Systemic Risk and the Statistical Physics of Falling Dominoes

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King's College London
University of London

NETADIS
Statistical Physics Approaches to Networks Across Disciplines

Outline

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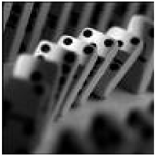
Risk and Falling Dominoes



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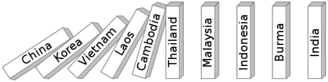
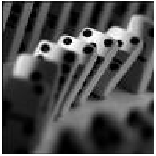


Domino Theory & Spread of Communism



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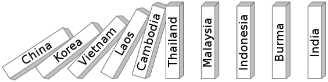

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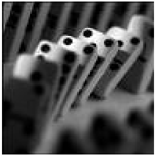

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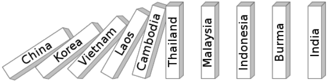

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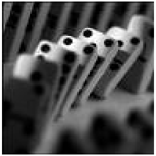

Blackouts in Power Grids

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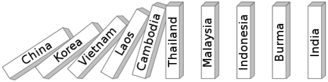

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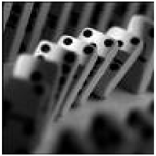

Blackouts in Power Grids

Financial Crisis

Outline

Fundamental Problem of Risk Analysis

- Estimate **likelihood of failures and potential losses**
- Main types of risk
  - negative fluctuation of portfolio-value (stock-prices, exchange rates, interest rates, economic indices) $\leftrightarrow$ **market risk**
  - change of credit quality, including default of creditor (asset values of firms, ratings, stock-prices) $\leftrightarrow$ **credit risk**
  - process failures (human errors, hardware/software- failures, lack of communication, fraud, external catastrophes) $\leftrightarrow$ **operational risk**
  - rare fluctuations in cash-flows, requiring short term acquisition of funds to maintain liquidity $\leftrightarrow$ **liquidity risk**

- Popular risk measure:
  Value at Risk

$$\mathrm{VaR}_q = \mathrm{e}^{-rT}\left(Q_q - \mathbb{E}[L]\right)$$

$\Leftrightarrow$ money to set aside now to cover extreme losses at $t = T$.

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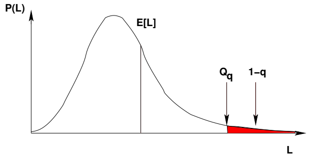

Main Interest and Concern: Interactions

- Traditional approaches treat risk elements as independent or *at best* statistically correlated
- Misses functional & dynamic nature of relations: terminal–mainframe/input errors–results/manufacturer–supplier relations ...
- Effect of interactions between risk elements
  - Can have of avalanches of risk events
    ⇔ **falling dominoes**
  - Fat tails in loss distributions
  - Volatility clustering in markets (intermittency)

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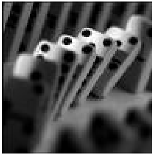

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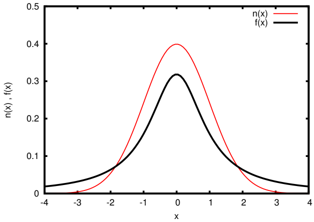

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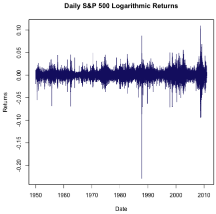
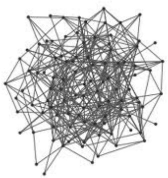


Outline

Operational Risks — Interacting Processes

- Conceptualise organisation as a network of processes
- Idealised two state model:
  - processes can be either up and running $(n_i = 0)$
  - or down $(n_i = 1)$
  - Reliability of processes heterogeneous across the set of processes
  - degree of interdependence heterogeneous across the set of processes
  - connectivity & concept of neighbourhood functionally defined

$\Rightarrow$model defined on random graph



- losses determined (randomly) each time a process goes down

Dynamics – Mathematics of Falling Dominoes

- Processes need support to keep running (energy, human resources, material, information, input from other processes, etc.)
- h_{it} total support received by process i at time t

$$h_{it} = h_i^* - \sum_j J_{ij} n_{jt} + x_{it}$$

  - h_i^* support in fully functional environment
  - J_{ij} support to process i provided by process j
  - x_{it} random (e.g. Gaussian white noise).

- Process i will fail, if the total support for it falls below a critical threshold (if $h_{it} \leq 0$ – domino falls, if kicked too strongly)

$$n_{it+1} = \Theta\big(-h_{it}\big) = \Theta\Big(\sum_j J_{ij} n_{jt} - h_i^* - x_{it}\Big)$$

- Because of the random noise x_{it}, failure is a probabilistic event.

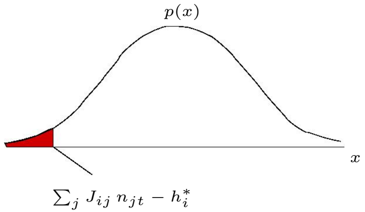
Probability that a Domino Falls

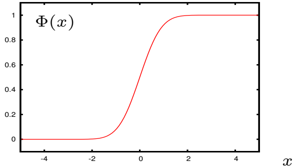
- Dynamics

$$n_{it+1} = \Theta\Big(\sum_j J_{ij} n_{jt} - h_i^* - x_{it}\Big)$$

- Probability of failure/probability of domino falling

$$\text{Prob}\big(n_{it+1} = 1\big) = \text{Prob}\big(x_{it} < \textstyle\sum_j J_{ij}\, n_{jt} - h_i^*\big) \equiv \Phi\big(\textstyle\sum_j J_{ij}\, n_{jt} - h_i^*\big)$$





- Unconditional and conditional probability of failure

$$p_i = \Phi(-h_i^*) \quad , \qquad p_{i|k} = \Phi\Big(J_{ik} - h_i^*\Big)$$

A Simple Homogeneous Process Network

- Recall dynamics
$$n_{it+1} = \Theta\Big(\sum_j J_{ij} n_{jt} - h_i^* - x_{it}\Big)$$
- Large homogeneous system $1 \leq i \leq N$; $(N \gg 1)$.
  - Uniform all-to-all couplings $J_{ij} = J_0/N$

$$\Rightarrow \quad \sum_j J_{ij} n_{jt} = \frac{J_0}{N} \sum_j n_{jt} = J_0 m_t$$

  - Dynamics depends only on fraction of failed nodes.

$$n_{it+1} = \Theta\Big(\sum_j J_{ij} n_{jt} - h_i^* - x_{it}\Big) = \Theta\Big(J_0 m_t - h_i^* - x_{it}\Big) .$$

- Then by Law of Large Numbers (assume $h_i^* = h^*$ indep. of i)

$$m_{t+1} = \frac{1}{N} \sum_{i=1}^{N} \Theta\Big(J_0 m_t - h^* - x_{it}\Big) \simeq \Phi\Big(J_0 m_t - h^*\Big)$$

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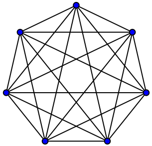

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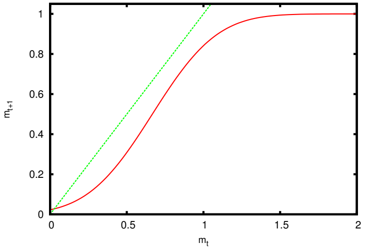
Analysis of the Dynamics

- Iterated function dynamics

$$m_{t+1} = \Phi\Big(J_0 m_t - h^*\Big)$$

- Analyze the behaviour as a function of the parameters J_0 and h^*



Graphical anlysis of stationary solution $m = \Phi(J_0 m - h^*)$ for $h^* = 2$ and $J_0 = 3$

- By increasing J_0, can change from system with only low-m, via system with coexisting low-m and high-m states, to system with only high-m states.

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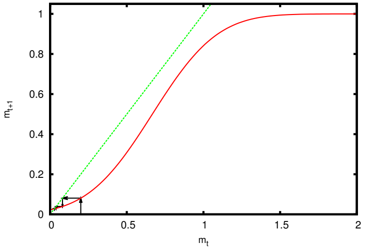

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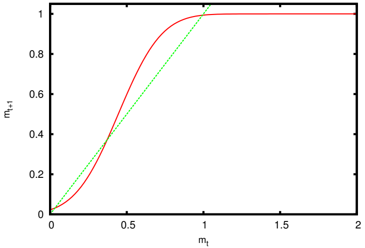

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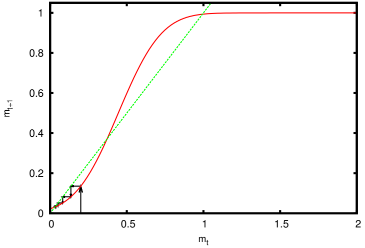

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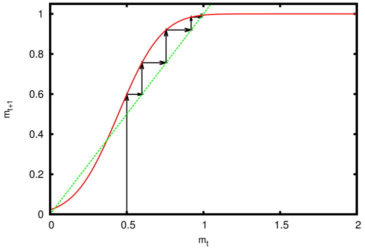

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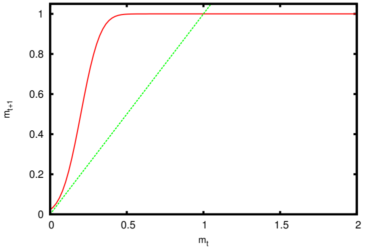

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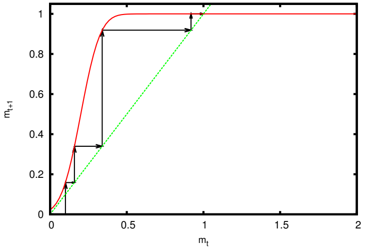

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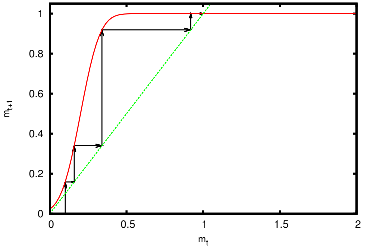

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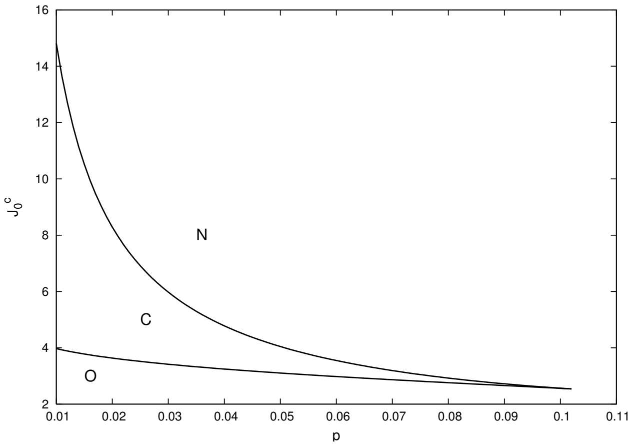
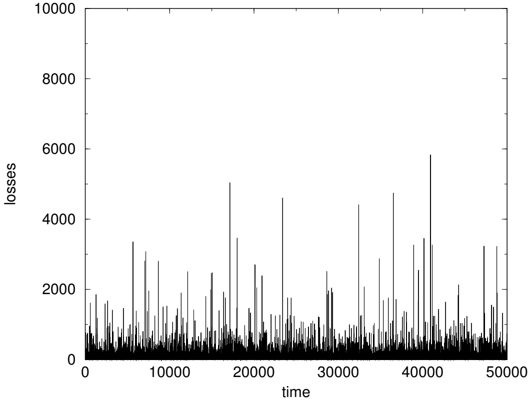


Phase diagram of the OR problem. From K Anand and RK, Phys Rev E**75** (2007)

Spontaneous Breakdown



Losses from operational risks in a network of 100 processes: J_0 such that low-m solution is stable

- Spontaneous breakdown of meta-stable functioning solution possible in finite systems

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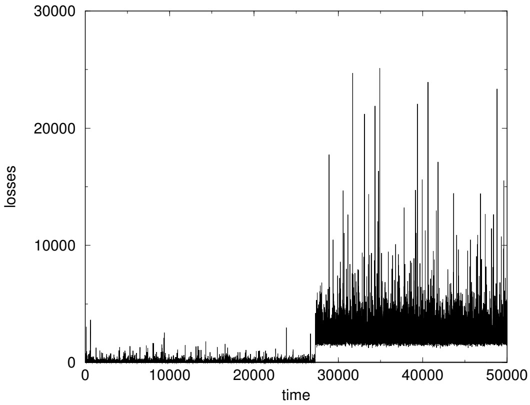

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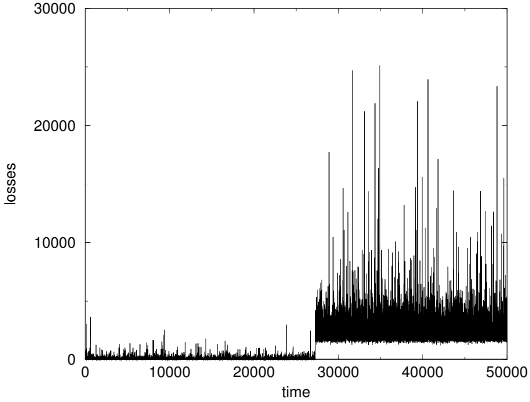

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Credit Risk — Interacting Companies

- Risk arising from the possibility of obligors going bankrupt or from changes in 'credit quality' (⇒ credit trading)
- Look at influence of defaults only ⇒ idealised two state model
  - company can be either up and running ($n_i = 0$)
  - or defaulted ($n_i = 1$)
  - Probability of default heterogeneous across the economy
  - mutual impacts of defaults heterogeneous across the economy
  - ⇒model defined on random graph
- Dynamics: Companies need "orders" (support, cash inflow) to maintain wealth and avoid default
  - h_{it} total wealth of company i at time t,

$$h_{it} = h_i^* - \sum_j J_{ij} n_{jt} - x_{it}$$

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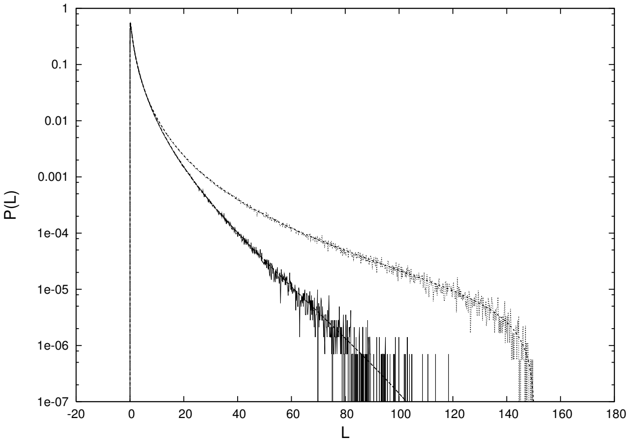
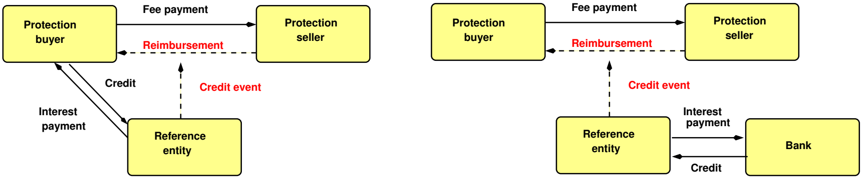


Loss distributions in the credit risk problem for a heterogeneos economy
with and without interactions taken into account. From: JPL Hatchett and RK, J Phys A**39** (2006)

Outline

Mechanics of CDS



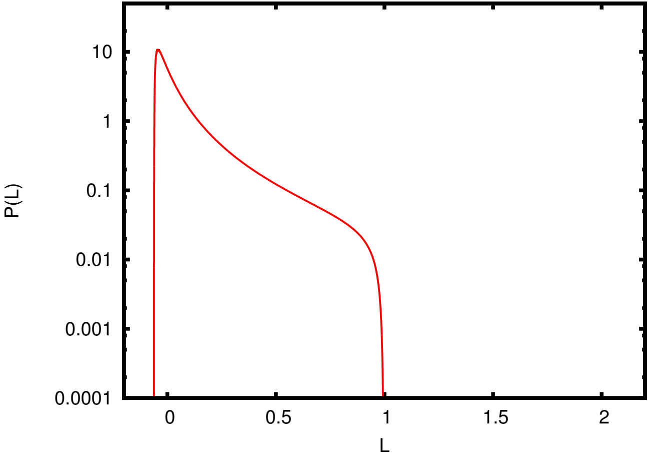
Mechanics of CDS contracts used for hedging and speculation.

CDS

- are used to manage credit risk (hedging), and for speculation
- are zero-sum games
- create additional 'three-particle' contagion channels
- amplify contagion in times of stress, and if used to expand loan books.

Unhedged Lending

- Starting point: no CDS



Unhedged lending: baseline scenario. From S. Heise and RK, Eur Phys J B **85** (2012)

Effect of doubling loan books with firms , doubling, but half-half firm & inter bank

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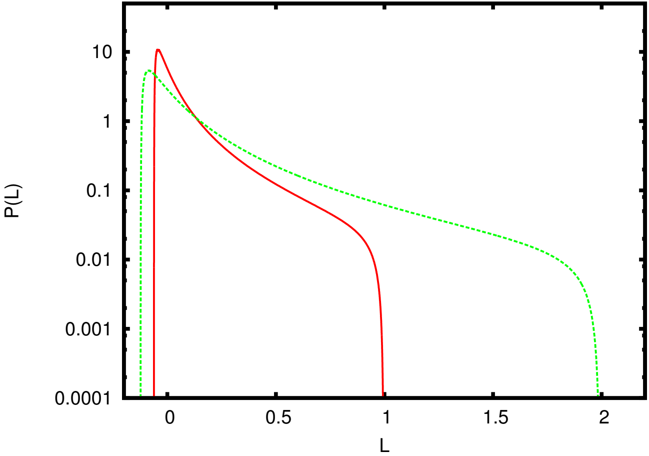

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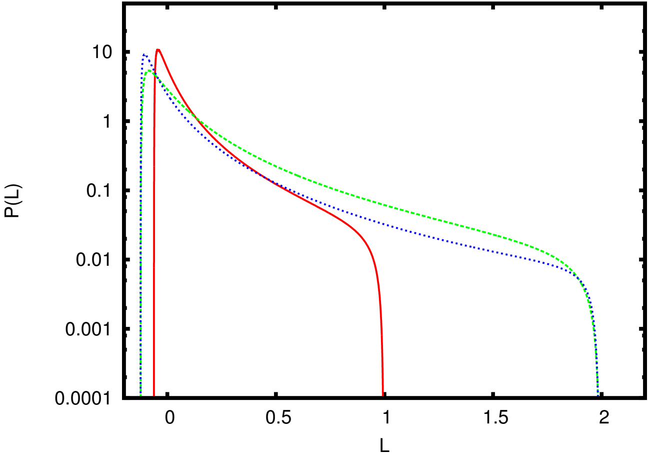

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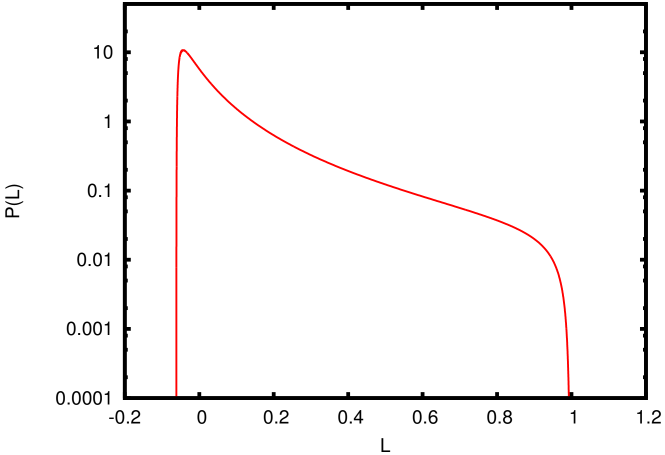
Hedging Exposures — Losses



Scenario 1: the effect of CDS, hedging exposures **within banking sector**, From S. Heise and RK, Eur Phys J B **85** (2012)

unhedged base-line scenario, 1/3 hedged, 2/3 hedged ⇔ **CDS are zero-sum game**.

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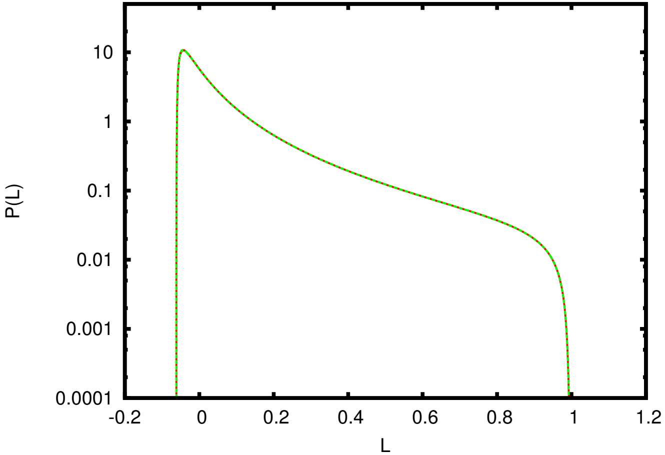

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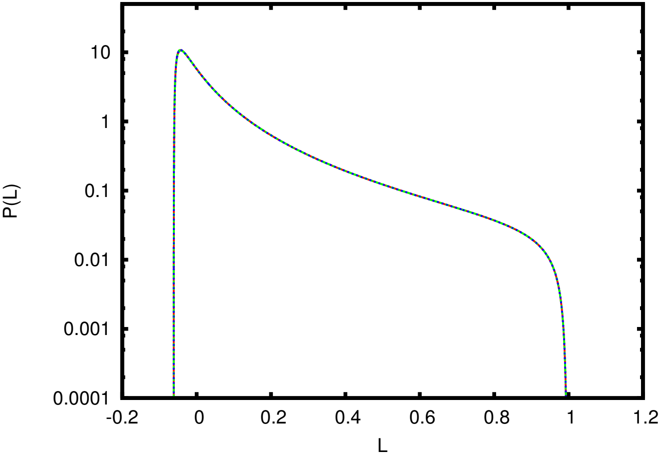

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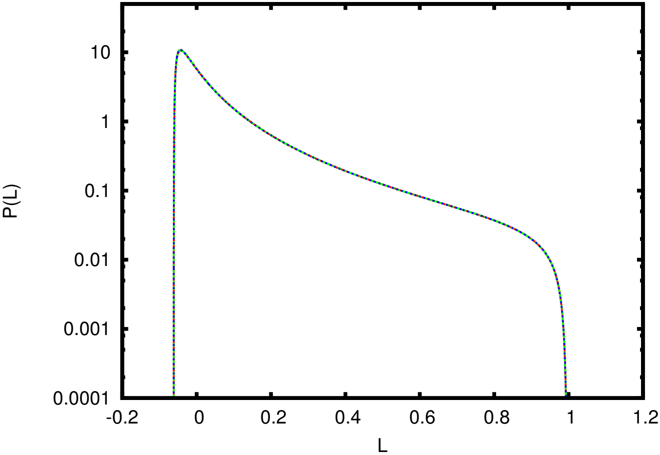

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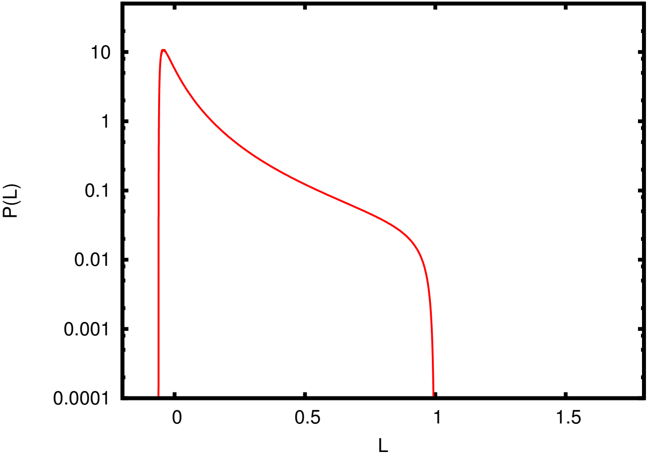
Hedging Increased Exposures with Insurers



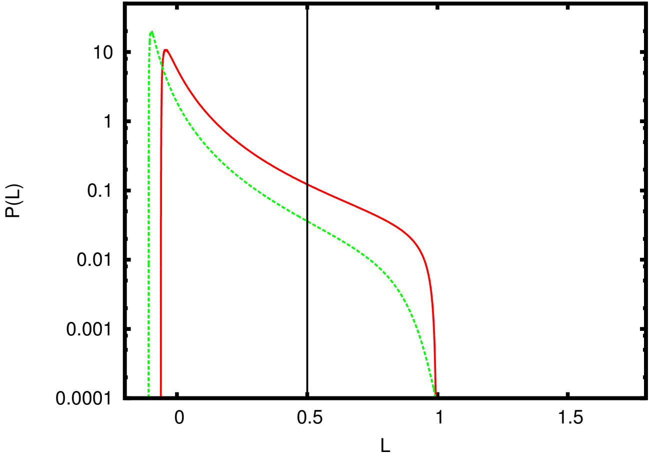
Scenario 2: Unhedged lending: baseline scenario (losses in banking sector). From S. Heise and RK, Eur Phys J B **85** (2012)

Effect of doubling the size of loan books, hedging half of original exposures with banks, the remainder with with insurers, and naively expected maximum loss.

Effect of tripling the size of loan books, hedging all additional exposures with insurers

Note: **incentives and dangers** of this strategy!

Hedging Increased Exposures with Insurers



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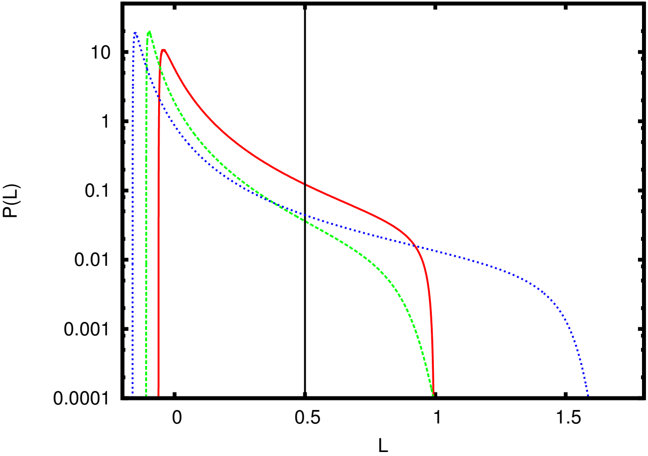

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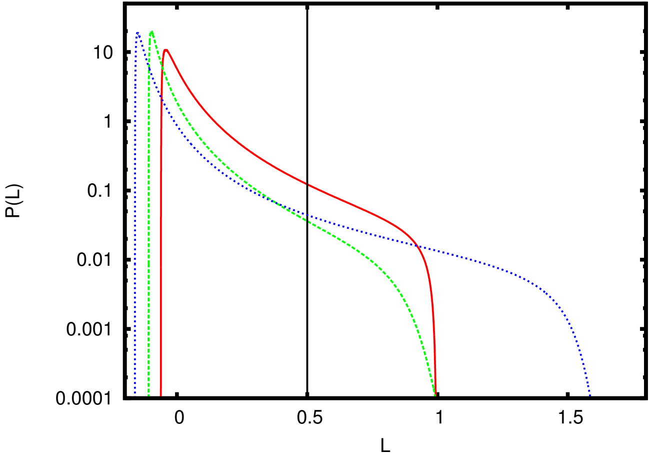

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Outline

Blackouts in Power Grids

North America Blackout - 14 August 2003, triggered 4:10 pm.

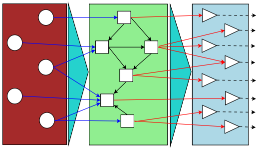
- Large blackouts extremely costly to econmies. Economic damage of North America blackout in 2003 estimated at \$7–10 bn.

Analysing Risk in Power Grids (DC)

- Power flows (currents I_{ij}) minimise Ohms dissipation

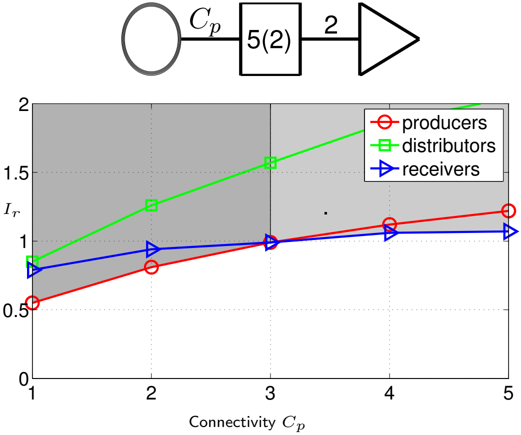
$$D = \sum_{(i,j)} R_{ij} I_{ij}^2 \qquad R_{ij} \text{ line resistance}$$

- with conventions for resistances $R_{ij} = R_{ji}$, and currents $I_{ij} = -I_{ji}$.
- Minimisation subject to constraints
  - production nodes: $\forall\ p$: $\sum_d c_{dp} I_{dp} = I_p$
  - distributon nodes: $\forall\ d$: $\sum_i c_{di} I_{di} = 0$
  - receiver nodes: $\forall\ r$: $\sum_d c_{rd} I_{rd} = I_r$
  - finite link capacity: $|I_{ij}| \leq I_{ij}^c$



Modular structure of a power grid.

Phase Diagram



Critical loads I_r as function of production recources (Connectivity C_p)

Outline

Summary

- Found that process networks can be destabilized by large degrees of interdependency (large J_0) even if all processes are very reliable (large h^*).
- For intermediate levels of dependency (intermediate J_0), functioning and dysfunctional states of the system coexist.
- In systems with finite N, a functioning state can spontaneously switch to the dysfunctional state (without an apparent 'big' perturbation.)
- Results qualitatively unchanged for heterogeneous networks (not all-to-all interactions, heterogeneous levels of reliability, heterogeneous mutual dependency)
- Similar methods for credit risk $\Rightarrow$ ('fat tailed' loss distributions). Crises much more frequent than anticipated if interactions are neglected.
- Credit derivatives (CDS) can destabilise a system.
- Can analyze capacity of power-grids (critical loads).

Thank you!

More on this:
http://www.mth.kcl.ac.uk/~kuehn/riskmodeling