\omega(z)]$, where $E$ is the expectation operator, the *availability factor*. Observe that also $\frac{E[\tilde{x}_N]}{k_N} = a$, and that expected production of a marginal generating unit is identical to marginal expected production of non-dispatchable capacity, i.e., $E[\omega(z)] = \frac{dE[\tilde{x}_N]}{dk_N}$. Let $\Omega = [0, k_N]$ be the sample space of $\tilde{x}_N$. For any interval $\Omega_c \subseteq \Omega$, the event $\tilde{x}_N \in \Omega_c$ realizes with probability $\Pr_c$. It will be convenient to denote *average conditional production* by $a_c := \frac{E[\tilde{x}_N|\Omega_c]}{k_N}$.

Our model considers two polar cases for the random variable: the *case of independence* (denoted by *ind*) and *case of perfect correlation* (denoted by *corr*). For *ind*, production of the marginal generating units $\omega(z)$ is stochastically independent. In contrast, for *corr*, production of marginal generating units is perfectly correlated, that is, each unit $z$ produces the same amount of output at a given point in time. While *ind* is standard in the literature (Chao, 1983; Kleindorfer and Fernando, 1993), the *corr* case *for supply* has not been considered to our knowledge so far.[1] In

[1] Chao (1983) considers the cases of marginal demand being independent from or perfectly correlated with with total demand.

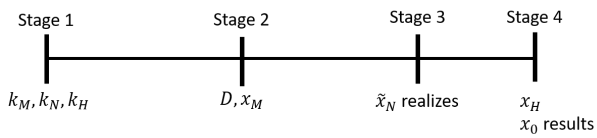



Fig. 1: Decision sequence of the model.

real-world cases, the situation is likely between these extreme cases. Yet, it is then difficult to obtain analytical results. If some results hold for both extremes, it might indicate robustness to more general conditions.

The technologies' unit production costs are denoted by the $c_j$, and the unit capacity costs by $b_j$. In line with the standard literature of peak-load pricing, we assume $c_H > c_M > c_N \geq 0$ and $\frac{b_N}{a}, b_M > b_H$; otherwise highly-dispatchables are obvious inferior to medium-dispatchables or vice verse, respectively. The order between $\frac{b_N}{a}$ and $b_M$ is not further specified to admit different conditions. In the basic model, we consider one time period (see Section 6 for the extension to multiple periods). Consequently, we further assume that (average) long-run marginal costs (LRMC) of the technologies follow $\frac{b_N}{a} + c_N$, $b_M + c_M < b_H + c_H$, where the term *average* refers to the random production of non-dispatchables. Otherwise, all other technologies would be inferior to highly-dispatchables. Again, the order between non- and medium-dispatchables' costs is not further specified. It will be convenient to denote the difference between medium- and non-dispatchables' LRMC by $\Delta C := b_M + c_M - \frac{b_N}{a} - c_N$.

Our model considers the possibility that total production is below (scheduled) load. To represent the costs of unserved load, we use the Turvey-Anderson-Chao approximation with constant marginal costs of unserved load $c_0$ (Turvey and Anderson, 1977; Chao, 1983). Accordingly, the utility ultimately obtained by consumers is 'scheduled utility' $U(D)$ net of losses $c_0 x_0$. This linear approximation is common in the literature to capture the combined effect of foregone utility, curtailment, rationing costs, and losses from disruption (for a discussion see Visscher, 1973; Kleindorfer and Fernando, 1993). Since we focus on the effect of dispatchability types, we do not need a higher-order approximation. We do not implement demand uncertainty in the model to isolate the effects of the dispatchability types. However, the effects of demand uncertainty are similar since costs of unserved load occur in our model as well. We assume that producing with highly-dispatchables is cheaper than accepting unserved load, $b_H + c_H < c_0$. Otherwise, unserved load would be so cheap that no highly-dispatchable capacity would be installed.

To capture the differences between non-, medium-, and highly dispatchable technologies, our model makes the following assumptions about the sequence of decisions (see Figure 1).

While we ultimately want to determine the optimal capacity choice $k_M, k_N, k_H$ (Stage 1), they are fixed in the short-term. Production decisions and load scheduling are made under these restrictions. The essential assumption from the peak-load pricing literature is that load $D$ is scheduled before production decisions are made (Stage 2). As core assumption of our model, also medium-dispatchable production $x_M \leq k_M$ needs to be scheduled in this stage due to their short-term rigidity justified by technological constraints. For example, large coal-fired power plants need several hours to ramp-up or ramp-down, and nuclear power plants need up to several days to start operation. In Stage 3, no decision is made due to the assumed short-term rigidity of non-dispatchables: random production $\tilde{x}_N$ realizes. For example, fluctuating wind generators cannot raise their production if weather conditions are unfavorable. Then, load needs to be served in Stage 4. If the actual non-dispatchable production then deviates from the expectation, it might be that total production of medium- and non-dispatchables is not sufficient to serve load. As load cannot be changed anymore, either highly-dispatchables can be used to increase production, or load remains partially unserved. That is to say, we assume highly-dispatchables have no dynamic constraints in the short-run.

In the following, we determine the load, production, and capacity decisions that maximize

$$J = U(D) - \sum_j b_j k_j - c_M x_M - c_N \tilde{x}_N - c_H x_H - c_0 x_0, \quad (1)$$

under the above technological constraints for the decision sequence. Optimization in Stages 1 and 2 require maximization of $E[J]$, where in Stage 4 no expectations are necessary since the random variable has already realized. This problem will be solved by backward induction in the next Section.

## 3 Production and Capacity Decisions

We start with some preparatory notation. Depending on the realization of the random variable $\tilde{x}_N$ we need to distinguish between four kind of events $\tilde{x}_N \in \Omega_c$, $c = 1, 2, 3, 4$. First, it may happen that production of non-dispatchables leads to excess production above scheduled load, so that part of the production cannot be utilized, i.e., $\tilde{x}_N \in \Omega_1 = [D, k_N]$. We call this the *non-dispatched* events. If non-dispatchable production is not sufficient to serve load, but there is enough medium-dispatchable production, we call all $\tilde{x}_N \in \Omega_2 = [D - x_M, D)$ *medium-dispatched* events. Obviously, highly-dispatchables are not employed in the medium-dispatched event because they are more expensive. Yet, if non-dispatchable and medium-dispatchable production together is not sufficient to serve load, but there is enough highly-dispatchable capacity to fill this gap, that is, if $\tilde{x}_N \in \Omega_3 = [D - x_M - k_H, D - x_M)$, we call this *highly-dispatched* events. Finally, events of *unserved load* occur if production of all three technologies is not sufficient to serve load, $\tilde{x}_N \in \Omega_4 = [0, D - x_M - k_H)$. In addition, we denote the interval of events with either *highly-dispatched* or *unserved load* by $\Omega_{34} = \Omega_3 \cup \Omega_4 = [0, D - x_M)$, and denote the probability of event $c$ by $\Pr_c$.

Now turn to solving the model. The following derivations assume that $k_N > 0$. The case with $k_N = 0$ will be considered separately below, as this removes the random components from the model.[2]

Start with Stage 4. If follows from the definition of unserved load (see Section 2) that $x_0 = D - \tilde{x}_N - x_M - x_H > 0$ only if non-dispatchable production is below a certain value, i.e., $\tilde{x}_N <$

[2] We can solve the whole program by using Kuhn-Tucker conditions. As several of the steps are common in the peak-load pricing literature, we keep them brief to concentrate on the intuition and the particularities of our model.

$D - x_M - x_H$. If $\tilde{x}_N \geq D - x_M - x_H$, there is no unserved load. Since $D, x_M, \tilde{x}_N$ are given in Stage 3, we can leave out expectations and the problem is to maximize welfare w.r.t. $x_H$. We obtain the derivative

$$\frac{\partial J}{\partial x_H} = \begin{cases} -c_H + c_0 & > 0 \text{ for } \tilde{x}_N < D - x_M - x_H, \\ -c_H & < 0 \text{ else.} \end{cases} \tag{2}$$

The signs of the derivatives follow from the production cost assumptions. Thus, highly-dispatchables are only employed if there would be unserved load otherwise. For all other events, reducing production of highly-dispatchables down to zero would increase the objective $J$. By taking the decisions and outcomes from the previous Stages into account, we obtain by the established line of argument in the literature, that optimal highly-dispatchable production is

$$x_H = \begin{cases} k_H & \text{for } \tilde{x}_N \in \Omega_4, \\ D - \tilde{x}_N - x_M & \text{for } \tilde{x}_N \in \Omega_3, \\ 0 & \text{else.} \end{cases} \tag{3}$$

In Stage 3, non-dispatchable production realizes, independently of all decision variables except $k_N$, so that

$$E\left[\tilde{x}_N\right] = a k_N. \tag{4}$$

By using the definition of unserved load, Equation (3), and conditional expectations, we obtain the expected outcome of Stage 3:

$$E\left[x_H\right] = k_H \Pr{}_4 + E\left[D - x_M - \tilde{x}_N | \Omega_3\right] \Pr{}_3, \tag{5}$$

$$E\left[x_0\right] = E\left[D - x_M - \tilde{x}_N - k_H | \Omega_4\right] \Pr{}_4. \tag{6}$$

Now turn to Stage 2. Inserting expected production of non-dispatchables, highly-dispatchables, and expected unserved load (Equations (4) to (6) into Equation (1)) yields

$$\begin{aligned} E\left[J\right] = {} & U\left(D\right) - \sum_j b_j k_j - c_M x_M - c_N a k_N \\ & - c_H \left(k_H \Pr{}_4 + E\left[D - x_M - \tilde{x}_N | \Omega_3\right] \Pr{}_3\right) \\ & - c_0 E\left[D - x_M - \tilde{x}_N - k_H | \Omega_4\right] \Pr{}_4. \end{aligned} \tag{7}$$

For both the case of independence (*ind*) and the case of perfect correlation (*corr*), derivatives of conditional expectations can be simplified by interchanging expectation and differentiation (see, e.g., Chao, 1983; Kleindorfer and Fernando, 1993).[3] In Stage 2, we thus obtain

$$\frac{\partial E\left[J\right]}{\partial D} = U' - c_H \Pr{}_3 - c_0 \Pr{}_4, \tag{8}$$

[3] We will use this in the following for other derivatives as well.

Since we have assumed Inada conditions for $U(D)$, optimal marginal utility in the optimum is given by

$$U^{'} \quad = \quad c_H \Pr{}_3 + c_0 \Pr{}_4. \tag{9}$$

Load is optimal if marginal utility $U'$ is equal to the weighted costs of producing such a marginal unit, which is either $c_H$ (in the highly-dispatched events) or $c_0$ (in the events of unserved load). We further obtain the derivative

$$\frac{\partial E[J]}{\partial x_M} \quad = \quad c_H \Pr{}_3 + c_0 \Pr{}_4 - c_M. \tag{10}$$

Depending on the sign of the derivative, either $x_M < k_M$ or $x_M = k_M$ in the optimum. Suppose that $x_M < k_M$ is optimal. Then $\frac{\partial E[J]}{\partial k_M} = -b_M < 0$. Installing medium-dispatchables would never be beneficial so that $x_M = k_M = 0$, a contradiction to $x_M < k_M$. Consequently, $x_M = k_M$. The result is intuitive. Excess capacity of medium-dispatchables has no benefits, but is associated with unnecessary capacity costs. If there are medium-dispatchables at all, they will always be employed at full capacity.

Finally, turn to the optimal capacities (Stage 1). By interchanging expectation and differentiation again, we obtain

$$\frac{\partial E[J]}{\partial k_N} \quad = \quad c_H \bar{a}_3 \Pr{}_3 + c_0 \bar{a}_4 \Pr{}_4 - b_N - c_N a, \tag{11}$$

$$\frac{\partial E[J]}{\partial k_M} \quad = \quad c_H \Pr{}_3 + c_0 \Pr{}_4 - c_M - b_M, \tag{12}$$

$$\frac{\partial E[J]}{\partial k_H} \quad = \quad -c_H \Pr{}_4 + c_0 \Pr{}_4 - b_H. \tag{13}$$

Here, the definition of $\bar{a}_3, \bar{a}_4$ depends on the case. In the case of independence (*ind*), they are equal to the availability factor, $\bar{a}_3 = \bar{a}_4 = a$, while in the case of perfect correlation (*corr*), they are equal to average conditional production, $\bar{a}_3 = a_3$ and $\bar{a}_4 = a_4$.

Now, it is crucial to observe that (for both *ind* and *corr*), the derivatives for non-dispatchables (Equation (11)) and medium-dispatchables (Equation (12)) cannot become zero at the same time. The only exception is the boundary case, where the difference between medium- and non-dispatchable LRMC becomes $\Delta C = \Phi$, with

$$\Phi := \frac{a - \bar{a}_3}{\bar{a}_3} \left( \frac{b_N}{a} + c_N \right) + \frac{\bar{a}_3 - \bar{a}_4}{\bar{a}_3} c_0 \Pr_4. \tag{14}$$

The parameter $\Phi$ is the difference in LRMC for which the first-order conditions for $k_M$ and $k_N$ hold simultaneously. For *ind*, note that $\Phi = 0$. For *corr*, $\Phi = \frac{a - a_3}{a_3} \left( \frac{b_N}{a} + c_N \right) + \frac{a_3 - a_4}{a_3} c_0 \Pr_4$. If $\Delta C \neq \Phi$, only one of the first-order conditions can hold. This leads to:

**Proposition 1.** *If $\Delta C \neq \Phi$, then $k_N \cdot k_M = 0$ in the optimum.*

The Proposition establishes that non- and medium-dispatchables exclude each other in the optimum, except for a certain costs constellation $\Delta C = \Phi$, which must be considered as an unlikely boundary case. If there is an internal solution to the first-order condition for some $k_N > 0$, then the first-order condition for $k_M$ will never hold, so that both capacities cannot be positive. The same holds if there is an internal solution to the first-order condition for some $k_M > 0$.

The next step is to determine whether a capacity decision with $k_M > k_N = 0$ or with $k_N > k_M = 0$ is optimal. One might be tempted to take $\Delta C \gtrless \Phi$ as a criterion. The analysis is yet complicated by the fact that the above derivation was based on the assumption that $k_N > 0$. For $k_N = 0$, however, the random event collapses: we have $\tilde{x}_N = 0$ with certainty. The maximand (Equation (1)) has a discontinuity at the boundary. Consequently, we need to scrutinize the two cases for possible capacity mixes separately, and then compare them by levels.

So we now need to characterize the case without any non-dispatchables, $k_M > k_N = 0$. The analysis simplifies considerably. We denote results for this case with the superscript $m$. Since load is fixed and unserved load is more costly than producing either with medium- or highly-dispatchable capacity, load is always served, $x_M^m + x_H^m = D^m$ (this yields $x_0 = 0$ and $\text{Pr}_4 = 0$). As the LRMC of medium-dispatchables are lower than the LRMC of highly-dispatchables, $b_M + c_M < b_H + c_H$, highly-dispatchables are not installed (i.e., $k_H = 0$ and $\text{Pr}_3 = 0$). Moreover, excess capacity has no benefits, so that medium-dispatchables must produce with full capacity, $x_M^m = k_M^m = D^m$. The maximand simplifies to $J^m := U(D^m) - (b_M + c_M) D^m$. Thus, optimal load $D^m$ is characterized by $U'^m = b_M + c_M$.

Now consider that case without any medium-dispatchables, $k_N > k_M = 0$. Where convenient, we denote results with the superscript $n$ in this case. The optimal probability of unserved load ($\Omega_4$) and highly-dispatched ($\Omega_3$) can be derived from setting the Equations (13) and (11) to zero, yielding

$$\text{Pr}_4 = \frac{b_H}{c_0 - c_H} \in (0,1), \tag{15}$$

$$\text{Pr}_3 = \frac{b_N + ac_N - \bar{a}_4 c_0 \text{Pr}_4}{\bar{a}_3 c_H}. \tag{16}$$

Positivity of Equation (15) implies that the condition of Equation (13) always yields $k_H > 0$. For *ind*, Equation (16) can be expressed as $\text{Pr}_3 = \frac{b_N/a + c_N - b_H}{c_H} - \text{Pr}_4$, and for *corr* as $\text{Pr}_3 = \frac{b_N + ac_N}{a_3 c_H} - \frac{a_4 c_0}{a_3 c_H} \text{Pr}_4$. Intuitively, as soon as $k_N > 0$, a positive probability of unserved load must be accepted. Highly-dispatchables moderate this as back-up capacity. If costs of unserved load are high in comparison to costs of highly-dispatchables, highly-dispatchables should prevent unserved load more frequently. If costs of unserved load $c_0$ are close to $b_H + c_H$, then unserved load is accepted more frequently in the optimum. Using Equations (15) and (16) in Equation (9) yields load $D^n$, characterized by marginal utility $U'^n = \frac{b_N}{a} + c_N + \Phi$. Marginal utility must be equal to the LRMC of non-dispatchables plus a *correlation mark-up* $\Phi$ (which vanishes for *ind*). Again, the maximand can be simplified by exploiting $k_M = 0$ and the vanishing first-order conditions of

Equations (4) and (13) to obtain $J^n := U(D^n) - U'^n D^n - \gamma k_N$, where

$$\gamma \quad := \quad \begin{cases} (a - a_3)\left(\frac{b_N}{a} + c_N\right) + (a_3 - a_4)\, c_0 \Pr_4 & \text{for } ind, \\ 0 & \text{for } corr. \end{cases} \tag{17}$$

We can now turn to the direct comparison of the case without any non-dispatchables and without any medium-dispatchables. By denoting $\Delta U := U^n - U^m$ and $\Delta D := D^n - D^m$, it can be verified that $J^n = J^m$ if $\Delta C = \Psi$, where

$$\Psi := \Phi + \frac{U'^n \Delta D - \Delta U + \gamma k_N}{D_m}, \tag{18}$$

and $U'^n = \frac{b_N}{a} + c_N + \Phi$. The fraction in Equation (18) can be considered as an extended mark-up that comes on top of $\Phi$. Based on these considerations, we now obtain the final result for the optimal decision.

**Proposition 2.** *Suppose that $\Delta C \neq \Phi$ and that production of generating units is either perfectly correlated or stochastically independent. If $\Delta C < \Psi$, then $x_M = k_M = D$, $k_N, k_H = 0$ with $U' = b_M + c_M$ and $\Pr_3 = \Pr_4 = 0$. If $\Delta C > \Psi$, then $k_M = 0$, $k_N, k_H > 0$ with $U' = \frac{b_N}{a} + c_N + \Phi$ and $\Pr_4 = \frac{b_H}{c_0 - c_H}$, $\Pr_3 = \frac{b_N + ac_N - \bar{a}_4 c_0 \Pr_4}{\bar{a}_3}$.*

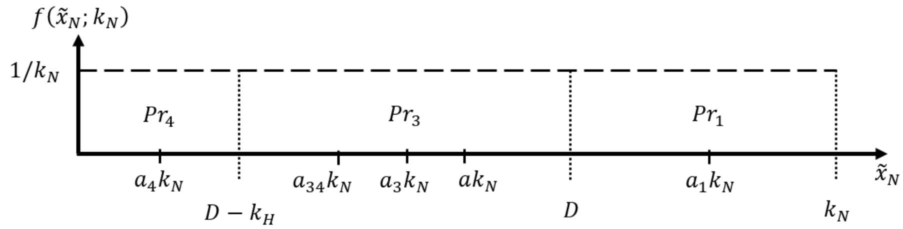
Thus, for $\Delta C < \Psi$, installing only medium-dispatchables is optimal, while for $\Delta C > \Psi$, it is optimal to install no medium-dispatchables at all. As soon as marginal generating units are neither perfectly correlated nor independent, the task to determine which capacity mix is optimal becomes more complex. In the case where $k_N > 0$, the optimal capacities $k_N, k_H$ are implicitly defined by Equations (15) and (16). For *ind*, we have $\Phi = 0$ so that $\Psi = \frac{(b_N/a + c_N)\Delta D - \Delta U + \gamma k_N}{D_m}$. Thus, if $\Delta C > \Psi$, then $U' = \frac{b_N}{a} + c_N$ and $\Pr_{34} = \Pr_3 + \Pr_4 = \frac{b_N/a + c_N - b_H}{c_H}$. For *corr*, we obtain $\Psi = \Phi + \frac{(b_N/a + c_N + \Phi)\Delta D - \Delta U}{D_m}$ and if $\Delta C > \Psi$, then $U' = \frac{b_N}{a} + c_N + \Phi$ and $\Pr_{34} = \frac{b_N + ac_N}{a_3 c_H} - \frac{b_H}{a_3 c_H}\frac{a_4 c_0 - a_3 c_H}{c_0 - c_H}$.

Suppose that $U^{'n} = U^{'m}$, then $\Delta D, \Delta U = 0$ and $\Delta C = \Phi$. For *corr*, we have $\Psi = \Phi$ and $J^n = J^m$ since $\gamma = 0$ (see Equation (17)). Thus, $\Delta C = \Psi$ be definition of $\Psi$.

It is illustrating to study the special case of perfect correlation where all marginal production units are uniformly distributed on $[0, 1]$, and where $k_M = 0$ (see Figure 2). Then, we have $a = \frac{1}{2}$ and some equations can be explicitly solved. The probability density function is plotted on the vertical axis (vertical dashed line). The vertical dotted lines separate the three possible events: unserved load ($\Omega_4$), highly-dispatched ($\Omega_3$) and non-dispatched ($\Omega_1$). For example, $a_4$ must be right in the middle of the respective area since the density is constant. The relation $a_4 < a_{34} < a$, $a_3$ holds for any proper distribution, but the relation between $a_3$ and $a$ is inconclusive. It can further be verified that $a_3 \Pr_3 + a_4 \Pr_4 = a_{34} \Pr_{34}$ holds. Using this in (14) and subsequently substituting Equations (16) and (15), we obtain

$$\Phi \quad = \quad \frac{a - a_{34}}{a_{34}}\left(\frac{b_N}{a} + c_N\right) + \frac{a_{34} - a_4}{a_{34}} b_H, \tag{19}$$

$$\Pr{}_{34} \quad = \quad \frac{a_3 \Pr_3 + a_4 \Pr_4}{a_{34}} = \frac{b_N + ac_N - a_4 b_H}{a_{34} c_H}, \tag{20}$$




| | $\Omega_4$ | $\Omega_3$ | $\Omega_{34}$ | $\Omega_1$ |
|---|---|---|---|---|
| $Pr_c$ | $(D - k_H)/k_N$ | $k_H/k_N$ | $D/k_N$ | $1 - D/k_N$ |
| $a_c$ | $0.5\,(D - k_H)/k_N$ | $0.5\,(2D - k_H)/k_N$ | $0.5\,D/k_N$ | $0.5\,(k_N - D)/k_N$ |

Fig. 2: Illustration of probabilities for a uniform distribution and *corr*

for the case of perfect correlations and a uniform distribution. Thus, the correlation markup $\Phi$ is strictly positive since $a_4 < a_{34} < a$ as argued before. However, without the assumption of a uniform distribution we might have $a_3 > a$, so that also $\Phi < 0$ is possible .

## 4 Comparative Statics

In this Section, we derive the comparative statics of capacity and load decisions. In the simple case where only medium-dispatchables are installed, we have $k_M = D$. Only the parameters $b_M, c_M$ are relevant. Marginal utility is equal to LRMC of medium-dispatchables, so, load decreases in $b_M, c_M$ and also $\frac{\partial k_M}{\partial b_M}, \frac{\partial k_M}{\partial c_M} < 0$.

What can be said about the propensity that the case without non-dispatchables occurs? Recall from Section 3 that for $\Delta C < \Psi$, installing only medium-dispatchables is optimal, while for $\Delta C > \Psi$, it is optimal to install no medium-dispatchables at all. If $b_M, c_M$ are lower, $\Delta C$ is lower as well and it is more likely that only medium-dispatchables are employed. If $b_N, c_N$ are lower, $\Delta C > \Psi$ is more likely, so that no medium-dispatchables are installed. The costs $b_H, c_H, c_0$ only influence $\Psi$. Intuitively, absence of medium-dispatchables is more likely if highly-dispatchables and unserved load are less expensive, since these parameters are all cost components in a system with non-dispatchables.

Now turn the comparative statics of the case where non-dispatchables are installed. It is helpful to determine how capacities and load affect the probabilities of different events (see Appendix for the proof).

**Lemma 1.** *Suppose there is non-dispatchable capacity in the optimum. Then, the comparative statics of optimal probabilities* $\mathrm{Pr}_c$ *for both the case of independence and perfect correlation are summarized by the following table.*

| | $k_N$ | $k_H$ | $D$ |
|---|---|---|---|
| $\Pr_4$ | $(-)$ | $(-)$ | $(+)$ |
| $\Pr_{34}$ | $(-)$ | $0$ | $(+)$ |
| $\Pr_3$ | $\#^1$ | $(+)$ | $\#^2$ |
| $\#^1$ | for $corr$: $(-)$ iff $f(D;k_N)\,D > f(D-k_H;k_N)\,(D-k_H)$ | | |
| $\#^2$ | $(-)$ iff $f(D;k_N) < f(D-k_H;k_N)$ | | |

Higher $k_N$ increases production of non-dispatchables so that it is less likely that unserved load ($\Omega_4$) occurs, and similarly for $k_H$. Higher load raises $\Pr_4$ since it becomes more likely that non- and highly-dispatchables do not produce enough together. The event of either unserved load or highly-dispatched ($\Pr_{34} = \Pr_3 + \Pr_4$) becomes more probable if $k_N$ decreases and $D$ increases by the same argument as before. Interestingly, the probability of $\Omega_{34}$ is unaffected by $k_H$. While highly-dispatchable capacity decreases $\Pr_4$, it increases $\Pr_3$ by the same amount. If $D$ rises, the effect on $\Pr_3$ is ambiguous. While, at the margin, rising demand increases the likelihood that additional highly-dispatchables are employed (thus increasing the probability), it also increases the likelihood of unserved load (thus decreasing the probability). It cannot be said without further assumptions which effect is larger. For $\frac{\partial \Pr_3}{\partial k_N}$, the situation is more complicated, since the capacity of non-dispatchables influences the probability distribution of $\tilde{x}_N$. We obtain a closed-form criterion for the case of perfect correlation if we additionally assume that the random variables $\omega(z)$ are uniformly distributed, so that $\frac{\partial \Pr_3}{\partial k_N} > 0$ and $\frac{\partial \Pr_3}{\partial D} = 0$ (see Figure 2).

Now turn to the comparative statics of $k_N, k_H, D$, which are summarized by the following Propositions (see Appendices B and C for the proofs).

**Proposition 3.** *In the case of independence, the comparative statics for capacities and load are summarized by the following table:*

| | $b_N, c_N$ | $b_H, c_H$ | $c_0$ |
|---|---|---|---|
| $k_N$ | $(-)$ | $(+)$ | $0$ |
| $k_H$ | $\#^3$ | $(-)$ | $(+)$ |
| $D$ | $(-)$ | $0$ | $0$ |
| $\#^3$ | $(-)$ iff $-U''/c_H < \frac{\partial \Pr_{34}}{\partial k_N} / \frac{\partial \Pr_4}{\partial k_N} \cdot f(D-k_H;k_N) - f(D;k_N)$ | | |

For the case of perfect correlation unambiguous results are more difficult to obtain. Here, we concentrate on an interesting special case.

**Proposition 4.** *In the case of perfect correlation, if all $\omega(z)$ are uniformly distributed, then the comparative statics for capacities and load are summarized by the following table:*

| | $b_N, c_N$ | $b_H, c_H$ | $c_0$ |
|---|---|---|---|
| $k_N$ | $(-)$ | $\#^4, \#^5$ | $(-)$ |
| $k_H$ | $\#^4$ | $(-)$ | $\#^4$ |
| $D$ | $(-)$ | $(-)$ | $(-)$ |
| $\#^4$ | $(-)$ iff $-U''/c_H < k_H/k_N\,(D-k_H)$ | | |
| $\#^5$ | $(-)$ iff $-U''/c_H < \left(D^2 - (D-k_H)^2\right)/k_N\left(D^2 + (D-k_H)^2\right)$ | | |

First, consider that higher costs of non-dispatchables $b_N, c_N$ lead to less non-dispatchables capacity in the optimum in both Propositions. This is not as straightforward as it appears because changes in costs also affect the probabilities of the events. However, lower capacities are not offset by a possibly larger share of highly-dispatchables, as the probability of unserved load $\mathrm{Pr}_4$ is unaffected due to Equation (15). In contrast, Equations (15) and (16) indicate that $\mathrm{Pr}_{34}$ increases in $b_N, c_N$, but this is consistent with lower capacity of non-dispatchables (see Lemma 1). Higher costs of non-dispatchables also lead to lower demand in both Propositions because marginal utility is higher accordingly (this follows from Proposition 2 by considering that the mark-up $\Phi$ is not decreased).

The effect for highly-dispatchable capacity $k_H$ indirectly depends on how two effects are balanced: reduced demand lowers the probability of unserved load, while reduced non-dispatchable capacity increases this probability (the conditions are different in both Propositions). In both Propositions, highly-dispatchable capacity decreases if it becomes more costly. This holds although there are second-order effects. By Equation (15), also the probability of unserved load $\mathrm{Pr}_4$ rises, which is in line with reduced $k_H$ (Lemma 1). On the other hand, the probability of highly-dispatched ($\mathrm{Pr}_3$) decreases due to Equations (15) and (16).

Next, consider changes in $b_H, c_H, c_0$ in the case of independence (Proposition 3). Load is unaffected, since optimal marginal utility only depends on the costs of non-dispatchables (Proposition 2). The probability of unserved load decreases in $c_0$ due to Equation (15), which can only be due to higher $k_H$. The probability $\mathrm{Pr}_{34}$ is not influenced by $c_0$, so that also non-dispatchable capacity is constant. If costs of unserved load rise, this is completely compensated by installing more highly-dispatchables, but not by more non-dispatchables.

Finally, consider the case of perfect correlation (Proposition 4). Marginal utility increases in all costs parameters since they all raise the correlation mark-up $\Phi$. So, load must decrease if any technology becomes more expensive. If unserved load is more costly, it is optimal to install less non-dispatchable capacity to avoid unserved load. It might be intuitive to also install more highly-dispatchables, but this is not necessarily the case. Since higher $c_0$ also leads to less load, there is less capacity needed.

Whether non-dispatchables and medium-dispatchables are substitutes complements is ambiguous for most cost parameters (see $\#^3, \#^4, \#^5$ in the Tables in Propositions 3 and 4). This crucially depends on the elasticity of $U^{'}$. If $|U^{''}|$ is very small, then a negative sign is more likely so that both capacity types are complements. In contrast, if marginal utility is very steep, it is more likely that both capacity types are substitutes. To sum up, non-dispatchables and highly-dispatchables are substitutes with respect to $b_H, c_H$ in the case of independence. If other cost parameters change, or in the case of perfect correlation, non-dispatchables and highly-dispatchables can be either substitutes or complements.

## 5 Costs Recovery

It is important to know whether capacity decisions can be decentralized by a price signal. This is only possible if an optimal price and capacity mix leads to zero profits for all technologies. Producers need to recover their fix costs. If the zero profit condition is violated, there would be incentives to leave or enter the market, so that optimal capacities cannot be an equilibrium outcome. We show in this Section that for prices equal to marginal utility, some technologies cannot recover costs, and others may yield a strictly positive surplus. Consequently, an efficient market with non-dispatchable technologies cannot be designed in a conventional way.

This is different to the deterministic settings, where the zero profit condition holds in the optimum (see, e.g., Williamson, 1966; Alayo and García, 2015). It is known that this is generally not the case when demand or supply uncertainty enters the stage, with mixed results depending on the specification (see, e.g., Brown and Johnson, 1969; Visscher, 1973; Crew and Kleindorfer, 1976; Carlton, 1977). In contrast, Helm and Mier (2018) show for electricity markets with (non-dispatchable) renewables and perfectly dispatchable fossils, but without medium-dispatchable technologies, that dynamic pricing leads to zero profits and efficient capacity decisions are an equilibrium outcome. The question is open in the presence of dispatchability types.

Assume that the price is determined from the inverse demand function by $p = U'$. We treat the case of independence and of perfect correlation together as long as the results are identical. Consumers will only pay for own consumption and not for overproduction in the non-dispatched ($\Omega_1$) or the medium-dispatched events ($\Omega_2$). Denote the production from technology $j$ sold to consumers by $D_j$. The expected profits from technology $j$ are

$$E[\pi_j] \quad = \quad pE[D_j] - c_j E[x_j] - b_j k_j. \tag{21}$$

We know from Section 3 that random events do not affect medium-dispatchables ($D_M = x_M$) and that highly-dispatchables can always sell their scheduled production ($D_H = x_H$) at the price $p$. If there is non-dispatchable capacity, we also know that $x_M = 0$, and that $x_H > 0$ if only if $\tilde{x}_N < D$, so that $D_N = \min\{\tilde{x}_N, D\}$.

For medium-dispatchables, it is easy to see that the zero profit condition holds. Proposition 2 shows that if there is a positive capacity of medium-dispatchables, then $p = U' = b_M + c_M$ and $x_M = k_M$. All uncertainty is removed, so that $\pi_M = (p - c_M - b_M) k_M = 0$.

Now turn to highly-dispatchable capacity in the case where $k_N > 0$ and expectations need to be considered. Setting Equation (13) to zero and substituting $c_0 \Pr_4 = b_H + c_H \Pr_4$ into Equation (9) yields $p = U' = b_H + c_H \Pr_{34} < b_H + c_H$, where we have used that $\Pr_3 + \Pr_4 = \Pr_{34}$. Since highly-dispatchables produce only at full capacity in the event of unserved load ($\Omega_4$), we have

$$E[D_H] \quad = \quad E[x_H] = E[D_H|\Omega_3] \Pr{}_3 + k_H \Pr{}_4 < k_H, \tag{22}$$

$$E[\pi_H] \quad = \quad (p - c_H) E[D_H] - b_H k_H < (b_H + c_H - c_H) k_H - b_H k_H = 0. \tag{23}$$

Thus, highly-dispatchables will never recover costs. Finally, turn to non-dispatchable capacity and note that $p = U' = \frac{b_N}{a} + c_N + \Phi$ from Proposition 2. We obtain

$$E[D_N] = E[\tilde{x}_N|\Omega_{34}] \Pr{}_{34} + E[D|\Omega_1] \Pr{}_1 < E[\tilde{x}_N] = ak_N, \tag{24}$$

$$E[\pi_N] = -(b_N/a + c_N)(E[\tilde{x}_N] - E[D_N]) + \Phi E[D_N]. \tag{25}$$

The first (negative) term in Equation (24) represents the costs from producing more than can be sold. The second term represents the adjustments from the correlation mark-up $\Phi$. Thus, if $\Phi \leq 0$, e.g. if generating units are independently distributed (*ind*), non-dispatchables will never recover costs.

In the case of perfect correlation, a positive mark-up might be sufficient to cover the losses from producing more with non-dispatchables than can be sold. Yet, the zero profit condition is only satisfied in the boundary case where

$$\frac{\Phi}{b_N/a + c_N} = \frac{E[\tilde{x}_N] - E[D_N]}{E[D_N]}, \tag{26}$$

that is, if the mark-up in relation to LRMC of non-dispatchables is exactly equal to the relative excess production. In the case of perfect correlation, if it is additionally assumed that marginal generation units are uniformly distributed, we obtain a stronger result (see Appendix D).

**Proposition 5.** *Medium-dispatchables exactly recover costs. Highly-dispatchables do not recover costs. Non-dispatchables do not recover costs in the case of independence or if* $\Phi$ *is not sufficiently large. For the case of perfect correlation and generation units being uniformly distributed, non-dispatchables make positive profits.*

## 6 Comparison and Extensions

One main finding so far is that non- and medium-dispatchable capacity exclude each other in the optimum if $\Delta C \neq \Phi$. This Section compares this finding against the results from standard peak-load pricing that disregards dispatchability types. We also study modifications of the model to further scrutinize which assumptions drive the results.

**Peak-load pricing without dispatchability types.** As recalled in the introduction, the established literature on peak-load pricing under uncertainty disregards production with different dispatchability types. It would yet be interesting to know how these results differ. Differences can then be attributed to the effect of medium-dispatchables being available.

We thus compare to another decision structure: In Stage 2, only load must be decided. Then, random production $\tilde{x}_N$ realizes in Stage 3. In Stage 4, medium- and highly-dispatchable production and possibly unserved load follows. The results of Chao (1983), adopted to this paper's notation, imply for Stage 4 that expected unserved load and expected production of highly-dispatchables are the same as in Equations (5) and (6), where expected medium-dispatchable production is given by

$$E[x_M] = k_M \Pr{}_{34} + E[D - \tilde{x}_N|\Omega_2] \Pr{}_2. \tag{27}$$

In Stages 1 and 2, we obtain

$$-\frac{\partial E[J]}{\partial D} = c_0 \Pr{}_4 + c_H \Pr{}_3 + c_M \Pr{}_2 - U', \tag{28}$$

$$\frac{\partial E[J]}{\partial k_H} = c_0 \Pr{}_4 - c_H \Pr{}_4 - b_H, \tag{29}$$

$$\frac{\partial E[J]}{\partial k_M} = c_0 \Pr{}_4 + c_H \Pr{}_3 - c_M \Pr{}_{34} - b_M, \tag{30}$$

$$\frac{\partial E[J]}{\partial k_N} = c_0 \overline{a}_4 \Pr{}_4 + c_H \overline{a}_3 \Pr{}_3 + c_M \overline{a}_2 \Pr{}_2 - c_N a - b_N. \tag{31}$$

In contrast to our main results, this program can be solved for $D, k_N, k_M, k_H > 0$ to obtain $U' = b_M + c_M \Pr_{234}$ with

$$\Pr{}_4 = \frac{b_H}{c_0 - c_H}, \tag{32}$$

$$\Pr{}_{34} = \frac{b_M - b_H}{c_H - c_M}, \tag{33}$$

$$\Pr{}_{234} = \frac{b_N + c_N a}{c_M \overline{a}_2} - \frac{c_H \overline{a}_3 - c_M \overline{a}_2}{c_M \overline{a}_2 (c_H - c_M)} b_M - \frac{(c_H - c_M)(c_0 \overline{a}_4 - c_M \overline{a}_2) - (c_H \overline{a}_3 - c_M \overline{a}_2)(c_0 - c_M)}{c_M \overline{a}_2 (c_H - c_M)(c_0 - c_H)} b_H. \tag{34}$$

In the case of independence, we can use $\overline{a}_c = a$ and simplify to $\Pr_{234} = \frac{b_N/a + c_N - b_M}{c_M}$, a standard result in peak-load pricing. The more complicated case of perfect correlation is, to our knowledge, not considered in the literature so far. In the case of independence, it is also well-known that the equation system has a positive solution for all probabilities if $0 < \frac{b_H}{c_0 - c_H} < \frac{b_M - b_H}{c_H - c_M} < \frac{b_N/a + c_N - b_M}{c_M} < 1$. This condition guarantees that all technologies are employed in the optimum. Importantly, this condition is not a boundary case like $\Delta C = \Phi$, but holds for a whole range of cost parameters. So, the consideration of medium-dispatchables leads to a less diverse mix of production technologies.

Note that one could expand our three technology case easily to a more diverse technology model. However, the insights gained from such an analysis are limited.

**Non-dispatchability.** So far, we assumed non-dispatchability in a rather strict way. This can cause additional costs if there is excess production above load. We now alleviate this assumption by allowing that in Stage 3 non-dispatchable production can be reduced below $\tilde{x}_N$ without costs. The remaining decision structure is as in our original model. The Proposition yet confirms that strict non-dispatchability is not the driving factor of our results (for a proof see Appendix E):

**Proposition 6.** *If production $x_N$ can be reduced below $\tilde{x}_N$, a strictly positive capacity of both medium- and non-dispatchables can only be optimal for a specific value for $\Delta C$.*

**Multiple periods.** The incompatibility of medium- and non-dispatchables in the model (possibly except for boundary cases) is a provocative result. Might this be a consequence of considering a one-period setup with constant load and time-independent expected production of non-dispatchables?

In the following, we thus extend the analysis to the multi-period case.

In the one-period setup, optimal medium-dispatchable production is always at full capacity. As is common in the established theory of peak load pricing, this changes in a multi-period setup: while medium-dispatchables might still produce at full capacity during periods of peak load, production will be below capacity in some periods with lower utility. If different types of dispatchability enter the scene, we might thus expect that there are periods where only medium-dispatchables are employed, and other periods where only non- and highly dispatchables are employed, that is, there is a technology mix.

We consider multiple periods $t$, where $T$ is the number of periods. Utility functions $U_t$ differ by time period so that there are times with higher or lower demand. After capacities are decided upon in Stage 1 at a fixed level for all periods, each period admits a new decision in Stages 2 and 4 (Stage 3 is random as before). In period $t$, production by technology $j$ is denoted by $x_{jt}$ and load $D_t$. Then, the following Proposition shows that non- and medium-dispatchables still do not coexist (for a proof see Appendix F).

**Proposition 7.** *In the case of multiple periods, a strictly positive capacity of both medium- and non-dispatchables can only be optimal for a boundary case with a specific relation of cost parameters.*

This result is due to each time period following the same rationale as in the one-period setup where, basically, medium-dispatchables are a perfect substitute for the composite of non-dispatchables and highly-dispatchables. As the objective function is additive separable in the $T$ time periods, medium-dispatchables being cheaper in one period extends to all periods – and vice versa for the composite. Consequently, medium- and non-dispatchables are also incompatible in the optimum with multiple periods.

The result can also be extended to multiple periods with different expected production of non-dispatchables, i.e. $E\left[\tilde{x}_{Nt}\right] = a_t k_N$ in period $t$, and conditional production $E\left[\tilde{x}_{Nt}|\Omega_{ct}\right] = a_{ct} k_N$. Furthermore, we can combine multiple periods with downward-dispatchability as in Proposition 6. Results do not differ qualitatively.

## 7 Application to Electricity Systems

This Sections applies the results to the case of integrating renewables into electricity systems and arrives at important policy implications for electricity market design that strongly differ from the implications of the standard peak-load pricing model.

World-wide, the share of wind, solar and other kinds of renewable energy supply in electricity production is rising (REN21, 2018). Capacity costs of renewables are anticipated to further falling in the future (Schröder et al., 2013; IRENA, 2016). This trend is primarily driven by the need to reduce the production of conventional power plants which emit greenhouse gases. The integration of a large share of renewables into the electricity system poses a major challenge for a transition to a low-carbon economy. Generation of crucial renewable technologies like wind generators or photovoltaic (PV) power stations can be highly fluctuating due to weather conditions in many parts of the world, and electricity is difficult to store (with current technologies, hydropower at appropriate geographical conditions being the exception; see, e.g., Luo et al. 2015; Gimeno-Gutiérrez and Lacal-Arántegui 2015; Sinn 2017). Thus, an increasing share of fluctuating renewables can cause additional electricity system costs (Lamont, 2008; Hirth, 2013; Reichelstein and Sahoo, 2015). When the share becomes significant, this might open a complete new chapter for how to organize the electricity system.

The general challenges for a new electricity system are well-recognized in present policy making (e.g., the EU winter package, see Hancher and Winters (2017) for a summary). There is a heated debate both in practice and in academia on the future electricity market design (Joskow and Tirole, 2007; Newbery, 2010; Hiroux and Saguan, 2010; Fabra et al., 2011; Cramton et al., 2013; Henriot and Glachant, 2013), for example whether to charge prices on energy or power (Ito, 2014; Borenstein, 2016), intraday market design (Borggrefe and Neuhoff, 2011) or how to auction balancing power (Hortaçsu and Puller, 2008; Müsgens et al., 2014). Another debate centers around the possibility of a missing money problem for remaining conventional power plants, that is, whether they can recover their fixed costs in the future (Joskow, 2008; Newbery, 2016).

The case of renewable integration can be matched to the assumptions of our model as follows. Renewables like wind turbines and PV power stations are non-dispatchables as they partially face random availability within their installed capacity. For convenience, we call those technologies simply 'renewables' subsequently. Technologies like gas turbines, spinning reserves or pump storage power plants are highly-dispatchables because they can (almost) instantly adjust to short-term fluctuations of other power plants. Pars pro toto, we call them 'gas turbines' in the following. Note that there are both fossil and renewables highly-dispatchables. Steam power from large scale coal or nuclear power plants qualify as medium-dispatchables. They usually require scheduling from several hours to days ahead of actual production and suffer from the unit commitment problem. They have minimum ramp-up or down times to keep them functioning and smaller short-term adjustments of production are associated with additional abrasion costs (Wang and Shahidehpour, 1995; Kumar et al., 2012; Van den Bergh and Delarue, 2015; Schill et al., 2017; Göransson et al., 2017). Thus, if renewables feed-in is lower than expected, steam power cannot balance much of this deviation, so that highly-dispatchables like gas turbines are required. The cost assumptions from our model fit real costs of steam power, renewables, and gas turbines. Steam power comes with high capacity and low production costs, where gas turbines come with low capacity and high production costs. Renewables have nearly zero production costs, but capacity costs are comparatively high. Costs of steam power are lower than those of gas turbines (IEA, 2015).

With these assumptions, the standard model (Chao, 1983; Kleindorfer and Fernando, 1993) that disregards dispatchability types can describe historical electricity production before renewables became competitive quite well. As outlined in the introduction, this model can accommodate fluctuating renewables, but is not able to represent how well other technologies are able to balance such fluctuations. The standard model suggests a mix of different technologies to serve different load levels throughout the year. Base-load was mostly served by steam power before renewables became subsidized. These power plants produced most of the time with full capacity. They are complemented by peak-load power plants like gas turbines. Under peak-load conditions, the latter are price-setting. These prices are, in the absence of uncertainty, sufficient to recover capacity costs of base-load power plants (theoretically, the zero-profit condition is fulfilled). With renewables becoming competitive, the standard model would place them before steam power in the merit order, even in the presence of random fluctuations. Steam power would then be used less frequently, but would partially remain in the electricity system. The implications for the optimal capacity of gas turbines are ambiguous.

The implications for the optimal capacity mix change fundamentally in our model. This also has policy implications for the probability of unserved load, prices, and cost recovery, as discussed in the following.

First, renewables and steam power exclude each other in the optimal capacity mix. The incompatibility is solely driven by the fact that steam power cannot react on random renewable production. Exceptions of this result are only possible for very unlikely boundary cases. This result holds for both the single and the multiple period case, and is robust if renewables can at least be dispatched downwards. The main reason is that the optimal composite of renewables and gas turbines is a perfect substitute for steam power. The model derives a condition for the long-run marginal costs of steam power and renewables where it is optimal to switch from a purely conventional to a purely renewable system with gas turbines.

Second, the probability of unserved load is independent from the costs of renewables, that is, renewables does not lead to more frequent outages. We find that unserved load is only more frequent if gas turbines are more costly or if the costs of unserved load are lower. If the latter costs are reduced, less gas turbine capacity is installed. This would have implications for demand response programs that reduce costs of unserved load. Of course, in comparison to a system with steam power only, there is more unserved load, but the root of this effect is a fundamentally different electricity system and not marginal changes in costs of renewables. In an electricity system with renewables, lower costs lead to higher load and less gas turbine capacity, being exactly compensated by additional renewable capacity.

Third, in the presence of renewables, electricity prices are not set by highly-dispatchable peak-load power plants. Instead, we find that if the price is set to marginal utility, it is equal to the long-run marginal costs of renewables, plus a mark-up if renewable generators do not produce stochastically independent.

Fourth, there is no missing-money problem for steam power, but capacity costs cannot be recovered for gas turbines and likely not for renewables. This is due to the price being independent from the costs of gas turbines and a system without steam power requiring excess renewable capacity. Only for special cases where renewable production is correlated, the resulting mark-up can lead to additional revenues to recover the costs of renewables. However, even in this case, the zero-profit condition would only hold in an unlikely boundary case.

Missing cost recovery has considerable implications for the electricity market design in the presence of a large share of renewables. If optimal capacities in light of the technical constraints of limited dispatchability do violate the zero-profit condition, marginal utility power prices cannot decentralize the optimum. To avoid gas turbines and renewable capacity leaving the market (or to avoid excessive market entry in the case of positive profits), subsidies are needed to achieve cost recovery. Such subsidies would, however, distort the partial equilibrium. We thus need to relinquish the idea of a simple price mechanism for power that works in the presence of an electricity system dominated by renewables. Finding an appropriate market design in light of this problems requires future research.

These institutional implications might be relieved by considering other technological aspects. Our model abstracts from load uncertainty, which is also a relevant issue in electricity systems without renewables. This would yet not principally favor such a system compared to a system with renewables only. In addition, load predictions work well due to decades of experience. We also abstract from production uncertainties of steam power, but we would not expect fundamental changes to our implications either. Although it is already known for the standard model that uncertainties can in principle lead to problems with cost recovery of steam power, this effects seem to almost negligible in terms of magnitude. Relieves might be expected, however, if the dispatchability of steam power is technically improved so that it is able to cheaply react to random fluctuation of renewables. Then they become highly-dispatchable and the result that they cannot coexist with renewables in the optimum breaks down. Alternatively, we can consider improvements in the predictability of renewables (see Iversen et al., 2016). Then, the problems from limited dispatchability of some technologies might vanish in practice (see Gowrisankaran et al., 2016).

## 8 Concluding Remarks

The analysis shows that considering different types of dispatchability changes the established analysis of welfare maximizing capacity decisions. This provides a more differentiated picture on when an increasing share of non-dispatchable capacity needs to be complemented by more highly-dispatchable back-up capacity to prevent unserved load, and informs deliberations about market designs. It is shown that non-dispatchables, once they become competitive, completely replace medium-dispatchable capacity so that a system with medium-dispatchables only is directly redeemed by a non-dispatchable dominated system, supplemented with highly-dispatchable capacity. Moreover, a rising capacity of non-dispatchables does not lead to a higher probability of unserved load. Unserved load occurs more often only if highly-dispatchable capacity becomes more expensive, or if unserved load becomes cheaper. Importantly, in the presence of competitive non-dispatchables, optimal capacity cannot be decentralized with prices equal to marginal utility. For example, this has crucial policy implications for the integration of a large share of renewables in an electricity system.

As for every model, the conclusions come with some caveats. First, we consider only three capacity types with constant marginal production and capacity costs. In general there are more capacity types but our analysis focuses on a discrete conceptualization of dispatchability. It solely matters whether a capacity type needs to be decided before random non-dispatchable production realizes, or afterwards. There might also be differences in degree. For example medium-dispatchable technologies might not be as inflexible as suggested, e.g., being more flexible at higher costs. The reverse might hold for highly-dispatchables. There might be some bundling effects between both capacity types. Applied to electricity systems, steam power can be adjusted to some degree. Nuclear power plants are restricted more, combined-cycle gas turbines or modern steam power plants with coal vaporization less. Nevertheless, a total shut-down of production is not possible since they cannot restart immediately. Considering linear marginal generation costs could enable non- and medium-dispatchables to coexists for a range of cost parameters, but this range is likely smaller than the standard models without dispatchability types suggest. Second, we focus on the cases where marginal generating units are identically distributed, either stochastically independent or perfectly correlated. Thus, there are no locational advantages from arranging generating units in space. Accounting for locational advantages and imperfect correlation between generators might lead to bundles of generators at the best locations. Those important extensions can likely only be studied with simulation methods that can build on and qualify our results. Third, the model disregards load uncertainty. Accounting for load uncertainty increases the need to balance deviations by highly-dispatchables also in systems with medium-dispatchables. For systems dominated by non-dispatchables, highly-dispatchable capacities could be used to balance both deviations from load and from non-dispatchable production. Such synergies might improve the value of non-dispatchables in comparison to medium-dispatchables. However, the size of this effect crucially depends on the correlation of load and production uncertainties.

The established literature on peak-load pricing under uncertainty disregards production with different dispatchability types. Our paper shows that giving consideration to the flexibility of production, in addition to reliability, fundamentally alters the optimal solutions to peak-load pricing problems. These results open avenues for further research, for example on the institutional design of future electricity markets.

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# Appendix

## A Proof of Lemma 1

We write the probability density function of the random variable $\tilde{x}_N$ as $f = f(\tilde{x}_N; k_N)$. We only need to consider the situation with $k_M = 0$. Then, $\Pr_4 = \int_0^{D-k_H} f(\tilde{x}_N; k_N)\, d\tilde{x}_N$, $\Pr_3 = \int_{D-k_H}^{D} f(\tilde{x}_N; k_N)\, d\tilde{x}_N$, and $\Pr_{34} = \int_0^{D} f(\tilde{x}_N; k_N)\, d\tilde{x}_N$. By the Leibniz rule, we obtain the signs of the derivatives of $\Pr_3, \Pr_{34}, \Pr_4$ w.r.t. $k_H, D$ as given in Table 1, as $f$ is independent from $k_H, D$.

For $\frac{\partial \Pr_{34}}{\partial k_N}, \frac{\partial \Pr_4}{\partial k_N}$ consider that they are determined over intervals bounded below by zero. Since $\tilde{x}_N \geq 0$ by definition and almost sure $\forall z : \omega(z) > 0$, both probabilities become lower if $k_N$ rises, i.e., $\frac{\partial \Pr_4}{\partial k_N} = \int_0^{D-k_H} \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N < 0$ and $\frac{\partial \Pr_{34}}{\partial k_N} = \int_0^{D} \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N < 0$. This argument does not apply for $\frac{\partial \Pr_3}{\partial k_N}$, since $\frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N}$ could be higher or lower at $D$ or $D - k_H$, i.e., $\frac{\partial \Pr_3}{\partial k_N} = \int_{D-k_H}^{D} \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N \gtrless 0$.

For the case of perfect correlation *corr*, a result can be obtained. Since we can write $\tilde{x}_N = \omega k_N$ for one representative $z$, we can transform the random variable. Denote the probability density function of $\omega(z)$ by $f_\omega$, which is independent from $k_N$. Then, $f(\tilde{x}_N; k_N) = f_\omega(\omega) \cdot \frac{1}{k_N}$. With the variable transformation we can write

$$\begin{aligned}
\Pr_3 &= \int_{D-k_H}^{D} f(\tilde{x}_N; k_N)\, d\tilde{x}_N \\
&= \int_{(D-k_H)/k_N}^{D/k_N} f_\omega(\omega)\, d\omega, \\
\frac{\partial \Pr_3}{\partial k_N} &= f_\omega(D/k_N)\left(-\frac{D}{k_N^2}\right) - f_\omega((D-k_H)/k_N)\left(-\frac{D-k_H}{k_N^2}\right) \\
&= f(D-k_H, k_N)\, k_N \frac{D-k_H}{k_N^2} - f(D, k_N)\, k_N \frac{D}{k_N^2}.
\end{aligned}$$

This expression is positive if and only if $f(D-k_H, k_N)(D-k_H) > f(D, k_N) D$. Thus, every component of the Table in Lemma 1 has been shown.

| | $\partial/\partial k_H$ | $\partial/\partial D$ |
|---|---|---|
| $\Pr_{34}$ | $0$ | $f(D; k_N) > 0$ |
| $\Pr_4$ | $-f(D-k_H; k_N) < 0$ | $f(D-k_H; k_N) > 0$ |
| $\Pr_3$ | $f(D-k_H; k_N) > 0$ | $f(D; k_N) - f(D-k_H; k_N)$ |
| | $\partial/\partial k_N$, for *ind* | $\partial/\partial k_N$, for *corr* |
| $\Pr_{34}$ | $\int_0^D \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N < 0$ | $-\frac{f(D;k_N)D}{k_N} < 0$ |
| $\Pr_4$ | $\int_0^{D-k_H} \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N < 0$ | $-\frac{f(D-k_H;k_N)(D-k_H)}{k_N} < 0$ |
| $\Pr_3$ | not possible to show | $-\frac{f(D;k_N)D - f(D-k_H;k_N)(D-k_H)}{k_N}$ |

Tab. 1: Partial derivatives of probabilities w.r.t. $k_N, k_H, D$

## B Proof of Proposition 3

The comparative statics can be derived from first-order conditions in Equations (8), (11), and (13). The total differential of these three conditions with respect to the dependent variables $(k_N, k_H, D)$ and one parameter of interest (here: one of $b_N, c_N, b_H, c_H, c_0$), principally yields an equation system. This needs to be solved to obtain the comparative statics of $k_N, k_H, D$ with respect to the parameter.

Before solving these equation systems, we note that the first-order conditions can be equivalently written in the case of independence as

$$F_H := (c_0 - c_H)\Pr_4 - b_H = 0, \tag{35}$$

$$F_D := \frac{b_N}{a} + c_N - U' = 0, \tag{36}$$

$$F_N := b_H + c_H \Pr_{34} - \frac{b_N}{a} - c_N = 0, \tag{37}$$

by considering the following: Equation (35) just rewrites Equation (13). Equation (36) is obtained by solving Equation (35) for $c_0 \Pr_4$, substituting into Equation (11), and using that $\Pr_3 + \Pr_4 =$

$\mathrm{Pr}_{34}$. Also Equation (37) is obtained directly from Equation (11), substituting $c_0\,\mathrm{Pr}_4$, and simplifying as before. Note that $\mathrm{Pr}_3$ does not show up here, thus, the gap in determining $\frac{\partial\,\mathrm{Pr}_3}{\partial k_N}$ (see Appendix A) is not relevant here. The partial derivatives can then be summarized in Table 2, where $\frac{\partial F_N}{\partial k_H} = 0$ is implied by Table 1 from Appendix A.

| | dependent variables | | |
|---|---|---|---|
| | $\partial/\partial k_N$ | $\partial/\partial k_H$ | $\partial/\partial D$ |
| $F_N$ | $c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}$ | $0$ | $c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial D}$ |
| $F_H$ | $(c_0 - c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial k_N}$ | $(c_0 - c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}$ | $(c_0 - c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial D}$ |
| $F_D$ | $0$ | $0$ | $-U''$ |

| | parameters | | | | |
|---|---|---|---|---|---|
| | $\partial/\partial b_N$ | $\partial/\partial c_N$ | $\partial/\partial b_H$ | $\partial/\partial c_H$ | $\partial/\partial c_0$ |
| $F_N$ | $-1/a$ | $-1$ | $1$ | $\mathrm{Pr}_{34}$ | $0$ |
| $F_H$ | $0$ | $0$ | $-1$ | $-\mathrm{Pr}_4$ | $\mathrm{Pr}_4$ |
| $F_D$ | $1/a$ | $1$ | $0$ | $0$ | $0$ |

Tab. 2: Partial derivatives of $F_N, F_H, F_D$.

This structure make the comparative statics of load $D$ easy to determine. Using implicit differentiation, we obtain $\frac{dD}{db_H} = -\frac{\partial F_D}{\partial b_H} / \frac{\partial F_D}{\partial D} = 0$, and in the same way yields $\frac{dD}{dc_H} = \frac{dD}{dc_0} = 0$, $\frac{dD}{db_N} = \frac{1}{aU''} < 0$, and $\frac{dD}{dc_N} = \frac{1}{U''} < 0$.

The total differential of Equation (37) becomes, for any parameter, a straightforward equation since $\frac{\partial F_N}{\partial k_H} = 0$. For $b_N$, we have

$$0 \quad = \quad c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N} dk_N + c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_H} dk_H + c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial D} dD - \frac{1}{a} db_N,$$

where $\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_H} = 0$ as shown by Table 1. Solving for $\frac{dk_N}{db_N}$ by using $\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}, \frac{\partial\,\mathrm{Pr}_{34}}{\partial D}$ as described by Table 1 and $\frac{dD}{db_N} = \frac{1}{aU''}$ yields

$$\frac{dk_N}{db_N} \quad = \quad \frac{1}{a} \frac{U'' - c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial D}}{U'' c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}} > 0. \tag{38}$$

We can repeat the same steps with the other parameters to obtain $\frac{dk_N}{dc_N} = a\frac{dk_N}{db_N} > 0$, $\frac{dk_N}{db_H} = \left(-c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}\right)^{-1} > 0$, $\frac{dk_N}{dc_H} = \mathrm{Pr}_{34}\left(-c_H \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}\right)^{-1} > 0$, and $\frac{dk_N}{dc_0} = 0$.

The comparative statics for $k_H$ is slightly more complicated. We start with $b_H$, so that the total differential of Equation (35) is

$$dF_H = (c_0 - c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial k_N} dk_N + (c_0 - c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial k_H} dk_H + (c_0 - c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial D} dD - db_H = 0,$$

so that

$$\frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{dk_N}{db_H} + \frac{\partial\,\mathrm{Pr}_4}{\partial k_H}\frac{dk_H}{db_H} \quad = \quad \frac{1}{c_0 - c_H},$$

where we have used that $\frac{dD}{db_H} = 0$. Now, the result for $\frac{dk_N}{db_H}$ and Table 1 can be used to determine

$$\frac{dk_H}{db_H} = \frac{c_H \frac{\partial \Pr_{34}}{\partial k_N} + (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_N}}{c_H \frac{\partial \Pr_{34}}{\partial k_N} (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_H}} < 0. \tag{39}$$

In the same way, we obtain $\frac{\partial k_H}{\partial c_H} = \Pr_{34} \frac{\partial k_H}{\partial b_H} < 0$, $\frac{\partial k_H}{\partial c_0} = -\Pr_4 \left( (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_H} \right)^{-1} > 0$, and

$$\frac{dk_H}{dc_N} = \frac{-U'' \frac{\partial \Pr_4}{\partial k_N} + c_H \left( \frac{\partial \Pr_4}{\partial k_N} \frac{\partial \Pr_{34}}{\partial D} - \frac{\partial \Pr_{34}}{\partial k_N} \frac{\partial \Pr_4}{\partial D} \right)}{U'' c_H \frac{\partial \Pr_{34}}{\partial k_N} \frac{\partial \Pr_4}{\partial k_H}} = a \frac{\partial k_H}{\partial b_N}, \tag{40}$$

where equation (40) has an ambiguous sign. The denominator is always negative, so that the numerator determines the sign. It follows that $\frac{dk_H}{db_N}, \frac{dk_H}{dc_N} < 0$ if and only if

$$\begin{aligned}
0 &< -U'' \frac{\partial \Pr_4}{\partial k_N} + c_H \left( \frac{\partial \Pr_4}{\partial k_N} \frac{\partial \Pr_{34}}{\partial D} - \frac{\partial \Pr_{34}}{\partial k_N} \frac{\partial \Pr_4}{\partial D} \right) \\
-U'' &< -c_H \frac{\frac{\partial \Pr_4}{\partial k_N} \frac{\partial \Pr_{34}}{\partial D} - \frac{\partial \Pr_{34}}{\partial k_N} \frac{\partial \Pr_4}{\partial D}}{\frac{\partial \Pr_4}{\partial k_N}} \\
&= c_H \left( \frac{\partial \Pr_{34}}{\partial k_N} / \frac{\partial \Pr_4}{\partial k_N} \cdot \frac{\partial \Pr_4}{\partial D} - \frac{\partial \Pr_{34}}{\partial D} \right) \\
&= c_H \left( \frac{\partial \Pr_{34}}{\partial k_N} / \frac{\partial \Pr_4}{\partial k_N} \cdot f(D - k_H; k_N) - f(D; k_N) \right).
\end{aligned}$$

## C Proof of Proposition 4

Similar to the comparative statics for the case of independence, the equation system following from the total differential of the first-order conditions in Equations (13), (8), and (11), here repeated as

$$\begin{aligned}
F_N &:= c_H a_3 \Pr_3 + c_0 a_4 \Pr_4 - b_N - c_N a = 0, \\
F_H &:= (c_0 - c_H) \Pr_4 - b_H = 0, \\
F_D &:= c_H \Pr_3 + c_0 \Pr_4 - U' = 0,
\end{aligned}$$

needs to be solved. In the case of perfect correlation and a uniform distribution, however, this becomes more complicated, since the Jacobian

$$J = \begin{pmatrix} \frac{\partial F_N}{\partial k_N} & \frac{\partial F_N}{\partial k_H} & \frac{\partial F_N}{\partial D} \\ \frac{\partial F_H}{\partial k_N} & \frac{\partial F_H}{\partial k_H} & \frac{\partial F_H}{\partial D} \\ \frac{\partial F_D}{\partial k_N} & \frac{\partial F_D}{\partial k_H} & \frac{\partial F_D}{\partial D} \end{pmatrix}, \tag{41}$$

does not has a diagonal form. On the other hand, since $\omega$ is uniformly distributed, we can use the explicit expressions from Table 3 (see also Figure 2), so that, e.g.,

$$F_N = c_H \frac{1}{2} \frac{2D - k_H}{k_N} \frac{k_H}{k_N} + c_0 \frac{1}{2} \frac{D - k_H}{k_N} \frac{D - k_H}{k_N} - b_N - c_N a = 0.$$

| | $\Omega_{34}$ | $\Omega_4$ | $\Omega_3$ |
|---|---|---|---|
| $a_c$ | $\frac{1}{2}\frac{D}{k_N}$ | $\frac{1}{2}\frac{D-k_H}{k_N}$ | $\frac{1}{2}\frac{2D-k_H}{k_N}$ |
| $\Pr_c$ | $\frac{D}{k_N}$ | $\frac{D-k_H}{k_N}$ | $\frac{k_H}{k_N}$ |

Tab. 3: Expressions for $a_c, \Pr_c$ assuming a uniform distribution

We obtain the derivatives with respect to the parameters in a straightforward way (see Table 4).

| dependent variables | $\partial/\partial k_N$ | $\partial/\partial k_H$ | $\partial/\partial D$ |
|---|---|---|---|
| $F_N$ | $-\frac{c_H k_H(2D-k_H)+c_0(D-k_H)^2}{k_N^3}$ | $-\frac{(c_0-c_H)(D-k_H)}{k_N^2}$ | $\frac{c_0 D-(c_0-c_H)k_H}{k_N^2}$ |
| $F_H$ | $-\frac{(c_0-c_H)(D-k_H)}{k_N^2}$ | $-\frac{(c_0-c_H)}{k_N}$ | $\frac{(c_0-c_H)}{k_N}$ |
| $F_D$ | $-\frac{c_0 D-(c_0-c_H)k_H}{k_N^2}$ | $-\frac{(c_0-c_H)}{k_N}$ | $\frac{c_0}{k_N}-U''$ |

| parameters | $\partial/\partial b_N$ | $\partial/\partial c_N$ | $\partial/\partial b_H$ | $\partial/\partial c_H$ | $\partial/\partial c_0$ |
|---|---|---|---|---|---|
| $F_N$ | $-1$ | $-a$ | $0$ | $a_3\Pr_3$ | $a_4\Pr_4$ |
| $F_H$ | $0$ | $0$ | $-1$ | $-\Pr_4$ | $\Pr_4$ |
| $F_D$ | $0$ | $0$ | $0$ | $\Pr_3$ | $\Pr_4$ |

Tab. 4: Jacobian and further partial derivatives of $F_N, F_H, F_D$.

For each parameter of interest, the total differential leads to a system of three equations. To determine the comparative statics with respect to $c_0$, for instance

$$
\begin{aligned}
-\frac{\partial F_N}{\partial c_0} &= \frac{\partial F_N}{\partial k_N}\frac{dk_N}{dc_0}+\frac{\partial F_N}{\partial k_H}\frac{dk_H}{dc_0}+\frac{\partial F_N}{\partial D}\frac{dD}{dc_0},\\
-\frac{\partial F_H}{\partial c_0} &= \frac{\partial F_H}{\partial k_N}\frac{dk_N}{dc_0}+\frac{\partial F_H}{\partial k_H}\frac{dk_H}{dc_0}+\frac{\partial F_H}{\partial D}\frac{dD}{dc_0},\\
-\frac{\partial F_D}{\partial c_0} &= \frac{\partial F_D}{\partial k_N}\frac{dk_N}{dc_0}+\frac{\partial F_D}{\partial k_H}\frac{dk_H}{dc_0}+\frac{\partial F_D}{\partial D}\frac{dD}{dc_0}.
\end{aligned}
$$

We solve these systems with Cramer's rule. The determinant of Jacobian evaluates to the following expression with an unambiguous sign

$$\det(J) = -\frac{c_H\,(c_0-c_H)\,D^2}{k_N^4}U'' > 0.$$

Since we are only interested in the signs of the solutions, we can focus the further proof on the signs of the determinants of the matrices where the appropriate column of the Jacobian is replaced by the negative partial derivatives w.r.t. the parameter of interest. We thus obtain, with $\doteq$ denoting equivalence in signs, the values in Table 5

| | $\partial k_N/\partial \doteq$ | $\partial D/\partial \doteq$ |
|---|---|---|
| $b_N$ | $-\frac{(c_0-c_H)(-U''k_N+c_H)}{k_N^2}<0$ | $-\frac{c_H(c_0-c_H)D}{k_N^3}<0$ |
| $c_N$ | $-\frac{(c_0-c_H)(-U''k_N+c_H)}{k_N^2}a<0$ | $-\frac{c_H(c_0-c_H)D}{k_N^3}a<0$ |
| $b_H$ | $-\frac{(c_0-c_H)(U''k_N(D-k_H)+c_Hk_H)}{k_N^3}$ | $-\frac{c_H(c_0-c_H)D}{k_N^3}k_H<0$ |
| $c_H$ | $-\frac{(c_0-c_H)[U''k_N\frac{1}{2}((D-k_H)^2+D^2)+c_H\frac{1}{2}(2D-k_H)k_H]}{k_N^4}$ | $-\frac{c_H(c_0-c_H)D}{k_N^3}\frac{\frac{1}{2}(2D-k_H)k_H}{k_N^2}<0$ |
| $c_0$ | $-\frac{(c_0-c_H)(-U''k_N+c_H)}{k_N^2}\frac{\frac{1}{2}(D-k_H)^2}{k_N^2}<0$ | $-\frac{c_H(c_0-c_H)D}{k_N^5}\frac{\frac{1}{2}(D-k_H)^2}{k_N^2}<0$ |

| | $\partial k_H/\partial \doteq$ |
|---|---|
| $b_N$ | $-\frac{(c_0-c_H)(U''k_N(D-k_H)+c_Hk_H)}{k_N^3}$ |
| $c_N$ | $-\frac{(c_0-c_H)(U''k_N(D-k_H)+c_Hk_H)}{k_N^3}a$ |
| $b_H$ | $-\frac{c_H(c_0-c_H)k_H^2-U''k_N(c_H(2D-k_H)k_H+c_0(D-k_H)^2)}{k_N^3}<0$ |
| $c_H$ | $-\frac{c_H(c_0-c_H)k_H^2(2D-k_H)-U''[c_H(2D-k_H)k_H+c_0(D^2+(D-k_H)^2)]k_N(D-k_H)}{2k_N^5}<0,$ |
| $c_0$ | $-\frac{(c_0+c_H)(U''k_N(D-k_H)+c_Hk_H)}{k_N^3}\frac{\frac{1}{2}(D-k_H)^2}{k_N^2}$ |

Tab. 5: Signs of the derivatives of $k_N, k_H, D$ w.r.t. $b_N, c_N, b_H, c_H, c_0$

The ambiguous cases can be further analyzed as follows. Note that $\frac{dk_N}{db_H}, \frac{dk_H}{db_N}, \frac{dk_H}{dc_N}, \frac{dk_H}{dc_0} < 0$ iff

$$0 \quad < \quad U''k_N\left(D-k_H\right)+c_Hk_H.$$

Using Table 3, we can resubstitute for $\mathrm{Pr}_3, \mathrm{Pr}_4$ to obtain

$$-U'' \quad < \quad \frac{c_Hk_H}{k_N\left(D-k_H\right)}=\frac{c_Hk_H/k_N}{k_N\left(D-k_H\right)/k_N}=\frac{c_H\,\mathrm{Pr}_3}{k_N\,\mathrm{Pr}_4}.$$

The expression on the right-hand side is increasing in $k_H$, but decreasing in $k_N, D$. We also obtain $\frac{dk_N}{dc_H}<0$ if and only if

$$0 \quad < \quad U''k_N\frac{1}{2}\left(\left(D-k_H\right)^2+D^2\right)+c_H\frac{1}{2}\left(2D-k_H\right)k_H,$$

which can be solved (using Table 3) to obtain

$$-U'' \quad < \quad \frac{c_H}{k_N}\frac{\frac{1}{2}\left(2D-k_H\right)k_H}{\frac{1}{2}\left(D-k_H\right)^2+\frac{1}{2}D^2}=\frac{c_H}{k_N}\frac{\frac{1}{2}\frac{2D-k_H}{k_N}\frac{k_H}{k_N}}{\frac{1}{2}\frac{D-k_H}{k_N}\frac{D-k_H}{k_N}+\frac{1}{2}\frac{D}{k_N}\frac{D}{k_N}}=\frac{c_H}{k_N}\frac{a_3\,\mathrm{Pr}_3}{a_4\,\mathrm{Pr}_4+a_{34}\,\mathrm{Pr}_{34}}.$$

Again, by differentiation we obtain that the right-hand side is increasing in $k_H$, but decreasing in $k_N, D$.

## D Proof of Proposition 5

The main part of the Proposition has already been shown in the main text. Here, we show the results for uniformly distributed production units in the case of perfect correlation. We start with some preparations. By setting Equation (13) to zero, we can solve for $c_0\,\mathrm{Pr}_4$, and substitute this into Equation (12) to obtain

$$\frac{\partial E[J]}{\partial k_M} = -b_M - c_M + b_H + c_H \Pr{}_{34}, \tag{42}$$

where $\Pr_{34} = \Pr_3 + \Pr_4$ and $a_3 \Pr_3 + a_4 \Pr_4 = a_{34} \Pr_{34}$ has been used. Thus, Equation (11) can be rewritten as

$$\frac{\partial E[J]}{\partial k_N} = -b_N - c_N a + b_H a_4 + c_H a_{34} \Pr{}_{34}. \tag{43}$$

This allows us to find an equivalent expression for $\Phi$. Recall that this parameter was derived from the first-order conditions of $k_M$ and $k_N$ holding both at the same time. Applying this to Equations (42) and (43) yields

$$\Phi = \frac{a - a_{34}}{a_{34}} \left( \frac{b_N}{a} + c_N \right) + \frac{a_{34} - a_4}{a_{34}} b_H.$$

With these preparations, we can alternatively express expected profits of non-dispatchables (25) as follows:

$$\begin{aligned}
E[\pi_N] &= -\left(\frac{b_N}{a} + c_N\right)(E[\tilde{x}_N] - E[D_N]) + \Phi E[D_N] \\
&= -\left(\frac{b_N}{a} + c_N\right) E[\tilde{x}_N] + \left(\frac{b_N}{a} + c_N\right) E[D_N] \\
&\quad + \left(\frac{b_N}{a} + c_N\right) \frac{a - a_{34}}{a_{34}} E[D_N] + b_H \frac{a_{34} - a_4}{a_{34}} E[D_N] \\
&= -\left(\frac{b_N}{a} + c_N\right) E[\tilde{x}_N] + \left(\frac{b_N}{a} + c_N\right) \frac{a}{a_{34}} E[D_N] + b_H \frac{a_{34} - a_4}{a_{34}} E[D_N] \\
&= -\left(\frac{b_N}{a} + c_N\right) \frac{a}{a_{34}} a_{34} k_N + \left(\frac{b_N}{a} + c_N\right) \frac{a}{a_{34}} E[D_N] + b_H \frac{a_{34} - a_4}{a_{34}} E[D_N] \\
&= \left(\frac{b_N}{a} + c_N\right) \frac{a}{a_{34}} (E[D_N] - a_{34} k_N) + b_H \frac{a_{34} - a_4}{a_{34}} E[D_N] \\
&= \frac{a}{a_{34}} \left(\frac{b_N}{a} + c_N\right) (E[D_N] - a_{34} k_N) + \frac{a_{34} - a_4}{a_{34}} b_H E[D_N].
\end{aligned}$$

This is positive since $a_{34} - a_4 = \frac{1}{2} \frac{k_H}{k_N} > 0$ and

$$\begin{aligned}
E[D_N] - a_{34} k_N &= E[\tilde{x}_N | \Omega_{34}] \Pr_{34} + D \Pr_{1} - E[\tilde{x}_N | \Omega_{34}] \\
&= (D - E[\tilde{x}_N | \Omega_{34}]) \Pr{}_1 \\
&= \frac{1}{2} D \Pr{}_1 > 0.
\end{aligned}$$

Thus, in the case of perfect correlation and uniform distribution, optimal capacities lead to positive profits for non-dispatchables.

# E Proof of Proposition 6

While $\tilde{x}_N \leq k_N$ is the available production of non-dispatchables, $x_N \leq \tilde{x}_N$ denotes the actual production decision. The decision structure in Stage 4 from Section 3 is extended to $\max_{x_N, x_H} J$ s.t. $x_N \leq \tilde{x}_N, x_H \leq k_H$. The optimal choice depends on the random event: If $\tilde{x}_N \in \Omega_1$ or $\tilde{x}_N \in \Omega_2$, then $x_N = D - x_M, x_H = x_0 = 0$; if $\tilde{x}_N \in \Omega_3$, then $x_N = \tilde{x}_N, x_H = D - x_M - \tilde{x}_N, x_0 = 0$; if $\tilde{x}_N \in \Omega_4$, then $x_N = \tilde{x}_N, x_H = k_H, x_0 = D - x_M - \tilde{x}_N - k_H$. This yields

$$
\begin{aligned}
E[x_0] &= E[D - x_M - \tilde{x}_N - k_H | \Omega_4] \Pr{}_4, \\
E[x_H] &= E[D - x_M - \tilde{x}_N | \Omega_3] \Pr{}_3 + k_H \Pr{}_4, \\
E[x_N] &= E[\tilde{x}_N | \Omega_3] \Pr{}_3 + E[\tilde{x}_N | \Omega_4] \Pr{}_4 + (D - x_M) \Pr{}_{12},
\end{aligned}
$$

so that for Stages 1 and 2:

$$
\begin{aligned}
E[J] = & \; U(D) - \sum_j b_j k_j - c_M x_M \\
& - c_N \left( E[\tilde{x}_N | \Omega_3] \Pr{}_3 + E[\tilde{x}_N | \Omega_4] \Pr{}_4 + (D - x_M) \Pr{}_{12} \right) \\
& - c_H \left( k_H \Pr{}_4 + E[D - \tilde{x}_N - x_M | \Omega_3] \Pr{}_3 \right) \\
& - c_0 E[D - \tilde{x}_N - x_M - k_H | \Omega_4) \Pr{}_4.
\end{aligned}
$$

We obtain the following derivatives for the remaining decision variables by using the same approach as in Section 3:

$$
\begin{aligned}
-\frac{\partial E[J]}{\partial D} &= c_N \Pr{}_{12} + c_H \Pr{}_3 + c_0 \Pr{}_4 - U', \\
\frac{\partial E[J]}{\partial x_M} &= c_N \Pr{}_{12} + c_H \Pr{}_3 + c_0 \Pr{}_4 - c_M, \\
\frac{\partial E[J]}{\partial k_H} &= -b_H - c_H \Pr{}_4 + c_0 \Pr{}_4, \\
\frac{\partial E[J]}{\partial k_M} &= c_N \Pr{}_{12} + c_H \Pr{}_3 + c_0 \Pr{}_4 - b_M - c_M, \qquad (44) \\
\frac{\partial E[J]}{\partial k_N} &= (c_H - c_N) \overline{a}_3 \Pr{}_3 + (c_0 - c_N) \overline{a}_4 \Pr{}_4 - b_N, \qquad (45)
\end{aligned}
$$

The second and fourth expression imply, as in Section 3, that $x_M = k_M$. Load $D$ can be determined from setting the first equation to zero if all probabilities are known, and $\Pr{}_4$ is directly determined from setting the third expression to zero. What remains to be determined is $\Pr{}_3$. Yet, as in Section 3, setting the last two expressions to zero and using $\Pr{}_{12} = 1 - \Pr{}_3 - \Pr{}_4$ yields an overdetermined equation system (two equations for $\Pr{}_3$). This can only be solved for a boundary case with specific cost parameters.

## F Proof of Proposition 7

The proof starts from the assumption that $k_M \cdot k_N > 0$, and shows that this implies specific values for the costs parameters $b_M, c_M, b_N, c_N$. Thus, this cost configuration is a necessary condition for both capacities being strictly positive. The objective is to maximize $E[J^{mp}] := \sum_t E[J_t] - \sum_j b_j k_j$, where

$$E[J_t] \;=\; U_t(D_t) - \sum_j c_j E[x_{jt}] - c_0 E[x_{0t}],$$

and unserved load at time $t$ is $x_{0t} = \max\left\{D_t - \sum_j x_{jt}, 0\right\}$. In Stages 2 to 4, this additive separable structure allows to maximize $E[J_t]$ separately for each period. We can thus rewrite (since, by assumption, $k_N > 0$) the one-period results from Equations (6) and (5) with time index:

$$\begin{aligned} E[x_{0t}] &= E[D_t - x_{Mt} - \tilde{x}_{Nt} - x_{Ht} | \Omega_{4t}] \Pr{}_{4t}, \\ E[x_{Ht}] &= k_H \Pr{}_{4t} + E[D_t - x_{M,t} - \tilde{x}_{N,t} | \Omega_{3t}] \Pr{}_{3t}. \end{aligned}$$

The derivatives for load and medium-dispatchable production are

$$-\frac{\partial E[J_t]}{\partial D_t} \;=\; c_H \Pr{}_{3t} + c_0 \Pr{}_{4t} - U_t', \tag{46}$$

$$\frac{\partial E[J_t]}{\partial x_{Mt}} \;=\; c_H \Pr{}_{3t} + c_0 \Pr{}_{4t} - c_M, \tag{47}$$

where, for convenience, $U_t' := \frac{\partial U_t(D_t)}{\partial D_t}$. Equation (46) must be zero in the optimum, so that $\forall t : U_t' = c_H \Pr_{3t} + c_0 \Pr_{4t}$. For an internal optimum Equation (47) is equal to zero as well, and for a corner solution medium-dispatchable production is at the capacity limit, $x_{M,t} = k_M$. Denote the subset of all periods with a corner solution by $L$, and $|L|$ is the number of periods in $L$.

In Stage 1, the first-order condition for medium-dispatchable capacity then simplifies to

$$\frac{\partial E[J^{mp}]}{\partial k_M} \;=\; c_H \sum_{t \in L} \Pr_{3t} + c_0 \sum_{t \in L} \Pr_{4t} - b_M - |L| c_M = 0.$$

Since we assumed a positive medium-dispatchable capacity to be optimal, this first-order condition is satisfied for some $k_M > 0$. Note that, since Equation (46) is zero in any period, $\sum_{t \in L} U_t' = c_H \sum_{t \in L} \Pr_{3t} + c_0 \sum_{t \in L} \Pr_{4t}$. We thus obtain $\sum_{t \in L} U_t' = b_M + |L| c_M$. Ultimately, also $\sum_{t \in L} U_t'$ only depends on the cost parameters. Consequently, optimality of positive capacities is only possible for such a specific relation of the parameters.

## Appendix for Referees

## G Calculation of Equation (38)

$$dF_N = c_H \frac{\partial \Pr_{34}}{\partial k_N} dk_N + c_H \frac{\partial \Pr_{34}}{\partial D} dD - db_N/a = 0$$

$$\begin{aligned}
\frac{1}{ac_H} db_N &= \frac{\partial \Pr_{34}}{\partial k_N} dk_N + \frac{\partial \Pr_{34}}{\partial D} dD \\
\frac{1}{ac_H} &= \frac{\partial \Pr_{34}}{\partial k_N} \frac{dk_N}{db_N} + \frac{\partial \Pr_{34}}{\partial D} \frac{1}{aU''} \\
\frac{dk_N}{db_N} &= \frac{\frac{1}{ac_H} - \frac{\partial \Pr_{34}}{\partial D} \frac{1}{aU''}}{\frac{\partial \Pr_{34}}{\partial k_N}} \\
&= \frac{1}{a} \frac{U'' - c_H \frac{\partial \Pr_{34}}{\partial D}}{U'' c_H \frac{\partial \Pr_{34}}{\partial k_N}}
\end{aligned}$$

## H Calculation of $\frac{dk_N}{db_H}$

$$dF_N = c_H \frac{\partial \Pr_{34}}{\partial k_N} dk_N + c_H \frac{\partial \Pr_{34}}{\partial D} dD + db_H = 0$$

$$\begin{aligned}
\frac{1}{c_H} db_H &= -\frac{\partial \Pr_{34}}{\partial k_N} dk_N - \frac{\partial \Pr_{34}}{\partial D} dD \\
\frac{1}{c_H} &= -\frac{\partial \Pr_{34}}{\partial k_N} \frac{dk_N}{db_H} - \frac{\partial \Pr_{34}}{\partial D} \frac{dD}{db_H} \\
\frac{1}{c_H} &= -\frac{\partial \Pr_{34}}{\partial k_N} \frac{dk_N}{db_H} \\
\frac{dk_N}{db_H} &= \left( -c_H \frac{\partial \Pr_{34}}{\partial k_N} \right)^{-1} > 0
\end{aligned}$$

## I Calculation of Equation (39)

$$\frac{\partial \Pr_4}{\partial k_N} \frac{dk_N}{db_H} + \frac{\partial \Pr_4}{\partial k_H} \frac{dk_H}{db_H} = \frac{1}{c_0 - c_H}$$

$$
\begin{aligned}
\frac{dk_H}{db_H} &= \frac{\frac{1}{c_0-c_H} - \frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{dk_N}{db_H}}{\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}} \\
&= \frac{\frac{1}{c_0-c_H} + \frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{1}{c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}}}{\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}} \\
&= \frac{c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N} + (c_0-c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial k_N}}{c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}(c_0-c_H)\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}}
\end{aligned}
$$

# J Calculation of Equation (40)

$$
\frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{dk_N}{dc_N} + \frac{\partial\,\mathrm{Pr}_4}{\partial k_H}\frac{dk_H}{dc_N} + \frac{\partial\,\mathrm{Pr}_4}{\partial D}\frac{dD}{dc_N} = 0
$$

$$
\begin{aligned}
\frac{dk_H}{dc_N} &= -\frac{\frac{\partial\,\mathrm{Pr}_4}{\partial D}\frac{dD}{dc_N} + \frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{dk_N}{dc_N}}{\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}} \\
&= -\frac{\frac{\partial\,\mathrm{Pr}_4}{\partial D}\frac{1}{U''} + \frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{U''-c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial D}}{U''c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}}}{\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}} \\
&= -\frac{c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}\frac{\partial\,\mathrm{Pr}_4}{\partial D} + U''\frac{\partial\,\mathrm{Pr}_4}{\partial k_N} - c_H\frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{\partial\,\mathrm{Pr}_{34}}{\partial D}}{U''c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}} \\
&= \frac{-U''\frac{\partial\,\mathrm{Pr}_4}{\partial k_N} + c_H\left(\frac{\partial\,\mathrm{Pr}_4}{\partial k_N}\frac{\partial\,\mathrm{Pr}_{34}}{\partial D} - \frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}\frac{\partial\,\mathrm{Pr}_4}{\partial D}\right)}{U''c_H\frac{\partial\,\mathrm{Pr}_{34}}{\partial k_N}\frac{\partial\,\mathrm{Pr}_4}{\partial k_H}}
\end{aligned}
$$

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