COMMON PART ESTIMATION OF ACOUSTIC FEEDBACK PATHS IN HEARING AIDS 
OPTIMIZING MAXIMUM STABLE GAIN
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ABSTRACT
The computational complexity and convergence speed of adaptive feedback cancellation algorithms depend on the number of adaptive parameters used to model the acoustic feedback path. To reduce the number of adaptive parameters it has been proposed to decompose the acoustic feedback path as the convolution of a (time-invariant) common part and a (time-varying) variable part. Typically the problem of estimating all the required coefficients has been formulated as a least-squares optimization problem. In contrast, in this paper we propose to formulate the estimation problem as a minmax optimization problem and show how this is associated with the maximum stable gain of a hearing aid. Experimental results using measured acoustic feedback paths from a two-microphone behind-the-ear hearing aid show that the proposed minmax optimization outperforms the least-squares optimization in terms of maximum stable gain. Furthermore, the robustness of proposed common part decomposition for different feedback paths is evaluated.

Index Terms— acoustic feedback cancellation, common part modeling, invariant part extraction, minmax optimization, hearing aids

1. INTRODUCTION
The number of hearing impaired persons supplied with open-fitting hearing aids has been steadily increasing over the last years. Although largely alleviating problems related to the occlusion effect, open-fitting hearings aids are especially susceptible to acoustic feedback, often perceived as whistling or howling. This demands for robust and fast-adapting feedback cancellation algorithms.

While different strategies can be used to reduce the acoustic feedback (see e.g. [1, 2]), adaptive feedback cancellation (AFC) is one of the most promising approaches. In AFC an adaptive filter is used to estimate the impulse response (IR) of the acoustic feedback path, theoretically allowing for perfect cancellation of the feedback signal [1]. In general, the convergence speed and the computational complexity of the adaptive filter is determined by the number of adaptive parameters [3, 4]. In order to reduce the number of adaptive parameters and hence reduce the complexity and improve the convergence speed it has been proposed [5, 6, 7, 8] to model the acoustic feedback path as the convolution of two filters: a time-invariant common part and a (time-varying) variable part. While the time-invariant common part models parts that are common in several acoustic feedback paths, e.g., transducer characteristics and individual ear canal characteristics, the time-varying variable part enables to track fast changes, e.g., in the presence of a handheld telephone.

For modeling the common part different filters have been proposed, i.e., an all-zero filter [6], an all-pole filter [9] and the general pole-zero filter [7]. For practical reasons the variable part is commonly modeled using an all-zero filter to enable easy and stable adaptation using standard adaptive filtering techniques.

Assuming that the IRs of at least two acoustic feedback paths are available (e.g., using measurements), the common and variable parts are usually estimated by minimizing a least-squares (LS) cost function [6, 7, 8, 9]. This corresponds to optimizing the misalignment between the true and estimated IRs, which is often used to quantify the performance of adaptive filters. However, although these approaches allow for good performance in terms of the misalignment, the closed-loop transfer function of the hearing aid may already be unstable for smaller gains. Therefore, in this paper we propose to directly optimize the maximum stable gain (MSG) [10], i.e., the maximum applicable gain that leads to a stable closed-loop response of the hearing aid. After reviewing the commonly used technical measures to assess the performance of AFC algorithms in Section 3, in Section 4 we show how the problem of maximizing the MSG can be formulated as a minmax optimization problem. The resulting non-linear minmax cost function is then minimized by an alternating optimization procedure. Experimental results using measured acoustic feedback paths from a two-microphone behind-the-ear hearing aid indicate that: 1) using the proposed minmax optimization procedure to estimate the common pole-zero filter of acoustic feedback paths yields an increase in MSG compared to existing LS techniques, and 2) a reduction of the number of variable part parameters can be achieved even for unknown feedback paths.

2. PROBLEM FORMULATION
Consider a single-input-multiple-output (SIMO) system with \( M \) outputs as depicted in Figure 1a. Such a SIMO system arises, e.g., in a single-loudspeaker multiple-microphone setup in a multi-microphone hearing aid. The \( m \)-th output signal \( Y_m(z) \) is related to the input signal \( X(z) \) by the \( m \)-th acoustic transfer function (ATF) \( H_m(z) \) as \( Y_m(z) = H_m(z)X(z) \). We assume that the true ATFs \( H_m(z) \) are causal all-zero filters of finite order \( N_m^v \) each. To reduce the number of coefficients required to model all ATFs, the following approximation (depicted in Figure 1b) is introduced:

\[
\begin{bmatrix}
H_1(z) \\
\vdots \\
H_M(z)
\end{bmatrix}
\approx
\begin{bmatrix}
\hat{H}_1(z) \\
\vdots \\
\hat{H}_M(z)
\end{bmatrix}
= \hat{H}^c(z)
\begin{bmatrix}
\hat{H}_1^c(z) \\
\vdots \\
\hat{H}_M^c(z)
\end{bmatrix},
\]

where \( \hat{H}^c(z) \) denotes the microphone-independent common part and \( \hat{H}_m^c(z) \) the microphone-dependent variable parts. Assuming the common part can be modeled as a pole-zero filter with \( N_m^c \) poles.

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and $N_c^v$ zeros and each of the $M$ variable parts can be modeled as an all-zero filter with $N_v^c$ zeros, their transfer functions are defined as

$$
\hat{H}^e(z) = \frac{B^e(z)}{A^e(z)} = \frac{\sum_{i=1}^{N_v^c} b^e_i z^{-1}}{1 + \sum_{i=1}^{N_c^v} a^e_i z^{-1}},
$$

$$
\hat{H}^v_m(z) = B^v_m(z) = \sum_{l=0}^{N^v} b^v_{ml} z^{-1},
$$

where $a^e[i]$, $b^e[j]$ and $b^v_m[j]$ denote the coefficients of the polynomials associated with the common poles, common zeros and variable zeros, respectively. Note that $a^e[0] = 1$, i.e., $A^e(z)$ is assumed to be a monic polynomial. The coefficients in vector notation are defined as

$$
h_m = [h_m[0], h_m[1], \ldots, h_m[N^v_c]]^T,
$$

$$
a^c = [a^c[1], a^c[2], \ldots, a^c[N^c_v]]^T,
$$

$$
b^c = [b^c[0], b^c[1], \ldots, b^c[N^v_c]]^T,
$$

$$
b^v_m = [b^v_m[0], b^v_m[1], \ldots, b^v_m[N^v]]^T,
$$

where $[\cdot]^T$ denotes transpose operation. We also define the concatenation of the variable part coefficient vectors $b^v_m$ as

$$
b^v = [(b^v_1)^T, (b^v_2)^T, \ldots, (b^v_m)^T]^T.
$$

### 3. TECHNICAL MEASURES OF FEEDBACK CANCELLATION PERFORMANCE

To characterize the performance of AFC algorithms two measures are commonly used [1, 6]: the normalized misalignment $\epsilon$ and the MSG $\mathcal{M}$. The normalized misalignment $\epsilon_m$ in the $m$-th microphone indicates the performance of the estimated feedback paths in terms of its Euclidean distance to the true feedback paths, i.e.,

$$
\epsilon_m = \frac{\|h_m - \hat{h}_m\|_2}{\|h_m\|_2},
$$

where $\hat{h}_m$ is the estimated IR.

The MSG $\mathcal{M}$ indicates the gain that can be applied in a hearing aid until instability of the closed-loop system and hence howling or whistling occur. Assuming a broadband hearing aid gain, the MSG $\mathcal{M}_m$ of the $m$-th feedback path can be computed as [10]

$$
\mathcal{M}_m = 20 \log_{10} \frac{1}{\max_{0 \leq \Omega \leq \pi} |H_m(e^{j\Omega}) - \hat{H}_m(e^{j\Omega})|},
$$

where $\Omega$ denotes normalized frequency. Note that the system is actually only unstable if also the phase at the frequency of the maximum absolute difference is a multiple of $2\pi$ [11] and hence (10) provides the worst-case assumption.

### 4. OPTIMIZING THE MAXIMUM STABLE GAIN

In [6, 7, 8, 9] LS procedures minimizing the misalignment in (9) have been presented to estimate the coefficients vectors $a^c$, $b^c$, $b^v$ of the common and variable parts. In contrast in this section we propose to estimate the coefficient vectors $a^c$, $b^c$, $b^v$ of the common and variable parts in order to maximize the MSG in (10). This leads to a novel optimization procedure for the approximate SIMO system depicted in Figure 1b. Maximizing $\mathcal{M}_m$ in (10) corresponds to minimizing the maximum absolute difference of the frequency response of the true and estimated ATFs. Thus maximizing $\mathcal{M}_m$ for all $M$ IRs of the considered approximate SIMO system can be formulated as a minmax optimization problem, where we aim to minimize

$$
J_{MM}(a^c, b^c, b^v) = \max_{0 \leq \Omega \leq \pi} \max_{1 \leq m \leq M} |\tilde{E}_m(e^{j\Omega})|,
$$

with the so-called output-error $\tilde{E}_m(e^{j\Omega})$ defined as

$$
\tilde{E}_m(e^{j\Omega}) = H_m(e^{j\Omega}) - H_m(e^{j\Omega})
$$

$$
= H_m(e^{j\Omega}) - B^v_m(e^{j\Omega}) B_m(e^{j\Omega}).
$$

For conciseness we will omit the variable $e^{j\Omega}$ in the remainder of this paper. Note that the cost function in (11) aims at maximizing the minimum $\mathcal{M}_m$ for the considered set of $M$ IRs, which is a reasonable assumption since this will presumably dominate the MSG in, e.g., multi-microphone hearing aids. The output-error $\tilde{E}_m$ is non-linear in $A^c$, $B^c$ and $B^v_m$ and thus minimization of (11) is not straightforward. To ease the optimization we rewrite the output-error as

$$
\tilde{E}_m = \frac{1}{A^c} E_m = \frac{1}{A^c} (A^c H_m - B^v B^v_m),
$$

where $E_m$ is the so-called equation-error which is non-linear in only $B^c$ and $B^v_m$. This formulation suggests the following iterative optimization procedure to approximate (11) where at iteration $i$ we aim to minimize

$$
J_W(a^c_i, b^c_i, b^v_i) = \max_{0 \leq \Omega \leq \pi} \max_{1 \leq m \leq M} \frac{1}{A^c_{m,i-1}} |E_{m,i}|,
$$

where the equation-error $E_{m,i}$ is weighted by the inverse frequency-response of $A^c_{m,i-1}$ from the previous iteration. Thus at convergence ideally $A^c_i \approx A^c_{i-1}$ and hence $(A^c_{i-1})^{-1} E_{m,i} \approx E_{m,i}$ approximating the desired output-error minimization. This approach is similar to the well-known Steiglitz-McBride method [12] for LS identification of single-input-single-output (SISO) systems, which was also successfully applied to LS estimation of the approximate SIMO system in [8]. It should be noted that the Steiglitz-McBride method [12] as well as the approach in [8] use iterative LS estimation whereas minimizing (15) yields an iterative minmax optimization problem. A similar iterative optimization procedure was presented in [13] in the context of SISO digital filter design.

The cost function in (15) is non-linear in $B^c$ and $B^v_m$ and hence yields a non-linear optimization problem. However, to minimize (15) the problem can be split into two separate convex subproblems. Hence, we employ a two-step alternating optimization procedure similar to the two-step alternating optimization procedure proposed to minimize the LS equation-error in [7] or the weighted LS equation-error in [8].
Step 1: We assume \( a^c \) and \( b^c \) fixed to be fixed to values from the previous iteration and estimate the coefficient vector \( b^e \) that minimizes

\[
J_{WM}(b^e) = \max_{0 \leq \Omega \leq \pi} \frac{1}{|A_{i-1}^{c}|} |E_{m,i}^{e}|
\]  
(16)

with

\[
E_{m,i}^{e} = A_{i-1}^{c} H_m - B_{i-1}^{c} B_{m,i}^{e}.
\]  
(17)

This equation-error can be written in the time-domain as [7]

\[
e_{m,i}^{e} = \tilde{h}_m + \tilde{H}_m a_{i-1}^c - \tilde{B}_{i-1}^{c} b_{m,i}^{e}
\]  
(18)

where \( \tilde{h}_m \) is the \((\tilde{N}_b^h + N_b^c + 1)\)-dimensional zero-padded version of \( h_m \) with \( \tilde{N}_b^h = \max\{N_b^h, N_b^c + N_b^c\} + 1 \), \( \tilde{H}_m \) is the \((\tilde{N}_b^h + N_b^c + 1) \times (N_b^c + 1)\)-dimensional convolution matrix of \( h_m \) and \( \tilde{B}_{i-1}^{c} \) is the \((N_b^h + N_b^c + 1) \times (N_b^c + 1)\)-dimensional convolution matrix of \( b_{i-1}^c \). The minimization of (16) can be reformulated as the following linear program (LP) [14]

\[
\begin{align}
\min_{t,b^c} & \quad t \\
\text{s.t.} & \quad t \geq 0 \\
& \quad |c^T(\Omega) \left( \frac{1}{A_{i-1}^{c}(q^{-1})} e_{m,i}^{v} \right) | \leq t \quad \forall m, \Omega \\
& \quad |s^T(\Omega) \left( \frac{1}{A_{i-1}^{c}(q^{-1})} e_{m,i}^{v} \right) | \leq t \quad \forall m, \Omega
\end{align}
\]  
(19a)

with \( q^{-1} \) the unit delay operator [1], i.e., \( q^{-1} h_m[k] = h_m[k-1] \), \( k \) the sample index, hence essentially a filtering operation on the equation-error \( e_{m,i}^{v} \) is performed and

\[
c(\Omega) = [1 \cos \Omega \ldots \cos(\tilde{N}_b^h + N_b^c)\Omega]^T,
\]  
(20)

\[
s(\Omega) = [0 \sin \Omega \ldots \sin(\tilde{N}_b^h + N_b^c)\Omega]^T.
\]  
(21)

The expressions in (19c) and (19d) essentially compute the absolute values of the real and the imaginary part of the frequency response of the weighted equation-error in (16). This LP can then be efficiently solved using convex optimization methods, e.g., CVX [15, 16].

Step 2: We assume \( b^e \) fixed to be fixed to the value from the previous step and estimate the coefficient vectors \( a^c \) and \( b^e \) that minimize

\[
J_{WM}(a^c, b^e) = \max_{0 \leq \Omega \leq \pi} \frac{1}{|A_{i-1}^{c}|} |E_{m,i}^{e}|
\]  
(22)

with

\[
E_{m,i}^{e} = A_{i}^{c} H_m - B_{i}^{e} B_{m,i}^{e}.
\]  
(23)

Note that stability of the IRs estimated by minimizing (22) is not guaranteed. Hence, the location of the poles, i.e., the roots of \( A_{i}^{c}(z) \), needs to be constrained. In general, stability of a causal pole-zero filter is guaranteed if all poles are located strictly inside the unit-circle. A sufficient condition for stability of \( \frac{1}{A_{i}^{c}(z)} \) is that the real part of its frequency response is strictly positive [17], i.e.,

\[
\Re\{A_{i}^{c}\} > 0, \quad \forall \Omega.
\]  
(24)

5. EXPERIMENTAL EVALUATION

In this section the proposed minmax estimation procedure maximizing the MSG is evaluated and compared to the LS estimation procedure of [8] minimizing the misalignment. Acoustic feedback paths were measured using a two-microphone behind-the-ear hearing aid with an open-fitting ear mold on a dummy head with adjustable ear canals [18]. The IRs were sampled at \( f_s = 16 \text{ kHz} \) and truncated to order \( N_b^h = 99 \). The frequency and phase responses of four acoustic feedback paths used in the evaluation are depicted in Figure 2. Two IRs \( (m = 1, 2) \) were measured without obstruction and two IRs \( (m = 3, 4) \) were measured with a telephone receiver in close distance.

The performance was evaluated in terms of the MSG defined in (10). Evaluations were made for the following set of parameters: \( N_m^c, N_m^e \in \{ 0, 4, 8, \ldots, 24 \} \), \( N_m^e \in \{ 6, 12, \ldots, 48 \} \). In the...
Fig. 3. MSG improvements of the proposed minmax optimization (MM) and LS optimization of [8] using $N^c = 8$ compared to using $N^c = 0$.

following $N^c$ denotes the number of common part parameters, i.e., $N^c = N_p^c + N_z^c$. Note that different combinations of $N_p^c$ and $N_z^c$ can lead to the same value of $N^c$. We used $N = 1025$ discrete frequencies and $\delta = 10^{-6}$ to control the stability margin in (27e). Since it was experimentally found that a good initialization for the proposed minmax optimization procedure is crucial, we used the solution obtained from the LS approach proposed in [8] to initialize the coefficient vectors $a^c$ and $b^c$.

5.1. Improved maximum stable gain

Figure 3 shows exemplary average MSG improvements for IRs $m = 1, 2$ when using $N^c = 8$ compared to using $N^c = 0$, i.e., not using any common part information, for the proposed minmax (MM) optimization and the LS optimization of [8]. The average has been computed for a given combination of common part parameters $N_p^c$ and $N_z^c$ across both IRs. Note that three different choices of $N_p^c$ and $N_z^c$ from the considered parameter set lead to $N^c = 8$ where only the combination with the maximum improvement is depicted in Figure 3. The results indicate that using common part knowledge improves the MSG for both the proposed minmax optimization and the LS optimization. In general these large improvements are expected due to the fact that by including common part information the total number of available parameters is increased. In addition improvements decrease towards larger $N^c$ since most the energy of the IRs is located within the first 50 samples. Nevertheless, the proposed minmax optimization clearly outperforms the LS optimization of [8] in terms of MSG.

To compare the two optimization procedures for different choices of $N^c$ Figure 4 depicts the MSG improvement of the minmax optimization compared to the LS optimization for different choices of $N^c$. Here the average improvement is computed across the different combinations of $N_p^c$ and $N_z^c$ leading to the same $N^c$ and both IRs ($m = 1, 2$). Furthermore the error bars indicate the minimum and maximum improvement. Again the minmax optimization procedure outperforms LS optimization procedure in terms of MSG with average improvements as large as 4dB. Improvements appear to be rather constant in the range of 2-3 dB across different values of $N_z^c$ and tend to increase for larger values of $N^c$ and $N_z^c$.

5.2. Robustness to unknown feedback paths

Modeling of the acoustic feedback path as the combination of a common part and variable part(s) is motivated by the goal to reduce the number of adaptive parameters, i.e., the number of parameters $N_p^v$ used to model the variable part, to reduce complexity and increase convergence speed.

Here we thus investigate the robustness of the common part estimate to unknown feedback paths, i.e., feedback paths that were not included during optimization, in terms of the minimum number of variable part coefficients $N_p^v$ required to achieve a desired MSG. Compared to the IRs depicted in Figure 2 four additional IRs were included in the evaluation. Two IRs ($m = 5, 6$) were measured after repositioning of the hearing aid without any obstruction (similar to IRs $m = 1, 2$), while two IRs ($m = 7, 8$) where measured with a telephone receiver at a distance of approximately 24cm. First the common part $H^c(z)$ is estimated with the proposed minmax optimization procedure using only IRs $m = 1, 2$. Keeping the common part fixed the variable parts for IRs $m = 3, 4, m = 5, 6$ and $m = 7, 8$ are then estimated using (16).

Figure 5 depicts the minimum number of variable part parameters $N_p^v$ as a function of $N^c$ given a desired MSG of 40 dB for different acoustic feedback paths.

In this paper the problem of estimating a common pole-zero filter from a set of measured acoustic feedback paths was formulated as a minmax optimization problem aiming to maximize the MSG. The resulting non-linear cost function was minimized using alternating optimization techniques. Experimental results using measured acoustic feedback paths indicate that the proposed estimation procedure leads to a higher MSG compared to existing LS optimization procedures minimizing the misalignment. Furthermore, for a desired MSG the proposed minmax optimization is able to reduce the number of the variable part parameters even for unknown feedback paths.
7. REFERENCES


