

## Force, Momentum, and Impulse of Force

### Keywords:

Force, resilience, HOOKE's law, NEWTON's axioms, momentum, impulse of force, harmonic oscillation, distance law for forces.

### Measuring program:

Calibration of a force sensor, force and displacement in a harmonic oscillation, force of attraction between magnets, impulse of force and change of momentum at an elastic collision on an air cushion track.

### References:

- /1/ DEMTRÖDER, W.: „Experimentalphysik 1 – Mechanik“, Springer-Verlag, Berlin among others
- /2/ MESCHÉDE, D. [Ed.]: "Gerthsen Physik", Springer, Berlin among others

## 1 Introduction

This experiment mainly deals with the quantitative measurement of forces. In the first part, the relationship between force and deflection in a mechanical harmonic oscillation is investigated. The second part of this experiment deals with the distance law in the dynamic effect between two magnets. The third and final part explains the relationship between force and momentum and impulse of force, respectively.

## 2 Theory

### 2.1 Harmonic Oscillation: Force and Deflection

Let us observe a set-up according to Fig. 1, which we have already got to know in the experiment “*Sensors for force...*” in a similar form. A spring FE is attached to a force sensor S. A sphere K is suspended on the lower end of the spring via a rod ST. Additionally, a reflective disc R is mounted on the rod.

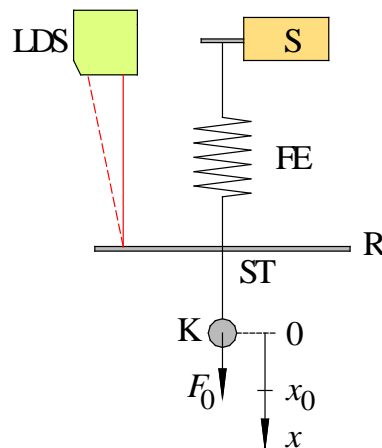


Fig. 1: Set-up for measuring the force and deflection in a harmonic oscillation. For notations cf. text.

When the sphere is at rest, there is an equilibrium in the force of gravity of ST, K and R that points downwards and the resilience pointing upwards. The center of the sphere is then in its equilibrium position at  $x = 0$ . An additional force  $F_0$ , which points downwards, deflects the sphere downwards by the distance  $x_0$ <sup>1</sup>. At time  $t = 0$ , the sphere is released and is accelerated upwards by the tensile force  $F$  of the spring. According to HOOKE'S law with the spring constant  $D$  we thus obtain for  $F$ :

$$(1) \quad F = -D x_0$$

<sup>1</sup> Since  $F_0$  and all other observed forces act only in  $x$ -direction, a description with scalar quantities is sufficient.

Subsequently, the sphere performs a harmonic oscillation in the  $x$ -direction. The deflection from its equilibrium position,  $x(t)$ , which can be measured using a laser distance sensor LDS, is described by the following equation neglecting friction effects<sup>2</sup>:

$$(2) \quad x(t) = x_0 \cos(\omega t)$$

where  $\omega$  is the angular frequency of the oscillation which is given by:

$$(3) \quad \omega = \sqrt{\frac{D}{m}}$$

$m$  is the decisive mass for the oscillation, given by:

$$(4) \quad m = m_K + m_{ST} + m_R + \frac{1}{3} m_{FE}$$

$m_K$ ,  $m_{ST}$ ,  $m_R$  and  $m_{FE}$  are the masses of the sphere, the rod, the reflective disc and the spring<sup>3</sup> respectively.

The velocity  $v(t)$  of the sphere is obtained by differentiating  $x(t)$  for time:

$$(5) \quad v(t) = -x_0 \omega \sin(\omega t)$$

The acceleration  $a(t)$  is obtained by differentiating the velocity for time:

$$(6) \quad a(t) = -x_0 \omega^2 \cos(\omega t)$$

Hence it follows for the force  $F(t)$  exerted on  $m$  by the spring:

$$(7) \quad F(t) = m a(t) = -m x_0 \omega^2 \cos(\omega t) = -F_0 \cos(\omega t)$$

with

$$(8) \quad F_0 = m x_0 \omega^2$$

**Question 1:**

- The force  $F(t)$  has a positive or a negative sign depending on the position  $x(t)$  of the sphere. Within which range does the spring act as a tension spring or as a pressure spring?

At the equilibrium position of the sphere, the force sensor S measures the weight  $G$  of FE, ST, R, and K. If the sphere is pulled down from its position of rest by the force  $F_0$ , S measures the force according to NEWTON'S 3rd axiom<sup>4</sup> *actio = reactio*

$$(9) \quad F_S = G + F_0 = G - F(t=0)$$

with  $F(t)$  according to Eq. (7). After the sphere has been released, S measures the force

$$(10) \quad F_S(t) = G - F(t)$$

After subtraction of  $G$ , S thus yields  $F(t)$  according to Eq. (7) with reversed sign.

<sup>2</sup> A detailed mathematical description of the oscillation is given later on in the experiment "Forced mechanical oscillations".

<sup>3</sup> The relationship according to Eq. (4) is explained in the experiment "Forced mechanical oscillations".

<sup>4</sup> ISAAC NEWTON (1643 – 1727)

## 2.2 Distance Law for Magnetic Force

As is generally known, the gravitational force  $\mathbf{F}_G$  between two masses  $m_1$  and  $m_2$  at distance  $r$  is defined by NEWTON's law of gravitation:

$$(11) \quad \mathbf{F}_G = G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}$$

where  $G$  is the gravitational constant and  $\hat{\mathbf{r}}$  is the unit vector parallel to the connecting line between the center of both masses<sup>5</sup>. As the force decreases with the square of the distance  $r$ , this is called a  $1/r^2$  law.

Also the COULOMB law, which describes the force  $\mathbf{F}_E$  between two electrical charges  $q_1$  and  $q_2$  in a vacuum, is a  $1/r^2$  law. It reads:

$$(12) \quad \mathbf{F}_E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

where  $\epsilon_0$  is the electric constant and  $r$  and  $\hat{\mathbf{r}}$  are defined in analogy to the law of gravitation. We obtain an attracting force for charges with unequal signs and a repulsive force for charges with equal signs.

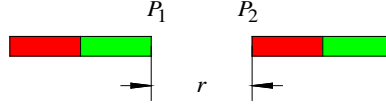


Fig. 2: The distance law for magnetic forces.  $P_1$  and  $P_2$  represent the pole strengths of the magnets with diameters that are small compared to their lengths. The distance  $r$  between the two magnets is measured between the poles.

Under certain conditions, a  $1/r^2$  law is valid for the force between two magnets, too. Let us consider two bar magnets according to Fig. 2, the diameters of which are small compared to their lengths. Under this condition, the “strength” of such magnets can be described by *pole strengths*  $P$  that act with different signs at the magnetic north and south pole of the respective magnet and can be assumed to be point-like<sup>6</sup>. The force between two magnets is then described by COULOMB'S law for magnets:

$$(13) \quad \mathbf{F}_M = \frac{\mu_0}{4\pi} \frac{P_1 P_2}{r^2} \hat{\mathbf{r}}$$

where  $\mu_0$  is the magnetic constant and  $r$  and  $\hat{\mathbf{r}}$  are defined in analogy to the law of gravitation. The force is attractive in cases of opposite poles and repulsive in cases of like poles.

## 2.3 Momentum and Impulse of Force

The state of motion of a body of mass  $m$  moving linearly at a uniform velocity  $\mathbf{v}$ , is described by the momentum

$$(14) \quad \mathbf{p} = m\mathbf{v}$$

According to NEWTON's 1st axiom, a change of the momentum is only possible, if a force  $\mathbf{F}$  acts on the body. According to NEWTON's 2nd axiom, the temporal change of the momentum caused by  $\mathbf{F}$  is given by

$$(15) \quad \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

Eq. (15) can also be written in the form

<sup>5</sup> To be exact, Eq. (11) is only valid for point-like masses at distance  $r$ . However, the same relationship is found in approximately sphere-like masses with symmetrical mass distribution around the center (e.g. in the system earth and moon). Then  $r$  describes the distance between both centers.

<sup>6</sup> The magnetic pole strength  $P$  is defined differently in the literature. Here, it denotes the quantity with the unit  $[P] = \text{A m}$ .

$$(16) \quad d\mathbf{p} = \mathbf{F} dt$$

It means that the force  $\mathbf{F}$  acting on a body during the time period  $dt$  causes a change of the momentum  $d\mathbf{p}$ . The product  $\mathbf{F} dt$  is called *impulse of force*. Hence, it holds:

$$\text{impulse of force} = \text{change of momentum}$$

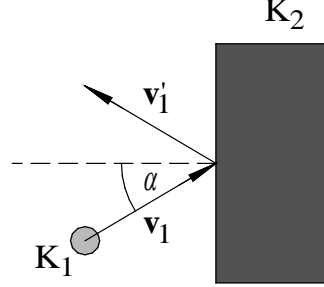


Fig. 3: On the elastic collision of two bodies  $K_1$  and  $K_2$ .

We now observe the elastic collision between a body  $K_1$  with the mass  $m_1$  and a resting body  $K_2$ , the mass  $m_2$  of which is much bigger than  $m_1$ , as illustrated in Fig. 3.  $K_1$  is initially assumed to move linearly at a velocity  $\mathbf{v}_1$  towards  $K_2$  ( $\mathbf{v}_2 = 0$ ) and to collide with  $K_2$  at the angle  $\alpha$ . Following the collision,  $K_1$  moves away from  $K_2$  at the velocity  $\mathbf{v}_1'$ . The prime (') denotes quantities following the collision here and in the following. In the case  $m_2 \rightarrow \infty$  that is observed here,  $\mathbf{v}_2' \rightarrow 0$  holds true. Hence, the momentum of  $K_1$  prior to and following the collision is:

$$(17) \quad \begin{aligned} \mathbf{p}_1 &= m_1 \mathbf{v}_1 \\ \mathbf{p}_1' &= m_1 \mathbf{v}_1' \end{aligned}$$

with  $|\mathbf{v}_1| \approx |\mathbf{v}_1'|$ . Therefore, the kinetic energy of  $K_1$  prior to and following the collision is almost identical.

$$(18) \quad \frac{1}{2} m_1 |\mathbf{v}_1|^2 \approx \frac{1}{2} m_1 |\mathbf{v}_1'|^2$$

Virtually, no kinetic energy is transmitted to  $K_2$  under the given conditions.

The change in momentum  $K_1$  experiences by the collision is:

$$(19) \quad \Delta\mathbf{p} = \mathbf{p}_1' - \mathbf{p}_1 = m_1 (\mathbf{v}_1' - \mathbf{v}_1)$$

It must be equal to the momentum received by  $K_2$ :

$$(20) \quad \Delta\mathbf{p} = \mathbf{p}_2' = m_2 \mathbf{v}_2'$$

Eq. (20) is not in contradiction to the fact that  $K_2$  does approximately not receive any kinetic energy by the collision due to  $\mathbf{v}_2' \rightarrow 0$ . This is quite evident from the relationship between kinetic energy and momentum:

$$(21) \quad E_{\text{kin},2} = \frac{1}{2} m_2 \mathbf{v}_2'^2 = \frac{\mathbf{p}_2'^2}{2 m_2}$$

For  $m_2 \rightarrow \infty$  the kinetic energy thus approaches zero even for a finite momentum  $\mathbf{p}_2'$ .

The change of momentum  $\Delta\mathbf{p}$  of  $K_1$  must be equivalent to the total (integral) impulse of force that  $K_2$  exerts on  $K_1$  during the collision process. According to Eq. (16) this impulse of force is given by:

$$(22) \quad \Delta \mathbf{p} = \int_0^{\tau} \mathbf{F}(t) dt$$

Here,  $\mathbf{F}(t)$  is the force acting during the time period  $\tau$  of the collision. Initially, it acts as a brake on  $K_1$  reducing its velocity  $\mathbf{v}_1$  to 0 and thereafter accelerates it to velocity  $\mathbf{v}_1'$ . In order to understand the collision process better, we can imagine  $K_2$  to be a spring pressed together by  $K_1$  until its velocity is 0 and then relaxing again thereby accelerating  $K_1$  to velocity  $\mathbf{v}_1'$ .

### 3 Experimental Procedure

#### *Equipment:*

Force sensor working on DMS principle (measuring range (0 - 5) N), measuring amplifier for force sensor, set of weights for calibrating the force sensor, 2 power units (PHYWE (0 - 15 / 0 - 30) V), digital oscilloscope TEKTRONIX TDS 1012 / 1012B / 2012C / TBS 1102B, spring with sphere, rod and reflective disc, laser distance sensor (BAUMER OADM 12U6460/S35), bar magnets in Al mountings with stay bars, PVC distance piece, guide for bar magnet, adjustable table, air cushion track, sled for air cushion track with screen and Al cubes, hybrid light barrier (BETA-SENSORIK GLS-30BP/R), hook for force sensor, stand on base, rods, pillar of stand, cross joints, caliper gauge.

#### 3.1 Calibration of the Force Sensor

The force sensor used in the following experiment works on the same principle as the sensor used in the experiment “*Sensors for force...*“. These two sensors differ only in their measuring range (here: 5 N, previously: 100 mN)<sup>7</sup>. The calibration of the sensor is performed using at least five mass pieces within the range  $m \approx (50 - 500)$  g.

The output voltage  $U$  of the force sensor<sup>8</sup> is measured with an oscilloscope<sup>9</sup>.  $U$  is plotted over  $G = mg$  using the value  $g = 9.8133$  m/s<sup>2</sup> for the earth’s acceleration  $g$  that can be assumed to be exact. A regression line is drawn through the data, whose parameters are later used for converting voltage to a force.

#### 3.2 Force and Deflection for a Harmonic Oscillation

The force  $F(t)$  and the deflection  $x(t)$  for a harmonic oscillation are to be measured using a set-up according to Fig. 1. First, the masses of the spring and of the system rod/reflective disc/sphere are determined by weighing. Then, all the components are attached to the force sensor, which is mounted on a stand as shown in Fig. 1. The laser distance sensor LDS is also attached to the stand and its position is adjusted such that the reflective disc remains within its measuring range during the oscillation. Subsequently, the sphere is manually pulled downwards by about 20 m as vertically as possible and then released.

The output voltage of the LDS,  $U_{LDS}(t)$ , and the output voltage of the force sensor,  $U_S(t)$ , are recorded with a storage oscilloscope triggered to  $U_{LDS}$ . The time-base of the oscilloscope is adjusted so that 5 - 10 oscillation periods are visible on the screen. Following the transient oscillation a typical oscillation process is stored in *single-sequence* mode. A screenshot is taken and attached to the report.

With the aid of the time cursors, the period of the oscillation is determined and the angular frequency  $\omega$  is calculated. The amplitudes (index 0) of  $U_{LDS}(t)$  and  $U_S(t)$  of a selected partial oscillation are measured using the voltage cursors:

$$(23) \quad U_{LDS,0} = (U_{LDS,max} - U_{LDS,min}) / 2 \qquad U_{S,0} = (U_{S,max} - U_{S,min}) / 2$$

<sup>7</sup> Because of this reason the damping must be switched off.

<sup>8</sup> To simplify matters, the term “output voltage of the force sensor“ is used here and in the following for the output voltage of the measuring amplifier connected with the actual force sensor.

<sup>9</sup> Since the voltage  $U$  is measured with the oscilloscope in the following measurements,  $U$  is also measured with the oscilloscope instead of a multimeter here, in order to avoid differences between the measured values caused by different calibrations of oscilloscope and multimeter.

With the calibration factor  $k$  of the LDS for voltage *differences*,

$$k = 0,0962 \text{ V/mm} ,$$

which is assumed to be exact, the oscillation amplitude  $x_0$  can be extracted and the force  $F_0$  can be calculated with Eq. (8). Mass  $m$  is calculated according Eq. (4) and  $\omega$  is measured by an oscilloscope. From  $U_{s,0}$  and the calibration function of the force sensor  $F_0$  can also be estimated. Both results should be compared.

**Question 2:**

- What could be the reasons for differences of both quantities?

### 3.3 Force of Attraction Between two Magnets

Using a set-up according to Fig. 4 we examine the validity of the distance law for the force of attraction between two magnets according to Eq. (13). Both magnets are mounted in aluminum tubes which are screwed to rods. Initially, only the upper magnet, with the pole strength  $P_1$ , is attached to a force sensor S together with the rod. An almost frictionless guide A confines its motion to a vertical direction. The output voltage  $U_{S,0}$  of the force sensor is measured. This voltage  $U_{S,0}$  corresponds to the case  $r \rightarrow \infty$ , hence only caused by the weight  $G$  of rod and magnet. From  $U_{S,0}$   $G$  is calculated with the data of the calibration function and subtracted from all other forces measured later on.

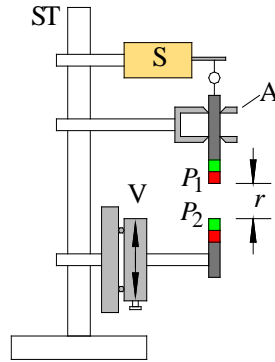


Fig. 4: Set-up for measuring the attraction force between two magnets with the pole strengths  $P_1$  and  $P_2$ . All components are mounted on one stand ST enabling a precise vertical mounting. For other notations refer to the text.

Subsequently, the lower magnet with the pole strength  $P_2$  is mounted on an adjustable table V, so that it can be moved in vertical direction. First, the lower magnet is adjusted so that the longitudinal axes of both magnets exactly superimpose each other. Then, a defined distance  $l$  is set between both magnets using a PVC piece (length  $l \approx 20$  mm, to be measured). The respective scale position of the shiftable table is noted down and defined as  $s = l$ . Then, the shiftable table is adjusted to the scale position  $s = 10$  mm and  $U_S(s)$  is determined<sup>10</sup>. Subsequently, we reduce  $s$  to about  $s = 2$  mm in steps of approx. 0.5 mm measuring again  $U_S(s)$  each time.

The values of  $U_S(s)$  are converted into forces  $F(s)$  with the data of the calibration function. After subtracting  $G$  we obtain the forces  $F_M(s)$  caused by magnetic attraction:

$$(24) \quad F_M(s) = F(s) - G$$

According to Fig. 5, each force  $\mathbf{F}$  acting on the bending rod in the force sensor leads to a minor deflection  $d$  of the rod in the direction of  $\mathbf{F}$  at the point of impact Q of  $\mathbf{F}$ .  $d(F)$  can be described by a polynomial of the 2<sup>nd</sup> order in good approximation:

$$(25) \quad d(F) = a_1 F + a_2 F^2$$

<sup>10</sup> Example: Let  $l = 20.1$  mm. At this distance between  $P_1$  and  $P_2$  the scale position 74.5 mm is read at the shiftable table V. This value is defined as  $s = l = 20.1$ mm. Afterwards V shall be moved to the position  $s = 10$  mm. Accordingly a value of  $(74.5 - 10.1)$  mm = 64.4 mm has to be adjusted at its scale.

The parameters  $a_1$  and  $a_2$  can be taken from the available work papers. Considering the deflection  $d$  the real distance  $r$  between both magnets at a position  $s$  on the shiftable table is:

$$(26) \quad r = s - d$$

$F_M$  is log-log plotted over  $r$  (Origin). A regression line is drawn into the diagram through the measured values for  $r > 5$  mm with the fixed slope - 2. If the  $1/r^2$  law expected from Eq. 13 is valid, all measured values must lie on this line. For small distances  $r$ , however, the measured values will distinctly deviate from the regression line, because the assumption of *point-like* pole strengths  $P$  is no longer justified for small  $r$ .

### Question 2:

- From which distance  $r$  upwards can we find a  $1/r^2$  dependence in good approximation?

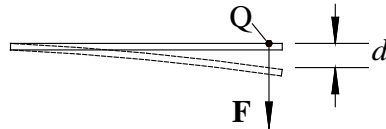


Fig. 5: Deformation of the bending rod in the force sensor upon impact of a force  $F$  at the contact point  $Q$ .  $Q$  is deflected by the distance  $d$  due to  $F$ .

## 3.4 Elastic Collision on an Air Cushion Track

The relationship *impulse of force = change of momentum* for an elastic collision is to be quantitatively investigated on an air cushion track LK (Fig. 6), on which a mounted sled can be moved almost frictionless. Since all motions proceed along the axis of the air cushion track ( $\alpha = 0^\circ$  according to Fig. 3), it is sufficient to observe the scalar quantities velocity ( $v$ ) and momentum ( $p$ ).

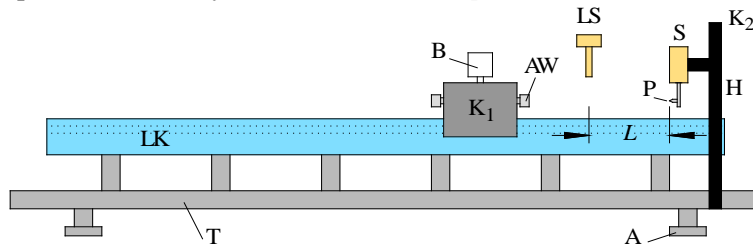


Fig. 6: Set-up for measuring the momentum and impulse of force during an elastic collision on an air cushion track LK.  $K_1$ : Sled with screen  $B$  and two Al cubes  $AW$ .  $S$ : Force sensor with spike  $P$  on mounting  $H$ .  $T$ : steel base for LK with regulating footing  $A$ .  $L$ : Length of measuring range.  $LS$ : hybrid light barrier.

The sled  $K_1$  with the mass  $m_1$  is manually accelerated to velocity  $v_1$ . It then moves towards the force sensor  $S$  that is connected to the massive steel base  $T$  of LK with a rigid mounting  $H$ . This set-up guarantees the conditions defined in Chapter 2.3 for the collision partner  $K_2$  (consisting of  $S$ ,  $H$  and  $T$ ), namely  $m_2 \rightarrow \infty$  and thus  $v_2' \rightarrow 0$  are met. The force  $F(t)$  acting between  $K_1$  and  $K_2$  during the collision is measured with the force sensor  $S$  and recorded with the aid of a storage oscilloscope.

A screen  $B$  of the width  $d$  is mounted in the center of the sled, which interrupts a hybrid light barrier  $LS$  for the period  $t_{LS}$  while passing it. By measuring  $d$  (caliper gauge),  $t_{LS}$  (storage oscilloscope), and  $m_1$  (laboratory scales) the velocities and momentums of  $K_1$  prior to and following the collision can be determined.

### Instructions for performance:

- For a precise measurement of  $F(t)$  the force sensor is provided with a spike  $P$  that enables an approximately point-like contact with an aluminum cube  $AW$  mounted on the sled<sup>11</sup>. Prior to the measurement the sensor must be adjusted in a way that the spike hits the center of the cube.
- In order to prevent the sled from being accelerated or decelerated along the track following the manual acceleration, it is necessary to adjust the track exactly horizontal. This cannot be achieved over its entire length, but can be achieved over a distance of  $L \approx (10 - 15)$  cm between  $LS$  and  $S$ , which is sufficient

<sup>11</sup> The sled must be provided with equal Al cubes on both sides in order to achieve a symmetrical mass distribution.

for the measurement. The track is adjusted by regulating the footing A of the steel base, exclusively. When the air supply has been switched on, A must be adjusted so that the mounted sled is not accelerated within the range  $L$ .

- The sled  $K_1$  may be accelerated only so strongly, that  $F(t)$  is always below 5 N in order not to exceed the measuring range of the force sensor. Thus, the velocity  $v_1$  of the sled must not become too high und hence  $t_{LS} > 120$  ms is a good rule-of-thumb value.

Fig. 7 shows a typical oscilloscope image following a collision process: The signals  $U_{LS}$  of the hybrid light barrier were recorded at CH1 (which is used for triggering), while the sled moved forth and back, and the signal  $U_S$  of the force sensor at CH2. A screenshot of the oscilloscope image is attached to the report. Then, the signals are stored on a Compact-Flash Card or USB stick and subsequently converted to ASCII format. For details see Appendix (Chapter 4). Subsequently, these data can be imported to Origin for further analysis.

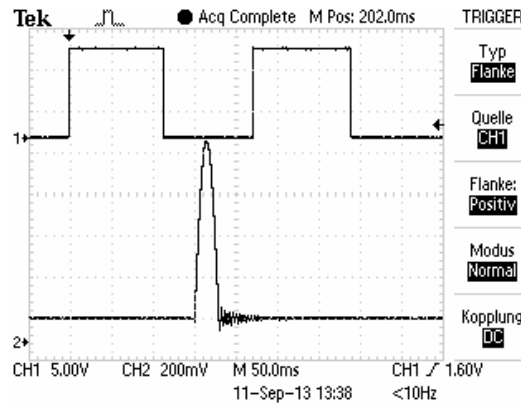


Fig. 7: Screenshot of the signals by the hybrid light barrier LS (CH1, top) and the force sensor S (CH2, below).

The pulse widths  $t_{LS}$  of the signals of the hybrid light barrier for the sled moving forth and back are determined either by means of the time cursors on the oscilloscope or in Origin using the Data Reader tool. Then, the absolute value of the entire change of momentum of the sled according to Eq. (19) can be calculated as follows:

$$(27) \quad |\Delta p| = |p_1| + |p_1'| = m_1 d \left( \frac{1}{t_{LS}} + \frac{1}{t_{LS}'} \right)$$

#### Question 4:

- Please demonstrate, how to get to equation (27).

For calculating the maximum error of the change of momentum  $m_1$  and  $d$  can be regarded as error-free.

For measuring the impulse of force the integral in Eq. (22) is replaced by a sum:

$$(28) \quad \Delta p = \sum_i F(t_i) \Delta t$$

where the  $F(t_i)$  are the discrete values of the force sensor at times  $t_i$  (*sampling points* of the oscilloscope) that can be calculated from the recorded voltage values  $U_S(t_i)$  by applying the calibration function. The sum over the  $F(t_i)$  is easy to determine using the Origin tool *Statistics on columns*<sup>12</sup>.  $\Delta t$  is the constant temporal distance between two consecutive measuring values at times  $t_i$  and  $t_{i+1}$ , which results from the adjusted time-base sweep on the oscilloscope ( $x$  SEC/DIV) and the number of recorded measuring values (2,500).

$$(29) \quad \Delta t = 10 \text{ DIV} \times (x \text{ SEC/DIV}) / 2500$$

<sup>12</sup> Right mouse-click on the marked column, choose the menu *Statistics on columns*, tick *Sum* under the submenu *Quantities*.



It is difficult to determine the start and end of the impulse of force, and thus its duration  $\tau$ , precisely. Therefore, the sum according to Eq. (28) is not added up over  $\tau$ , but over the complete time interval recorded, i.e. over all of the 2,500 measured values recorded with the oscilloscope. At this procedure we must take the following into account:

Outside the time interval  $\tau$ ,  $F(t_i) = 0$  should be valid. Actually,  $F(t_i) \neq 0$  can be caused by a small offset in the force signal and by electronic noise, which may lead to a significant error for summation over many  $t_i$ . Therefore, we first calculate the mean  $F_0$  of the force signal over the time interval, which *surely* lies before the start of the collision<sup>13</sup>. Subsequently, this value  $F_0$  is subtracted from all measured values  $F(t_i)$  and only then the sum is calculated according to Eq. (28).

In order not to make the calculation of the maximum error of the impulse of force too elaborate, a maximum error of 5 mN may be assumed for each single force value  $F(t_i)$  and the maximum error of  $\Delta t$  can be neglected.

Finally, we have to examine, whether the change of momentum according to Eq. (26) corresponds to the impulse of force according to Eq. (28). The reasons for possible deviations between both quantities are discussed.

## 4 Appendix

The following steps are necessary for storing data from the oscilloscope on an USB stick or Compact-Flash card, and for later converting them to ASCII data files:

First, the following keys of the oscilloscope are pressed once:

Basic adjustments:

```
SAVE/RECALL → Action          → Save all
PRINT                → Save all
Choose directory    → Select GPRnn → Change directory → Back
```

Storage:

```
SAVE / PRINT
```

After pressing the SAVE / PRINT key, four files are stored in the sub-directory ALLnnnn, where nnnn is a consecutive number (starting at 0000) that is increased by 1 each time the key SAVE / PRINT is pressed. The four files are:

FnnnnTEK.SET	ASCII file with operational parameters of the oscilloscope
FnnnnTEK.TIF	Image file with screenshot
FnnnnCH1.CSV	Data of CH1 (voltage $U_1$ as a function of time $t$ among others)
FnnnnCH2.CSV	Data of CH2 (voltage $U_2$ as a function of time $t$ among others)

For quantitative analysis, only the two last files available in CSV format are important.<sup>14</sup> With the aid of the available Matlab script<sup>15</sup> GPRTOOLS.m, there option Tektronix CSV to ASCII, the signal courses  $U_1(t)$  for CH1 and  $U_2(t)$  for CH2 are extracted from these files and converted into ASCII format<sup>16</sup>. Subsequently, these data are stored in the directory ALLnnnn under the following file names:

FnnnnCH1_all.txt	Column 1: $t$ , column 2: $U_1(t)$ for CH1
FnnnnCH2_all.txt	Column 1: $t$ , column 2: $U_2(t)$ for CH2

These files can be imported to Origin via File → Import → ...

<sup>13</sup> Mark lines in the column with values for  $F(t_i)$ , in which  $t_i$  lies before the start of the collision. Then choose → Statistics on columns, there choose submenu Quantities and tick Mean.

<sup>14</sup> CSV is the abbreviation of *character separated values*. This means that the single entries in the file (numerical values, character strings,...) are separated by a defined *character*. Here, the comma is the separating character.

<sup>15</sup> After logging onto a PC in the laboratory the Matlab script is available under the working directory Q: MatlabSkripte\$ (\gpr00.gpr.physik.uni-oldenburg.de).

<sup>16</sup> With newer versions of Origin, CSV files can be directly imported.