The Gauss Map and Moduli of Abelian Varieties

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Abelian varieties play an important role in algebraic geometry. They are smooth, algebraic group varieties with a rich structure. For example, for a curve X over a field k of genus g, there exists an abelian variety J such that $J(X_K) = \mathrm{Pic}^0(X_K)$ for all extensions K/k with $X(K) \neq \emptyset$, called the *Jacobian* of X, which encodes all the properties of X itself. The problem of determining which abelian varieties are in fact Jacobians of curves has become known as the *Schottky problem.* Since this problem deals with all (principally polarized) abelian varieties, it is natural to consider it within the framework of $\mathcal{A}_g$, the moduli stack of principally polarized abelian varieties (ppav), but for my talk it suffices to restrict attention to its analytic equivalent, which is a coarse moduli space. The approach of Andreotti and Mayer to the Schottky problem was to study the singular locus of the *theta divisor* of a ppav, a more or less canonical effective Cartier divisor coming from an ample line bundle. This led to the definition of $N_k \subset \mathcal{A}_g$, the locus of ppav whose theta divisor has a singular locus of dimension at least k. It turns out that N_0 (the locus of ppav whose theta divisor is not smooth) has two irreducible components, one of which is the object of study of my master's thesis: the locus θ_{null} of ppav whose theta divisor has a singularity at a point of order 2. It has a stratification

$$\theta_{null}^0 \subset \theta_{null}^1 \subset \cdots \subset \theta_{null}^{g-1} \subset \theta_{null}^g = \theta_{null},$$

and I will demonstrate a result I found on the dimension of the closure of these loci in the so called *partial toroidal compactification* $\overline{\mathcal{A}}_g^1$ of $\mathcal{A}_g$, using the geometry of the *Gauss map* of a theta divisor.