Reliability Assessment of Coalitions for the Provision of Ancillary Services

Dissertation zur Erlangung des Grades eines Doktors der Ingenieurwissenschaften

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To Frida
Foreword

This thesis has resulted during my time at OFFIS – Institute for Information Technology in Oldenburg. During this time, I was able to gain a lot of knowledge and skills not only because of the interesting tasks in which I was involved but also because of the freedom and confidence that has been shown to me. This was the basis for an environment which enabled me to work on my PhD-project. The realisation has not always been easy and would not have been possible without the support of my supervisors, colleagues, family and friends.

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Zusammenfassung

Die Integration kleiner, verteilter Anlagen basierend auf erneuerbaren Energien (EE) hat Veränderungen zur Folge, die den Betrieb des elektrischen Energieversorgungssystems betreffen. Diese Einheiten verdrängen konventionelle Kraftwerke und müssen in Folge dessen deren Aufgaben übernehmen. Dazu gehört neben der Energieversorgung auch die Bereitstellung netzstützender Systemdienstleistungen, wie beispielsweise Primärregelleistung.


Die RelACs-Methode kann in den Prozess der Verbundbildung integriert werden, da sie entsprechende Daten aus der Verbundbildung verarbeitet und eine Bewertung ausgibt, die wiederum von der Verbundbildung interpretiert werden kann. Auf diese Weise ist es möglich sicherzustellen, dass ein Verbund bestimmte Anforderungen an die Zuverlässigkeit erfüllt. Außerdem kann die RelACs-Methode dafür genutzt werden, Empfehlungen für die Verbundbildung zu geben, die die Eingaben für den Verbundbildungsprozess unter verschiedenen Bedingungen betreffen.
Abstract

The introduction of distributed, small-scale renewable power units changes the operation of electrical power systems. These units take over the tasks of conventional power plants in terms of energy supply. This also holds for the provision of system-stabilising ancillary services such as primary control reserves for frequency control.

In order to comply with the vast number of renewable power units, the paradigm of dynamic virtual power plants or coalitions is suitable in order to achieve high scalability. This means units are virtually aggregated in order to mime power plants and as such provide energy or ancillary service products. The ancillary service provision must be reliable in order to guarantee secure system operations, especially if they are provided by units depending on fluctuating resources such as solar irradiation or wind power.

In this thesis, the RelACs-method is introduced to assess coalitions of renewable power units with respect of how reliably they are able to provide ancillary services, particularly for the use case of providing primary control reserves. The RelACs-method has a modular and hierarchical structure. It takes into account different factors influencing the reliability. First, it incorporates uncertainties of individual units as induced by unit failures, the volatility of the units’ power feed-in as they rely on renewable power resources, and uncertainties induced by forecasts. Second, on the level of a coalition, the dependencies between units being subject to the same primary energy carriers are incorporated as well as dependencies resulting from their position in the power grid and failures of operational equipment.

The RelACs-method integrates into the process of coalition forming as it processes the necessary data provided by the coalition and returns a measure that can be handled by the coalition forming process. Thus, it can be guaranteed that the resulting coalition fulfils specific reliability requirements. Furthermore, the RelACs-method is used to derive recommendations for the choice of inputs to coalition forming given different conditions.
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1. Introduction

“The ability to integrate low-carbon and sustainable energy resources is essential to reducing the environmental impact of electricity supply systems.” [11] Fossil fuels or nuclear power plants cannot provide a sustainable power supply as the external costs produced by carbon gas emissions or health risks are high. Furthermore, in case of fossil fuels, scarce and limited resources are used. Thus, Smart Grid policies have emerged in order to establish a framework for the integration of Renewable Power Units (RPUs), i.e. electrical power generating units whose primary energy carrier is a Renewable Energy Source (RES) as e.g. biomass, solar irradiation, or wind speed. The German government for example, has set the political goal of reducing the emission of carbon dioxide by 80% in 2050 compared to 1990 [53]. For this reason, the share of RPU in gross power generation has to be increased. Already in 1990 with the so-called Electricity Feed-In Act the first law in Germany has been enacted to regulate feed-in tariffs and to oblige system operators to purchase power generated by RPU. It founded the basis for the German renewable energy act (EEG)¹ introduced in 2000. Thus, incentives have been created with great success to connect and operate power generating units based on renewable sources.

1.1. Background: Decentralized Power Supply

The share of renewable power units has increased during the past decades. In Germany 2014, the share of RPU in gross electricity production was about 25%, where 9.6% were wind power and 6% power generated by photovoltaic (PV) units [85]. The installed capacity of wind and PV units in German power grids in 2014 was 40 GW and 38 GW, respectively, which for both cases means almost 25% compared to 180 GW of totally installed capacity [1]. The development in other countries has been similar [32]. With the high penetration of RPU in the power system certain challenges arise: the integration of those units in the power system, energy supply and provision of ancillary services by them.

The integration of RPU in the power system is a challenge in order to maintain the security of supply. Many of those units have been connected to the distribution grid that had not been designed to take power by generating units. Moreover, distribution grids are usually operated as a radial system thus not satisfying the n-1 principle. This means that if operational equipment fails, e.g. due to overloading, the units connected to grid nodes in the affected grid section are disconnected from the system and power supply. To overcome these problems, grids might have to be expanded in order to cope with the increase of power transmitted.

¹from German Erneuerbare Energien Gesetz
Another challenge is to provide energy products by RPU and other distributed energy resources (DER). The number of conventional power plants that have been responsible for energy provision has decreased and will decrease further being substituted by RPU and DER. Energy products are tendered at energy markets (see e.g. [94, 95]). Units providing energy products must deliver power according to a setpoint schedule in order to supply the negotiated amount of energy during the stipulated time. Especially in case of RPU exhibiting volatile and stochastic power production like PV and wind power units (or wind units) this becomes a challenge. Those units are dependent on the availability of the underlying RES and thus nondispatchable and subject to uncertainties [39]. Furthermore, market entry barriers must be overcome by units in order to be allowed to participate in tendering on energy markets. This results in the need for coordination of those units. The concept of virtual power plants (VPPs) has been introduced to logically aggregate DER as well as load units and storage devices thus overcoming market barriers or supporting stable grid operation [5]. In the research network Smart Nord² the concept of VPPs has been extended to the concept of dynamic virtual power plants (DVPPs) [26]. “Smart Nord -- Intelligente Netze Norddeutschland” stands for “Smart Grids in northern Germany” and has been an interdisciplinary research network [59].

Within DVPPs units are represented by agents. These are software programs that perceive information via sensors about their surroundings and use this knowledge to make decisions and take actions [92] regarding the operation of the respective unit. DVPPs are not a fixed aggregation of units but product-related. This means units are clustered to DVPPs or synonymously (agent-) coalitions according to the current market situation and their available flexibilities. [25, 27]

Besides the schedule-based energy provision, another essential task is the provision of ancillary service (AS), for example services supporting frequency control, voltage control, system restoration and system control [34, 95]. ASs are services provided to support stable and uninterrupted operation of the system to keep quality, availability and security standards. According to [34] there are the following three options to ensure this: grid connection codes, flexible operational equipment, or “bilateral agreements or market mechanisms”. The last option refers to services provided by system users – incorporating RPU – that must be procured by system operators. The focus of this thesis lies on the topic of the provision of ancillary services by RPU.

1.2. Challenge: Ancillary Services by Renewable Power Units

As pointed out in the previous section, ASs are services provided by system users. As conventional power plants are substituted by DER those units have to provide ancillary services, as well. The frequency and voltage control ancillary services have been established in order to maintain the system state within feasible limits or mitigate the state to a feasible state in case of violations of boundaries whereas the service of system restoration restores the system after faults and interruptions. The objective of system control is

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²The Lower Saxony research network ‘Smart Nord’ acknowledges the support of the Lower Saxony Ministry of Science and Culture through the “Niedersächsisches Vorab” grant programme (grant ZN 2764).
the coordination and operation of the system. [95]

In [34] it has been shown that already today there exist technical solutions for all types of ancillary services to provide a secure and reliable power supply in the future and in [10] it is been shown that DER are technically capable of providing ancillary services. However, it has been suggested to adapt regulations as well as alter or extend prequalification schemes, market settings, and grid connection codes [34].

In order for units to participate in providing ancillary services, they have to fulfil certain prequalifications. The market rules and procurement methods vary for different countries [90]. In case of frequency control it is necessary that power reserves are provided that can be activated as soon as needed – so-called control reserves. Thus, different products for control reserves are tendered at dedicated markets. To ensure these power reserves, units that want to participate in the market competition must first fulfil certain prequalifications, and second, satisfy certain market entry barriers [15]. In case of voltage control it must be assured that compensation for voltage deviations is locally available. Above that there are grid connection codes, amongst others, specifying the behaviour of units connected to the power system in case the frequency or voltages are outside the feasible limits [31].

In the research network Smart Nord, agent-based strategies have been investigated and developed for coordination and control of RPU for providing both schedule-based energy products and ancillary service products [59]. In case of ASs the focus has been laid on system stabilizing ancillary services, i.e. services needed to keep or to restore a stable system state, a state that maintains certain boundaries. These services are frequency and voltage control ancillary services.

As in the case of schedule-based energy products, a challenge is the volatile and stochastic character of RPU whose sources are fluctuating, e.g. solar and wind power. During the process of forming coalitions, i.e. when planning what units are able to contribute to AS products, they must determine their available reserves and flexibilities. However, this can only be done based on forecasts that inherently contain errors. Thus, the contributions are subject to uncertainties. Furthermore, temporal restrictions must be fulfilled regarding both the coalition forming for AS products as well as the delivery of AS products.

In order to guarantee reliable system operations, it must be assured that ASs can be provided in a reliable way as well. Checking prequalifications of a unit is done to prove that it can follow a certain profile or behaviour. This can easily be done with conventional power plants as their primary energy carriers can be controlled and dispatched. However, this is not the case for RPU. Furthermore, many RPU are connected to the distribution grid that is not n-1 secure. Hence, failures of operational equipment result in the disconnection of grid users – consumers and generators. As a consequence, the reliable provision of ancillary services of RPU connected to distributions grids is also subject to the reliability of operational equipment. Thus, new methods are necessary to assess the reliability of ancillary service products provided by RPU that take account the mentioned issues into account.
1.3. Objective: Reliability Assessment of Coalitions

In the previous sections it has been argued that in order to form DVPPs or aggregations of DER that are able to reliably provide ancillary services, certain aspects have to be taken into consideration. The following items sum up these aspects and corresponding presumptions made within the research project that is presented in this thesis.

**Uncertainties** resulting from forecast errors: Those uncertainties increase with increasing time horizons. The provision of ancillary services is system and time-critical. Thus, the contributions that units make must be guaranteed to a certain extent. Taking into account uncertainties when determining units’ contributions reduces the risk of not being able to provide services to the demanded extent.

**Dependencies** between units: The power feed-in of RPU like PV and wind units are dependent of weather conditions. Units of the same technology that are spatially close to each other are subject to similar weather conditions. Thus, their feed-in behaviour is not independent. This also applies for predictions of these units’ behaviour and consequently for the prediction errors.

**Unit failures** of units contributing to ancillary service products: If units are subject to failures they cannot provide their contribution to the extent demanded.

**Grid reliability** with respect to failures of operational equipment: Since distribution grids are usually operated as radial systems they do not comply with \( n-1 \) security. Thus, failures of operational equipment result in the disconnection of units. As a consequence, they cannot provide ancillary services.

A measure to describe the reliability of a coalition should incorporate these aspects. Such a measure must be designed such that it can be incorporated in the process of forming a DVPP or coalition in terms of both interpretability and computational time. First, this means that reliability is a distinct measure that can be processed and interpreted by a coalition forming algorithm. Second, the reliability assessment must be finished within a certain amount of time defined by the real-time requirements of the coalition forming process.

The top-level research question of the presented research project is as follows:

How can coalitions be assessed with regard to how reliably they can provide ancillary services?

This research question can be separated into the following questions in order to take into consideration the aspects introduced above.

**RQ1** How can reliability be defined in the context of ancillary service provision by RPU?

**RQ2** How can forecast uncertainties be incorporated into the reliability assessment?

**RQ3** How can dependencies between units be incorporated into the reliability assessment?
RQ4 How can unit failures be incorporated into the reliability assessment?

RQ5 How can the reliability of operational equipment be incorporated into the reliability assessment?

RQ6 How can these aspects be integrated into one measure?

In summary, the goal of the presented research project is to develop a method for assessing aggregations of RPU, e.g. coalitions, with regard to the provision of a defined ancillary service product. The properties and requirements vary for different ancillary service products such that the reliability assessment method may have to be adapted for different products. The use-case of this thesis is the provision of primary control reserves for frequency control.

1.4. Procedure and Thesis Structure

In order to achieve the goal stated in the previous section the design science research process of Peffers et al. [40] has been chosen. This is a “process for carrying out design science research in information systems”. The corresponding process model is depicted in Figure 1.1. It consists of the following six steps.

Problem identification and motivation “Define the specific research problem and justify the value of a solution.” This is the starting point for developing an artefactual solution. The problem may be separated to atomic problems in order to reduce complexity. For problem identification the problem state must be known.

Objective of a solution “Infer the objectives of a solution from the problem definition.” The objective can be formulated quantitatively or qualitatively. The objectives should be inferred from the problem identification. In addition to the problem state, current solutions must be known.

Design and development “Create the artefactual solution”. Within this step, the functional requirements and the artefact’s architecture must be determined. Subsequently, the artefact is developed.

Demonstration “Demonstrate the efficacy of the artefact to solve the problem.” The artefact is used to demonstrate its functioning. This can be done by e.g. experimentation or simulation.

Evaluation “Observe and measure how well the artefact supports a solution to the problem.” This corresponds to a comparison of the results from the demonstration step with the objective where relevant metrics and techniques for analysis must be used. At the end of this step, an iteration step back to development can be made in order to improve the artefactual solution. Otherwise, the results are forwarded to the next steps. In that case improvements are left for subsequent projects.
Communication “Communicate the problem and its importance [...] to researchers and other relevant audiences [...] .”

The process can be entered at steps one to four depending on the problem, the preconditions and the state of a solution.

For the presented research project, the objective centered solution has been suitable. The step of problem identification and motivation had been given by the research network Smart Nord Work, Package 2. The objective of this Work Package 2 has been the guaranteed provision of system-stabilising ancillary services by decentralised units. The resulting motivation for the presented research project is that coalition forming for providing system-stabilizing ancillary services has to include a measure reflecting the reliability with which a coalition is able to provide an ancillary-service product. Thus – as pointed out above – the objective is to develop a method to assess a coalition's reliability and consequently returning a measure that can be incorporated in the coalition forming process.

The structure of this thesis reflects the process of design science. First in Chapter 2, the necessary background for the research project is given in order to depict the problem to be solved. Moreover, related work and existing solutions are discussed. Thus, the foundation for the steps problem formulation and objective are given.

In Chapter 3, the identified problem is derived in more detail. To this end, first a general framework is introduced to formally describe ancillary service products for maintaining a stable system state. Second, the use case of the presented research project is introduced. This is the provision of primary control reserves by RPU with volatile RES. The objective is inferred in more detail.

The design and development step is explained in Chapter 4 by deriving the requirements for an artefactual solution and presenting the artefact developed during the presented research project. The artefact has been termed Reliability Assessment of Ancillary-Service Coalitions (RelACs).

In Chapter 5, the implementation of the artefact is briefly presented and the environment for demonstration and evaluation is derived. Furthermore, evaluation metrics to assess remaining risk is presented. The results of the demonstration and evaluation steps are presented in Chapter 6.

The first iteration between design and demonstration and evaluation is taken within this research project giving a proof of concept. The step communication has been assured by presenting the developed method and first results at conferences.

In Chapter 7, a summary and discussion of the presented research results are given. This includes the research process, the developed artefact as well as the evaluation results. Furthermore, an outlook on extensions of the artefact and future research is presented.
Figure 1.1: Process model adopted from [40]
2. State of the Art and Related Work

In this chapter the background is given which is relevant for the presented research project. However, here only a brief overview can be given but references for relevant literature for further information is given. The background represents the state of the art of relevant fields. Furthermore, related work to the research project is presented and discussed.

The first section provides an introduction to the electrical supply system with respect to both the technical and organisational structure. Moreover, the concept of ancillary services is introduced. The services relating to frequency and voltage control are presented in more detail. The first section finishes with an overview of coordination strategies for DER. The second section introduces the basic terminology if reliability theory of technical systems. The reliability assessment in electrical grids is presented subsequently. Related work regarding the assessment of DER under consideration of uncertainties and dependencies, especially in case they provide ancillary services, is presented in the third section. The chapter concludes with a summary and a discussion of related work.

2.1. Electrical Power Supply

2.1.1. Electrical Power System

Historically, the power system has developed as a hierarchical system. Figure 2.1 schematically shows the hierarchical structure. The structure is according to different voltage levels that has resulted due to different capacities of power plants and different large- and small-scale consumers.

The transmission level comprises the extra-high voltage (EHV) levels 380 kV and 220 kV. Mainly large-scale power plants are connected to these voltage levels where they feed-in the produced power. The power from the transmission system level is transported to the downstream 110 kV high voltage (HV) or subtransmission level. The power is passed to the distribution grids or local large-scale consumers, e.g. industrial consumers.

The distribution grid consists of the medium voltage (MV) levels of 10 kV and 20 kV and the low voltage (LV) level of 0.4 kV. The medium voltage grid is the link between the transmission grid and industrial consumers or substations. In the low voltage grid the power is locally distributed to the end-users. [17, 71, 95]

There are different types of system topologies depending on how the grid nodes are connected. The two main variants are a radial and network configuration. In a radial system the feeder branch from the source to the nodes and thus power flows in one direction – downstream. Figure 2.2 visualizes the tree structure of a radial system. Radial systems
are usually found in distribution grids. One disadvantage of this configuration is that in case of a circuit all loads are disconnected from power supply. [12, 79, 95]

The security of supply of radial systems is increased by loop systems. With this configuration the feeder of a radial system are interconnected by a switch that is usually open. In case of a circuit the affected part can be disconnected. Loops are usually operated as a radial system. [79, 95]

In a network system all nodes and lines are connected by more than one path. Some lines form loops within the system. Such a configuration is also referred to as meshed configuration or system. The power flows in several directions. Transmission systems are usually operated as network systems. The subtransmission level is either structured as a radial system or shows a network topology depending on whether they are operated for distribution or transportation matters. Both, extra-high and network high voltage level grids are referred to as transmission grids. Network systems have the advantage of redundancy. In case of a failure or disturbance the loads can still be supplied without overloading of other operational equipment. This is also referred to as $n$-$1$-principle or $n$-security. Radial systems lack the $n$-$1$-principle. [79, 95]

Since the 1990s the organisational structure of the European electrical power systems has been changing due to the liberalization of energy markets. This was the consequence of the Directive 96/92/EC of the European Parliament and of the Council “concerning common rules for the internal market in electricity” which in Germany led to an amendment of the so-called Energiewirtschaftsgesetz¹ in 1998. The aim has been to disintegrate monopolistic structures and enable free competition on European energy markets. From an economic point of view only the physical components of a power system and corresponding services (transmission and distribution) can be regarded as a natural monopoly but

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¹ German for: law of the energy industry
not the sectors of generation or sales. The consequence was an unbundling of vertically integrated energy supply companies to separated companies. [18, 95]

Transmission grids and distribution grids are operated, maintained and developed by transmission system operators (TSOs) or distribution system operators (DSOs), respectively. Furthermore, they register market participants and their contractual relationships. TSOs operate on a national or regional level in control areas. Usually, TSOs operate their control area in an autarkical way. However, neighboured control areas are connected in a synchronous way and thus form a power pool or synchronous grid. This means that the whole synchronous grid is operated at the same system frequency. On the European level, the Union for the Co-ordination of Transmission of Electricity (UCTE) coordinates TSOs of 24 countries. In Germany there are four TSOs that are organized in the Federal Association of the Energy and Water Industry (BDEW)² with respect to economic concerns of the power grid. Technical aspects like harmonic operations are covered by the Network Technology / Network Operation Forum (FNN)³ of the Association for Electrical, Electronic & Information Technologies (VDE)⁴. [17, 71, 95, 101]

2.1.2. Ancillary Services

The system operators (TSOs and DSOs) are responsible for the quality of supply. According to the Council of European Energy Regulators (CEER) “Quality of service in electricity supply has a number of different dimensions, which can be grouped under three general headings: ”

Commercial quality “[... ] commercial relationships between a supplier and a user” [14]. It incorporates e.g. metering, billing or emergency services.

²from German Bundesverband der Energie- und Wasserwirtschaft
³from German Forum Netztechnik/Netzbetrieb
⁴from German Verband der Elektrotechnik
Continuity of supply  The continuity of supply “is characterised by the number and duration of interruptions” [14]. It is also referred to as reliability of the system (see Section 2.2.1 for details).

Voltage quality  The main parameters are “frequency, voltage magnitude and its variation, voltage dips, temporary or transient overvoltages and harmonic distortion.” [14]. The term voltage quality is interchangeably used with the terms power quality or voltage power quality.

Complying definitions can be found e.g. in [95].

In summary, regulators specify requirements and monitor system operators with respect to how they maintain the quality of supply. In order to fulfil this tasks from a technical perspective, certain services are necessary. The electricity industry on European level represented by the Union of the Electricity Industry (EURELECTRIC) distinguishes between system and ancillary services. The relationship is shown in Figure 2.3.

System services  “are services provided by network operators to users connected to the system in order to ensure required power quality and the stability of the distribution grid.” [86]

Ancillary services  are “[all] services procured by the transmission or distribution system operator from system users to enable them to maintain the integrity and stability of the transmission and distribution system as well as power quality [...]” [86]

This distinction makes sense because the unbundling of energy supply companies (see above) system and network operators cannot control the generating units. However, system users such as generators are needed to maintain power quality (see details below). Consequently, system users provide ancillary services to the system such that the system operated by system operators is able to supply system users with power in a stable and secure way. Ancillary services comprise services supporting frequency control, voltage control, system restoration, and system control [34, 95].

2.1.3. Frequency Control and Voltage Control

In this section, the frequency and voltage control ancillary services are briefly presented, i.e. ancillary services supporting the system services of frequency and voltage control. In
the electrical power system a balance between power generation and consumption must be kept in order to keep the system frequency stable at the nominal value or setpoint. The frequency is nearly constant throughout the whole system. In the European system the nominal value is at 50 Hz. If the generation exceeds the consumption the frequency increases. Conversely, if consumption exceeds generation the frequency drops. In order to keep the system frequency stable, actions must be taken to counteract the imbalance. If generation is too high, either generation must be reduced or consumption increased. On the contrary if consumption is too high, either consumption has to be decreased or generation increased. The purpose of frequency control is to restore the balance between consumption and generation to stabilize system frequency. To this end, there are reserves that must be constantly available such that they can be activated in case of frequency deviations.

There are three different types or qualities of frequency control reserves (see Figure 2.4a) with different temporal requirements shown in Figure 2.4b. The realiseation of providing these reserves varies in different countries. Depending on local properties, reserves can be organized differently or omitted [90]. Furthermore, inertia control is a special case. In the following, the reserves as provided in Germany are briefly presented (according to [15, 19, 95]).

**Inertia control** Due to inertia in the rotating mass of generators an automatic reserve is available at any time to compensate frequency deviation within few seconds. This is neither regulated nor traded at energy markets.

**Primary control** Primary control reserves (PCR) are automatically activated in case of frequency deviations exceeding ± 20 mHz. The goal is to counteract frequency deviations by changing the power feed-in or consumption. Frequency deviations are either decelerated or stopped. A remaining frequency deviation is resolved by secondary control.

Primary control is activated disregarding the position of the cause of the frequency deviation. It is provided by the whole interconnected European grid. It is each TSO’s responsibility to provide reserves relative to the size of its control area. The total amount of reserves of 3000 MW corresponds to the power of the two biggest generating units in the European synchronous grid [101]. In case of frequency deviation the activation of reserves is distributed to units taking part in providing PCR. To this end, each unit is obliged to follow a droop control, thus reducing or increasing production or consumption relative to the magnitude of the frequency deviation. The reserves must be activated within 30 seconds.

**Secondary control** The objective of secondary control reserves (SCR) is two-fold. First, primary control is replaced in order to release reserves and to balance the remaining frequency deviation. Second, the power warranted at coupling points between different control areas is restored. Thus, SCR are activated in the control area responsible for frequency deviations. To this end, the TSO monitors both the system frequency and the power at coupling points. The reserves are activated automatically and must be activated to their full extent within 15 minutes.
2.1. Electrical Power Supply

Table 2.1.: Main product characteristics of control reserve qualities tendered in Germany, extract from [15]

<table>
<thead>
<tr>
<th></th>
<th>PCR</th>
<th>SCR</th>
<th>TCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>tender period</td>
<td>weekly</td>
<td>weekly</td>
<td>daily</td>
</tr>
<tr>
<td>tender time</td>
<td>as a rule on Tue.</td>
<td>as a rule on Wed.</td>
<td>as a rule Mon.-Fri. 10 a.m.</td>
</tr>
<tr>
<td>product differentiation</td>
<td>none (symmetric)</td>
<td>positive/negative</td>
<td>positive/negative</td>
</tr>
<tr>
<td>minimum bid amount</td>
<td>1 MW</td>
<td>5 MW</td>
<td>5 MW</td>
</tr>
<tr>
<td>increment of bid</td>
<td>1 MW</td>
<td>1 MW</td>
<td>5 MW</td>
</tr>
</tbody>
</table>

**Tertiary control** tertiary control reserves (TCR) are activated to optimize the power plant dispatch within a control area. Reserves are activated via telecommunication and must be fully activated within 15 minutes and remain until up to one hour.

Units that want to provide control reserves must assure that they are technically capable of providing reserves. To this end, they must fulfill prequalifications for each reserve type. These are on the one hand technical properties and on the other hand it must be given evidence that they are able to follow certain control schemes under operational conditions. The prequalification is done by the TSO in which control area the units are connected. [15]

In Germany, control reserves are tendered on an internet platform operated by the TSOs [88]. For the three reserve qualities of PCR, SCR, and TCR different product requirements are given concerning e.g. tender period, product differentiation, minimum bid amount or increment of bid [15]. An overview is given Table 2.1. If the control reserves do not suffice to stabilize the system frequency or if a sudden decrease of the frequency results from disturbances, actions in power plants are taken and pumps are shed. After a frequency drop below 49.0 Hz further automatic load shedding is activated according to a step-wise scheme [52, 101]. In order not to shut down DER in the course of load shedding, a non-discriminatory scheme has been developed to take account of the current feed-in situation [52]. In case of an increase of system frequency above 50.2 Hz a FNN project group developed a preliminary control strategy to supplement the guidelines of [3] for the most urgent contingency. This concerns the behaviour of generating units which has become a challenge due to the high increase of PV units in the distribution grid [51]. According to [3], generating units must be turned off within 200 ms in case of system frequency above 50.2 Hz. Since this bears the risk that several GW of generated power are shut down at once two voluntary schemes have been proposed in [51]. These are (1) to shut down units at a frequency between 50.3 and 51.5 Hz and reconnect them if the frequency drops below the respective frequency, and (2) to shut down units according to a response curve and increase the power feed-in at a frequency below 50.05 Hz.
2. State of the Art and Related Work

(a) Overview of frequency control adopted from [15]

(b) Temporal sequence of different types of control reserves adopted from [15]

Figure 2.4.: Qualities of frequency control reserves and their interrelation
Contrarily to frequency which is nearly the same throughout the whole system, the voltage may strongly vary from grid node to grid node. The voltage at a node drops if consumption prevails and conversely increases if production prevails. Moreover, the voltage at grid nodes strongly depends on reactive power. The purpose of voltage control is to keep voltage within a margin of five to ten percent (depending on the voltage level) at all grid nodes.

Currently, there are no markets for voltage control. Instead, there are regulations – so called grid-connection codes – by system operators requiring units to participate in voltage control according to predefined curves [31, 21]. In the transmission level the voltage can be controlled by adapting the feed-in of reactive power. However, especially in the low voltage level, the active and reactive power cannot be considered separately. Thus, the active power fed in or taken from the grid has high influence on the local voltage.

A formal description of ancillary services with the objective of maintaining a stable system state – such as frequency and voltage control ancillary services – is given in Chapter 3.

2.1.4. Coordination of Distributed Energy Resources

As mentioned in the introduction, the concept of VPP has been introduced to logically aggregate DER [5]. This concept has been extended to also incorporate controllable consumers and storage units. A VPP is an aggregation or pool of units such that it can operate and act at energy markets like a conventional power plant. One may distinguish between commercial and technical VPPs depending on the objective of either overcoming market entry barriers or stable grid operations. In summary, the tasks of a VPP are the interconnection of units with information technology, the integrated optimisation, forwarding of operational set points, and supervision and control. [25]

In the research network Smart Nord the concept of VPP has been extended to the concept of DVPP [26]. A special characteristic of DVPPs is that they are formed for a specific product and a specific product horizon. The advantage is that units can be aggregated to pools depending on actual conditions, e.g. weather conditions or market prices, thus being able to utilise flexibilities in an optimal way. Units within a DVPP are represented by agents. These are software-programs with the ability to perceive information about their environment via sensors and – based on that information – to make decisions about actions of the units they represent [92]. Furthermore, agents have the ability to communicate with other agents, hence to negotiate and cooperate. A system consisting of different agents is referred to as multi-agent system (MAS). An aggregation of agents is also referred to as coalition. This is used interchangeably with the term of aggregation of units throughout this thesis. A detailed definition is given in Chapter 3.

By now, agent-based approaches constitute an essential part in research and development in the field of VPP [25]. There already is a big diversity of agent-based concepts with applications in Smart Grids. In [25] the P-CASIT³ framework has been introduced that allows a classification and comparison of different agent-based solutions in Smart

Grids. This indicates that the methodology of MASs is suitable for solving problems of different qualities. In the context of Smart Grids the objectives of MASs are e.g. the local supply-demand matching, schedule-based operation or stable grid operation. There are different coordination mechanisms concerning both the search of a solution and the decision-making. In both cases the coordination structure may be centralised, hierarchical, or decentralised/distributed. The search of a solution relates to the location of information about the system and its components. For instance, in a centralised structure a central instance has global information whereas in a decentralised system information about the state of a component is only locally available. The decision of a solution to a problem is made based on the information about the system and its components. For details refer to [25].

In [41, 72] – in the context of Smart Nord Work Package 2 – an approach for a decentralised search of a solution with the application of finding a coalition for the provision of PCR has been proposed with the aim of finding an optimal solution, i.e. a centralised decision-making process has been conducted. In [75] a heuristic approach has been developed to solve the same problem. Both approaches have had the objective of finding a coalition under the constraint of stable operation and reliable provision. The research project presented in this thesis integrates into these approaches in the sense that the reliability assessment is incorporated as a constraint into the optimisation or search process. A more elaborate description of this relationship is given in Section 3.2.

2.2. Reliability Theory

In this chapter, basic terminology from reliability assessment theory is presented as well as a discussion of related work regarding reliability assessment in electrical power system.

2.2.1. System Reliability Theory – Terminology

In this section, an overview is given of basic terminology (mainly from [89] and [81]) for the assessment of system reliability needed for this thesis.

Reliability assessment theory offers methods to investigate and estimate the reliability of a technical system. It is used to compare different systems that are built of identical components, for optimising the operation of a system, for maintenance of a system, as well as during the engineering design process since weaknesses in the system may be identified and possible improvements may be derived. Furthermore, methods may be utilized for risk analysis.

For the course of this thesis, it is important to define and distinguish between the terms of reliability, availability, and dependability, because these terms are often used interchangeably. According to the Electrotechnical Vocabulary by the International Electrotechnical Commission (IEC), part 191 Dependability and quality of service [61], dependability is “the collective term used to describe the availability performance and its influencing factors: reliability performance, maintainability performance and maintenance
2.2. Reliability Theory

![Figure 2.5: Dependability and its influencing factors in accordance with [61]](image)

support performance”. Figure 2.5 visualises this concept. The term dependability is not used for quantitative descriptions.

**Availability** is the “the ability of an item to be in a state to perform a required function under given conditions at a given instant of time or over a given time interval, assuming that the required external resources are provided” [61]. This is composed of the **Maintainability** which means that the item may be retained resp. restored by maintenance [61], the **Maintenance Support**, i.e. the ability of a maintenance organisation to “provide [...] the resources required” for maintenance [61], and reliability. **Reliability** is the “ability of an item to perform a required function under given conditions for a given time interval” [61]. The item is assumed to be in a state where it can perform its required function. An appropriate measure for reliability is to give a probability for the item to perform its required function. The definition for reliability by International Organization for Standardization (ISO) basically states the same: “the ability of an item to perform a required function, under given environmental and operational conditions and for a stated period of time” [89].

Often, the term reliability is used in the global meaning of dependability. The focus of this thesis, however, lies on reliability as defined above. For this reason, the main aspects related to reliability are introduced in the following whereas aspects concerning availability etc. are only touched.

The functioning of an item is terminated by a fault or outage. Hence, in order to assess a system with regard to its functioning, the terminology of faults and related terms must be characterised. According to [61] a **failure** is “the termination of the ability of an item to perform a required function”. On the contrary, a **fault** is “the state of an item characterized by inability to perform a required function, excluding the inability during preventive maintenance or other planned actions, or due to lack of external resources” [61]. The term **error** relates to “a discrepancy between a computed, observed or measured value or condition and the true, specified or theoretically correct value or condition” [61]. Hence, the term failure specifies an event whereas the term fault is used to describe a state that occurs after a failure. An error is not a failure but an event that deviates from a target situation as long as it lies within acceptable limits of the item. An **outage** is “the state of an item of being unable to perform its required function” [60] incorporating external events leading to the non-functioning of an item.
The definitions given above refer to the general term of an item. An *item* is a component, a subsystem of a system of interest or the system itself. A system may be divided into subsystems and components. The components’ reliability is assessed and from that conclusions are drawn on the reliability of the whole system. The required function may be a single function but also a combination of functions. Of course, the item and its required function must be specified before the assessment is conducted.

Technical systems are divided into *repairable* and *non-repairable systems*. A non-repairable system is a system where one is only interested in assessing the item until a first failure occurs. It does not necessarily mean that the system cannot be repaired; it means that even if the system is repairable it is treated as non-repairable. If the system is repairable, not only the time before the first failure but also the maintenance is of interest. Thus, concepts for repairable systems are an extension of non-repairable systems. The focus here lies on non-repairable systems as the provision of ancillary services must be available at any time during the product horizon (see Chapter 3). In Appendix A.2.1 formal concepts are introduced for reliability assessment of technical systems.

### 2.2.2. Reliability in Electrical Power Systems

As indicated previously, one of the pillars of quality of service in electricity supply is continuity of supply also referred to as reliability of the system or service reliability. In [60] service reliability is defined as “the ability of a power system to meet its supply function under stated conditions for a specified period of time”. This maps the general definition of reliability to the context of electrical power supply. The Association of the European Electricity Industry (UNIPEDE) group of experts (DISQUAL) introduced indices to assess availability and provide a metric to support comparability in [22]. Considered measures are interruption frequency, supply unavailability, and interruption duration. In case that these measures refer to customer interruptions they are referred to as System Average Interruption Frequency Index (SAIFI), System Average Interruption Duration Index (SAIDI) and Customer Average Interruption Duration Index (CAIDI) (According to the Institute of Electrical and Electronics Engineers (IEEE) Guide for Electric Power Distribution Reliability Indices 1366). With these indices system reliability may be described. Table 2.2 gives an overview about these indices. Note, that this is not an exhaustive list but only gives the most common indices. For a more elaborate explanation refer to [4]. Reliability indices yield average reliability estimates and may be used for benchmarking. “Reliability is primarily concerned with customer outages” [12] and relates to equipment outages. There are different interruption causes e.g. equipment failures or severe weather conditions. However, utilities usually exclude weather conditions like storms when calculating reliability indices. During normal weather conditions failures of equipment may be considered as independent. [12]

System reliability indices are computed ex post based on data on customer interruptions. However, for strategical decisions in operation and power system planning, e.g. to minimize interruptions, historical data is not sufficient and predictions are necessary [70]. In order to enable an ex ante estimation analytical methods or simulations are utilized. Analytical methods are e.g. Markov-methods whereas simulative methods refer to
Table 2.2.: Interruption indices according to [22] with regard to customer interruptions

<table>
<thead>
<tr>
<th>Index</th>
<th>IEC notation</th>
<th>Definition</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interruption frequency</td>
<td>SAIFI</td>
<td>average number of interruption per year and customer</td>
<td>$\sum_j n_j/N_s$</td>
</tr>
<tr>
<td>Supply unavailability</td>
<td>SAIDI</td>
<td>average number of minutes without supply per year and customer</td>
<td>$\sum_j (n_j \cdot t_j)/N_s$</td>
</tr>
<tr>
<td>Interruption duration</td>
<td>CAIDI</td>
<td>average duration of interruptions</td>
<td>$\sum_j (n_j \cdot t_j)/\sum_j n_j$</td>
</tr>
</tbody>
</table>

$n_j$ number of customers in group $j$ with interrupted customers  
$t_j$ duration of interruption of customers in group $j$  
$N_s$ total number of served customers  

(e.g. Monte Carlo (MC)-simulations.)

The basis for analysis form component and systems models. With regard to components at least the respective failure rates and component repair times must be known. For valuable results a good data base for component modelling should be at hand. The system model reflects the topology of the system and connections of components. Furthermore, a description of system response to contingencies and disturbances are needed.

According to e.g. [70] and [12] the basic idea is to select possible contingencies or interruptions and derive the response of the system and the impact on components. In case simulation techniques are used, the system state is chosen randomly considering its probability of occurrence. This allows to estimate a distribution of reliability indices.

In case of the analytical approach all combinations of possible contingencies are regarded. However, the number of contingencies to take into account may be reduced in order to reduce complexity. With the analytical approach, expected values of the state of the whole system are derived but not its distribution.

In [12], a procedure is presented for the analytical approach for radial systems. To this end, the concept of minimal cut sets and Markov-models are utilized. For each contingency the probability of occurrence is determined as well as its impact on each component. The impact is weighted by the probability of occurrence of the contingency. The sum over all contingencies yields the expected number of interruptions. For details see [12].

Additionally, modelling loads and generation in the system may be advantageous for estimating reliability indices since they have influence on how the system is stressed but also on the process of restoring a system state [96, 12]. To this end, e.g. load duration curves are utilized.
2.3. Ancillary Services by Renewable Power Units

So far the reliability estimation of technical systems has been introduced. This yields techniques that may be used to model failures of technical systems including RPU and operational equipment. Furthermore, basics related to reliability estimation of electric power grids have been briefly introduced. With this, a background on the state of the art and related work has been established to deal with research questions RQ4 and RQ5. In what follows, a background regarding uncertainties and dependencies is provided along with their consideration and application in the context of power systems, in particular with respect to RPU, in order to cover research questions RQ2 and RQ3.

2.3.1. Uncertainties

The integration of RPU such as PV and wind units induces uncertainty in the power system. One reason is the volatile character of weather-dependent power units. The second is that planning can only be conducted based on forecasts. “The need for forecast information on the expected solar and wind power production is increasing with the amount of installed power.” [74] In the literature the most important areas concerning system operation that are influenced by uncertainties are those of determining requirements for operating reserves, unit commitment, and real-time operations.

Forecasts for power injection of PV and wind units are based on forecasts of the respective primary source, i.e. solar irradiance or wind speed, other environmental measures, e.g. temperature, and the conversion to power. The forecast may be a point or probabilistic forecast. The first one is a single value of a prediction of the expected power output. The latter gives a probability distribution function of power values. Predictions are available for different prediction horizons or lead times. An uncertainty measure that is commonly used is the root mean square error (RMSE) or relative or normalised RMSE of prediction giving a measure for the accuracy of forecast that may also be utilised for comparing different prediction methods. For details refer to e.g. [33, 74]. The RMSE increases with time (see e.g. [56, 74]) meaning that predictions are less accurate with increasing prediction horizon. Reversely, this indicates that uncertainties diminish the closer in time the operational hour is.

Modelling of Uncertainties

Models of uncertainties represent the overall forecast error by using a parametric representation, e.g. a standard deviation, thus providing a constant measure for uncertainty for a given time horizon [33]. It is sufficient to determine uncertainties for different situations and weather conditions for reasons of the dynamic nature of weather phenomena [33, 74]. For example in [56, 43, 74] results on forecast uncertainties are presented and the representations by parametric probability distribution functions are discussed.

Using probabilistic forecasts, the uncertainty is given as probability for the occurrence of a power value, i.e. not the error is specified but the certainty of a prediction. In order to represent also temporal dependencies of uncertainties, so-called scenarios are utilised.
With this method the development of forecast errors can be described, i.e. the magnitude of a forecast error depending on the previous one. In [46, 65], prediction errors of different prediction horizons are transformed to have Gaussian distribution with mean zero and temporal interdependencies between the time steps are represented as a covariance matrix. This yields a multivariate Gaussian distribution from which samples are drawn yielding scenarios of the development of prediction errors.

Considering Uncertainties in Electrical Power Supply

Due to the integration of RPU into the power system, uncertainties are introduced that must be considered for both planning and operation of the system [77]. For maintaining a stable system frequency, control reserves are provided (see Section 2.1.3). The reserves for PCR are dimensioned as 3000 MW which is in accordance with the capacity loss if two power plants fail [101]. In contrast to this deterministic approach, in the cases of SCR and TCR probabilistic approaches are utilised. The influencing factors leading to a demand of control reserves are failures of power plants, load variation, load forecast errors, jumps in schedules and forecast errors of weather-dependent power feed-in. The uncertainties are modelled as probability distribution functions. A joint model is determined by convolution of the density functions and thus yielding a distribution function of summed deviations within the region. Therefore, independence between the influencing factors is assumed. The magnitude of reserves is determined using a so-called loss-of-load probability. An accepted deficit probability is given, i.e. a probability that the reserves provided are not sufficiently high. Given this deficit probability the demand for reserves is derived from the convoluted density function. [16, 28, 67] Besides the analytical methods, simulative approaches have been proposed using MC-simulations (e.g. [13]). With this approach correlations between wind feed-in and the activation of control reserves are taken into account.

The unit commitment problem is an optimisation problem used in operation planning in order to make decisions about the scheduling of units in an economically optimal way [35]. In [38] the unit commitment problem has been extended such that uncertainties of wind power forecasts are incorporated. In order to achieve this, the methodology of sampling scenarios is used and the scenarios are considered during solving the optimisation problem thus introducing more complexity to finding the optimal solution. A similar approach has been chosen in [36].

In [37] the uncertainty of DER is quantified by prediction errors distinguishing between different types of errors: systematic and residual errors. The errors are calculated as difference in point prediction and actual production. The use case is a VPP of different wind parks within a geographical area or PV-panels in an extended neighbourhood, i.e. spatially close units of the same technology. The objective of this approach is to support the decision process which units to consider for membership in a cooperative VPP. The systematic error relates to external information about the environment that is not in the control of the units. The residual error is due to internal factors in the control of each unit and the fact that predictions cannot be made without errors. The statistical method developed in [37] gives a measure to distinguish between these two error types. To do
so, the so-called Pearson correlation is utilised in order to determine whether prediction errors deviate from the average error of units of the same technology within the same area. This implies the assumption that there is a relationship between forecast errors of units of the same type.

The project “Regelenergie durch Windkraftanlagen”\textsuperscript{6} had the aim of developing a proof method for the provision of control reserves by wind turbines, more specifically wind parks, as well as a method for determining an offer for control reserves \cite{45, 63, 64}. The calculation of an offer is based on probabilistic forecasts of the power feed-in of the wind park. “The offer is the minimum of the probabilistic day-ahead forecast during each time step.” \cite{64} Concepts for calculating offers of conventional power plants based on probabilistic forecasts are presented in \cite{63}, too. In Figure 2.6 two proof methods are schematically presented. The first method (Figure 2.6a) has been implemented in the TWENTIES Project Demonstration project 1 - System services provided by wind farms (SYSERWIND) \cite{105}. In this case, the wind turbines are operated in a throttled way at a level according to their offer. In case of an activation of reserves, the units’ power output is reduced accordingly. In the second case (Figure 2.6b), the units inject power according to the possible maximum amount of active power. In case reserves are activated, the units reduce their feed-in accordingly but at most to the extent specified by their offer. The methods presented are primarily designed for SCR or TCR.

\subsection{2.3.2. Dependencies}

The power injection of RPU such as PV and wind units are weather dependent, i.e. their power feed-in is restricted by the uncontrollable primary energy carrier and other external measures, e.g. temperature. Units may be subject to the same or similar external influences and may therefore show dependent behaviour regarding their power feed-in. Power generation can be considered as a time series or stochastic process, i.e. a vector or sequence of realisations of random variables. The same holds for describing forecasts. Given this description of power injection and their predictions the following methods can

\textsuperscript{6}German for: balancing energy by wind turbines
be utilized to quantify dependencies between two random variables.

**Modelling of Dependencies**

Dependency can be described using scalar measures. Two of the most common measures are listed below. Details and further properties can be found for example in [78].

**Pearson correlation** The Pearson correlation measures pairwise linear dependence between random variables and returns a scalar between $[-1, 1]$ indicating positive or negative linear dependence. An important fact is that “the concept is only really a natural one in the context of multivariate normal or, more generally, elliptical models.” [78]

**Rank correlation** The rank correlation yields a scalar measure for dependence where the ordering or ranks of the values is relevant rather than their numerical values. Two measures for rank correlation are the Spearman or Kendell rank correlation. Both of these measures take values between $[-1, 1]$, as well. (Refer to [78] for details.)

As already mentioned, the Pearson correlation may only be used to identify linear dependencies. The rank correlation relaxes this restriction. However, the distributions of the random variables, also referred to as marginal distributions, in addition with the (rank) correlation does not suffice to fully describe the joint distribution of the random variables and thus it cannot fully explore the dependencies. **Multivariate distribution functions** or **joint distribution functions** of two or more random variables fully describe the relationship between the variables. With the help of so-called **copulas** a description of a joint distribution function is possible separating the dependence structure and the marginal distributions of the random variables. More details are given in Appendix A.2.3 or [78].

**Considering Dependencies in Electrical Power Supply**

In [102, 103] the importance has been investigated that correlations between wind parks within the same region have. For this purpose, a distribution of wind speed for one region has been determined based on historical data. The effect of correlations on reliability studies have been determined by comparing perfectly dependent wind parks with wind parks that are not dependent at all. In the latter case, the convolution product of the wind parks has been utilized for sampling feed-in constellations. In the former case, the power injection amounts to the same for all wind parks, i.e. they are assumed to simultaneously be in the same state.

Investigations of the relationship between dependencies of feed-in as well as variability of feed-in in different temporal resolutions and the distance between units for both PV and wind units have been conducted in [30, 57, 58, 97]. The dependencies are represented by pairwise correlations between different units. The purpose of investigations has been to determine a decorrelation distance, i.e. a distance between units where feed-in and variability can be assumed to be uncorrelated. This indicates that within a certain radius the feed-in and volatility can be assumed to be linearly dependent. However, investigations are restricted to linear dependencies that can be captured using correlations.
In [55] the methodology of copulas has been used to represent dependencies between different wind parks in Germany. A multivariate copula model for dependencies between different wind sites has been decomposed to bivariate copulas for a more flexible representation of pairwise dependencies.

As mentioned in the previous section, the methodology of so-called scenarios is a sampling-based approach to obtain forecasts of power generation [65]. The samples are drawn from a multivariate distribution function containing information not only about the feed-in of units but also temporal and spatial interdependencies. In [65] copulas are used to model the dependency structure given marginal predictive densities. This shows that there is a strong relation between uncertainties and dependencies for which copulas provide a sufficient modelling tool.

### 2.4. Summary and Discussion

The objective of the presented research project is to develop an assessment method for aggregations of RPU with the objective of providing ancillary services. In this chapter first, the background necessary for the presented research project has been briefly presented. This includes the technical and organisational structure of the electric power system. Subsequently ancillary services have been introduced, in particular frequency and voltage control ancillary services – the services to maintain a stable state of the electric power system. A brief overview of the methodology for coordinating DER has been presented.

In order to answer the research questions introduced in Section 1.3, a literature research has been conducted to obtain an overview of the state of the art and work related to the research topic. The terminology of reliability theory has been presented that forms a basis for Research question 1. Furthermore, concepts of reliability assessment for technical systems are applicable to answer Research questions 4 and 5. The state of the art for reliability assessment in electrical power system has been introduced as background for Research question 5.

A challenge of the provision of ancillary services by RPU especially PV and wind units are the uncertainties of predictions that must be considered for planning of operation. Furthermore, dependencies between different units of the same technology play an important role as well. For this reason, results on how uncertainties and dependencies can be modelled and incorporated for planning and provision of ancillary services and related domains have been given. In what follows, the findings of state of the art and related work are discussed with regard to their applicability for the RelACs-method. To this end, the discussion is structured according to the research questions. Table 2.3 gives an overview of the stated aspects with their advantages or disadvantages. Note that that ancillary services must be activated within certain time frames as stated in Section 2.1.3. Thus, the applicability is also discussed with respect to their incorporation of real-time applications. The requirements for the RelACs-method are derived in Section 4.1 in detail.

**RQ1 (Definition)** The term of reliability of technical systems is defined in a general way such that it can be adapted for the context of AS provision by RPU. Because AS have
to be activated under real-time restrictions units cannot compensate for failures of other units. For this reason, methods for non-repairable systems are suitable for the RelACs-method, i.e. systems that are investigated until a first failure occurs.

**RQ2 (Uncertainty)** Different measures of uncertainties have been presented as well as their usage in the context of electrical power systems.

**Point forecast** Point forecasts have the disadvantage that the prediction is the expected power output. However, with the RMSE a quality measure is available that may be used for comparing different forecast methods. Moreover, based on historical data or given by literature values an error model can be derived as probability distribution function. This has been done e.g. for the application of dimensioning control reserves where uncertainties have been modelled as the probability distribution of prediction errors.

**Probabilistic forecast** The methodology of probabilistic forecasts allows a flexible description of predictions. For a predicted power output a probability is given. However, data is needed in order to derive the predictions. In case the data needed is not available an error model based on point forecasts and literature values can be used instead for modelling forecast errors.

**Scenarios** Using scenarios, temporal and spatial dependencies can be incorporated into the prediction and planning process. However, the method to obtain scenarios is sampling-based which might be too time-consuming to fulfil real-time restrictions.

**Dimensioning of control reserves** As pointed out earlier, in the context of determining the needed magnitude of control reserves, prediction errors are considered as probability distribution functions. This has been done for the total PV and wind generation in a control area but can be adapted for single units as well. This way it is possible to assess heterogeneous aggregations of units.

**Proof of control reserves** A proof of control reserves has been proposed in [63] based on probabilistic forecasts for wind generation of a wind park or wind park pool. The concept may be adapted for the assessment of heterogeneous pools consisting of units of different technologies and point forecasts.

**RQ3 (Dependency)** Different measures of dependency have been presented as well as their usage in the context of electrical power systems.

**Correlations** Correlations between power injection or variability between different units can be expressed using correlations. The correlations can be calculated based on historical data. However, using the Pearson correlation it is only possible to describe linear dependencies. The concept of rank correlation generalises this but still the overall dependence structure cannot be captured.

**Correlations of wind parks** In [102, 103] the effect of correlations between wind parks on system reliability has been investigated. However, only the case of no de-
2. State of the Art and Related Work

Pendence and perfect dependence has been regarded. Thus the concept cannot be adapted in case of non-perfect dependencies.

**Multivariate models** With multivariate distribution functions behaviour of all units as well as interdependencies can be fully described. With the methodology of copulas the multivariate distributions can be flexibly modelled.

**Convolution** The method of convolution is a common technique used in order to determine the demand for ancillary services for a whole region, e.g. a control area, where the forecast errors of, especially renewable energy resources, are of interest. The demand for an ancillary service product is calculated depending on the probability for an accepted deviation from the predicted value for the *whole* region. Thus, one is interested in the distribution of the sum of errors. In case of stochastically independent variables - which is assumed for the error distributions in this context - the distribution of the sum of random variables equals the convolution of those random variables. However, for the reliability assessment of aggregations of RPU for the provision of ancillary services, the probability for the deviation from the predicted value must be known for *each* unit, because each unit must adhere to its contribution. Thus, the sum of random variables does not suffice since the information is lost about what each unit’s contribution is. Moreover, the units considered may be dependent. For these reasons, convolution is not suitable for the RelACs method.

**RQ4 (Unit failures)** Concepts for reliability assessment of technical systems have been introduced. The failure rate models known from reliability theory of technical systems are applicable for the presented research project.

**RQ5 (Operational equipment failures)** Reliability assessment techniques for the electrical power system have been presented including probabilistic methods with both an analytical and simulative approach. Reliability assessment of the power system is primarily concerned with finding reliability indices related to customer interruptions. While the simulative approach might be too time-consuming for real-time applications, the analytical approach may be applicable for the context of AS provision by RPU.

The methods presented may serve as basis for the RelACs-method. Well-known and proven concepts may be adapted and extended to be applicable in the context of AS provision by distributed RPU. Especially the cases of heterogeneous unit aggregations and the provision under real-time restrictions as well as the reliability of the power system under the perspective of generation interruptions constitute new challenges. Furthermore, the different approaches must be interrelated and connected.
### Table 2.3: Discussion of related work

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Aspect</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 (Definition)</strong></td>
<td>reliability theory</td>
<td>general definition</td>
<td></td>
</tr>
<tr>
<td><strong>2 (Uncertainty)</strong></td>
<td>point forecast</td>
<td>quality measure with RMSE</td>
<td>only expected value predicted</td>
</tr>
<tr>
<td></td>
<td>probabilistic forecast</td>
<td>flexible modelling of forecasts with distribution function</td>
<td>data needed</td>
</tr>
<tr>
<td></td>
<td>scenarios</td>
<td>representation of temporal dependencies</td>
<td>sampling based</td>
</tr>
<tr>
<td></td>
<td>dimension-ing of control reserves</td>
<td>uncertainties modelled as prediction errors</td>
<td>consider total production within whole control area</td>
</tr>
<tr>
<td></td>
<td>proof of control reserves</td>
<td>concept for control reserves by RPU</td>
<td>no single units, only wind parks</td>
</tr>
<tr>
<td><strong>3 (Dependence)</strong></td>
<td>correlations</td>
<td>modelling based on historical data or literature value</td>
<td>only linear or ranked dependencies</td>
</tr>
<tr>
<td></td>
<td>correlations of wind parks</td>
<td>only modelling of no correlation or perfect correlation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>multivariate models</td>
<td>complete dependence structure</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>flexible modelling with copulas</td>
<td></td>
</tr>
</tbody>
</table>
### Discussion of related work – continued

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Aspect</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (Unit failures)</td>
<td>failure rate models</td>
<td>reliability theory: proven methods</td>
<td></td>
</tr>
<tr>
<td>5 (Op. equipment failures)</td>
<td>reliability indices</td>
<td>proven methods from viewpoint of customer interruptions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>probabilistic analytic approach</td>
<td>proven methods from viewpoint of customer interruptions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>probabilistic simulative approach</td>
<td>proven methods from viewpoint of customer interruptions</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>simulative (time consuming)</td>
</tr>
</tbody>
</table>
3. Ancillary Services by Distributed Unit Coalitions

As introduced in Chapter 2, the system operators are responsible for providing system services. According to [49] “system services refers to the services essential to the proper functioning of the system which system operators provide for their customers in addition to the transmission and distribution of electric energy [...]” In order for the system operator to meet the responsibility to ensure safe, secure and reliable operation of the power system as well as quality of supply ASs are necessary. “Ancillary services are services attributable on the provider’s side which are made available by the relevant system users [...]” [49].

Services such as tendering, contracting, accounting or customer relations are not considered here. The presented research project has emerged from the context of the research cluster Smart Nord, Work package 2. There the focus had been laid on ASs utilized for stable system operations, in particular to maintain a feasible operational state. Thus this focus has been chosen for this thesis, as well.

The operational state must stay within certain system boundaries according to system operating standards and specifications of quality of supply. The demand of ASs depends on the system state with regard to the system boundaries and must be supplied fulfilling certain requirements. The formal model introduced subsequently serves as a framework for the description of ASs for maintaining a feasible operational state. From that framework, requirements may be formalized, similarities and differences of different services may be identified and solutions for AS provision may be transferred or adapted. Brief examples of the framework are given for the mapping to the specific AS of frequency and voltage control ancillary services. The formal model may be extended to other types of ASs, as well.

As mentioned earlier, ASs are provided by system users. The electrical power supply system is subject to substantial changes since more and more small power plants such as PV or wind units are integrated into the grid. This leads to a more important role of these units in the power system and their utilization for the procurement of AS as conventional large power plants are substituted. However, the amount of power such a unit is able to provide is usually too small to cover the demand for an AS product required by the system. Thus, concepts for aggregating those units to virtual power plants or coalitions help to enable small-scale distributed units to provide ASs.

After the formal model is introduced for the description of ASs in the subsequent section, the use case of providing primary control reserves is specified that is being discussed in more detail throughout this thesis.
3.1. Formal Model for Ancillary Service Provision

In this section, a formal model is introduced that serves as a framework for the description of ASs maintaining a feasible operational state. This general model holds for the provision of reserves by conventional power plants as well as aggregations of RPU. To this end, system boundaries, AS products as well as their requirements are formalized. Furthermore, the section introduces the concept of AS provision by coalitions of distributed energy resources that serves as the use case for this research project.

3.1.1. System Boundaries

There are ASs that have common attributes. This allows for a generic description of ASs of different types. The following formal specification serves as a framework to describe ASs to maintain a feasible operational state. This model is exemplarily mapped to frequency and voltage control ancillary services, but it may be extended to describe other service types as well.

The ASs considered here are required by the system in order to maintain a feasible operational state. The feasibility of the state is evaluated with respect to the quality of supply given by a certain measurement and its state with respect to system boundaries. This is specified in the following definition.

\textbf{Definition 3.1.1 (Quality Type and Properties)}

The quality type is determined by a measurable quantity $\nu$. This quantity has a nominal value $\nu_{\text{nom}}$. Furthermore, the value of $\nu$ is allowed to vary within a feasible region $A_\nu$, which is an interval or a union of intervals. The safety margin $D_\nu$ with respect to $\nu$ is defined as the region within $A_\nu$ within which $\nu$ is allowed to vary without consequences. It holds that $\nu_{\text{nom}} \in D_\nu$.

The connection of the properties are visualized in Figure 3.1. An overview of the quality aspects taken into account as well as the corresponding properties is given in Table 3.1 (see e.g. [88, 90]). The quality type is evaluated by measuring the respective quantity $\nu$. This can be done locally in case of nodal voltages or within the whole synchronous system in case of frequency. Its value must be kept at a certain level, termed the nominal value $\nu_{\text{nom}}$. Due to unforeseeable changes in the system, e.g. change in demand and supply of electric power, the level of the considered quantity cannot be kept to the nominal value in an exact way. Thus, there is a region $A_\nu$ where the measured value is accepted to vary in, though actions must be initialized to mitigate and reverse deviations. Furthermore, there is a safety margin $D_\nu$ that determines in which area around the nominal value it may vary without triggering a demand for AS, i.e. a control to influence the deviation of $\nu$.

The demand for ASs depends on the operational state given as the quality quantity with respect to system boundaries. The supply of ASs is formally introduced in the subsequent section.
3. Ancillary Services by Distributed Unit Coalitions

![Figure 3.1. Quality Properties with respect to a measurable quantity](image)

Table 3.1.: Quality aspects and corresponding properties

<table>
<thead>
<tr>
<th>quality aspects</th>
<th>frequency stability</th>
<th>voltage stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurable variable ( \nu )</td>
<td>frequency ( f )</td>
<td>voltage ( U )</td>
</tr>
<tr>
<td>nominal value ( \nu_{nom} )</td>
<td>( f_{nom} = 50Hz )</td>
<td>( U_{nom} = 1pu )</td>
</tr>
<tr>
<td>feasible region ( A_{\nu} )</td>
<td>( A_f = [49.8Hz, 50.2Hz] )</td>
<td>( A_U = [0.9pu, 1.1pu] )</td>
</tr>
<tr>
<td>safety margin ( D_{\nu} )</td>
<td>( D_f = [49.98Hz, 50.02Hz] )</td>
<td>( D_U = [0.98pu, 1.02pu] )</td>
</tr>
</tbody>
</table>

3.1.2. Ancillary Service Product

In order to maintain the quality of supply, the measured quantity \( \nu \) must be kept within the feasible region \( A_{\nu} \). To this end, ASs are employed. The measured value \( \nu \) can be influenced by providing a certain quantity, here termed AS quantity.

**Definition 3.1.2 (Ancillary Service Quantity)**

The **ancillary service quantity** \( q_{\nu} \) with respect to the measurable quantity \( \nu \) must be provided in order to regulate \( \nu \). The magnitude of \( q_{\nu} \) needed for regulation, i.e. the demand from the system, depends on the magnitude of deviation of \( \nu \).

With this formalism, several types of ASs may be described. In case of frequency the AS quantity is active power, in case of voltage this is reactive power, or (as voltage control is becoming more important in the distribution grid) a combination of active and reactive power, i.e. \( q_{\nu} \) can be a real or a complex number. In Table 3.2 an overview of instances of ASs is given.

In order to guarantee the provision of ASs with a necessary amount of AS quantity, the concept of AS products\(^1\) is introduced that allows for the annotation of costs for the provision of ASs. It specifies an AS quantity (with positive or negative sign) or a margin of AS quantity (i.e. the option of providing positive or negative power) within a given time interval that must be provided. This quantity is annotated with costs regardless of whether the service is monetarily rewarded or e.g. specified in the grid connection codes.

\(^1\)Here, the notion of ‘product’ does not necessarily mean a commodity that is dealt with at a marketplace.
Table 3.2.: Overview of instances for ancillary service types

<table>
<thead>
<tr>
<th>primary control</th>
<th>voltage control</th>
</tr>
</thead>
<tbody>
<tr>
<td>measurable quantity ( v )</td>
<td>( f )</td>
</tr>
<tr>
<td>ancillary-service quantity ( q_v )</td>
<td>( q_f = P )</td>
</tr>
<tr>
<td>(active power)</td>
<td>( q_u = Q ) or ((P, Q))</td>
</tr>
<tr>
<td>( U_{cont} )</td>
<td>(reactive power or combination of active and reactive power)</td>
</tr>
</tbody>
</table>

**Definition 3.1.3 (Ancillary Service Product)**

An ancillary service product \( AS_v \) with respect to \( v \) is a triple of a **product horizon** \( T_{pr}^v \), i.e. the time span during which the service must be available, an amount or margin \( e_{pr}^v \) of the ancillary service quantity \( q_v \), and corresponding costs \( c_{pr}^v \), i.e.

\[
AS_v = (T_{pr}^v, e_{pr}^v, c_{pr}^v).
\]

An AS product with respect to \( v \) determines the amount \( e_{pr}^v \) of a certain quantity \( q_v \) that must be provided within a specified product horizon \( T_{pr}^v \). The amount is usually measured by the maximum necessary amount of \( q_v \) to compensate a maximum tolerable deviation from the nominal value \( v_{nom} \). System users may commit themselves to provide an AS product. For example, this commitment may be specified in connection rules or the result of a market clearing where power products are contracted.

It must be made sure that the ASs considered can be activated to their full extent, i.e. \( e_{pr}^v \), during the whole specified time horizon, i.e. the product horizon \( T_{pr}^v \). The activation and delivery of an AS product \( AS_v \) with respect to \( v \) has to fulfil certain properties that are required by the system. The following list is not exhaustive but relevant to the research project.

**Vicinity** The vicinity \( v_{ic} \) according to \( v \) is the maximum distance to an event triggering a demand for an AS within which an AS product must be provided.

**Time of activation** The time of activation \( \partial t_v \) according to \( v \) is the time limit within which the demand of an AS must be fully supplied after it has been called.

**Duration of activation** Duration of activation \( t_{act,v} \) of an AS product \( AS_v \) with respect to \( v \) is the time span (within the product horizon) the AS must be supplied after it has been activated.

The vicinity specifies whether an AS product is needed locally and should be provided within a certain distance to the location where the event occurred that triggered the AS demand. This is e.g. the case for the provision of secondary control reserves where the service must be activated in the control area within which an imbalance between production and consumption occurred. In case of voltage control the vicinity is very close to the voltage deviation and the AS should be activated within the same section of the grid.
Furthermore, the activation of an AS quantity is time critical since it must be supplied to its full extent within a certain amount of time. It must be activated for a certain time horizon, too. For example secondary control must be activated within 15 minutes and be activated for up to an hour (see Figure 2.4a).

As mentioned before, for a time horizon a certain amount \( e_{pr} \) of the AS quantity \( q_v \) must be provided. This is usually the maximum necessary amount to compensate a maximum tolerable deviation of \( v \) from the nominal value \( v_{nom} \). The amount that actually must be supplied depends on the variation in \( v \) denoted as \( \Delta v = v_{nom} - v \), i.e. \( \Delta e_{pr} = \Delta e_{pr}(\Delta v) \), where \( \Delta e_{pr} \) is a deviation from a setpoint of the AS quantity \( q_v \) activated for AS provision. In other words, the deviation in \( v \) determines the deviation in \( e_{pr} \) necessary to counteract the deviation of \( v \).

The measure \( v \) is being checked in a continuous manner (e.g. in regular time steps). As long as \( v \) lies within the safety margin \( D_v \) no action needs to be taken. If \( v \) leaves \( D_v \) an ancillary service is required. Thus, the respective quantity \( q_v \) is called according to the deviation of \( v \), i.e. \( \Delta e_{pr}(\Delta v) \). This behaviour must be specified beforehand and relates to the maximum quantity \( e_{pr} \) that must be possible to activate within the time of activation. In case that \( v \notin A_v \) a specific emergency plan must be initiated, such as load shedding depending on the quality aspect considered.

In the subsequent section a scheme to provide an ancillary service product by DER is introduced. Furthermore, the use case of primary control reserve is regarded in more detail since it is the basis of this thesis.

### 3.1.3. Concept of Provision by Renewable Power Units

As mentioned before, ASs are provided by system users. Due to a decentralisation of the power system and the integration of a vast amount of RPU in the power system those units must take over tasks of conventional power plants. This also holds for the case of AS provision. However, the amount of power small-scale distributed units are usually able to provide is too small to cover the demand for an ancillary service product required by the system.

In [72], a method has been proposed in order to enable distributed units to providing ASs product by forming coalitions such that they are able to act as virtual power plants on energy markets in order to overcome market entry barriers. This serves as the use case of the presented research project. The basic idea presented in [72] is as follows. As mentioned in the previous section, the system has certain requirements for ASs regarding its amount \( e_{pr} \) for a given time horizon \( T_{pr} \). Hence, the system operator calls for bids for the provision of those services or specifies them in system connection rules (cf. Chapter 2). The units or agents (see below for details) that want to contribute to service provision determine the amount they are able to provide. If this contribution is not sufficient, agents must negotiate with other agents in order to form coalitions that fulfil system requirements. In case a coalition is chosen to provide an AS product, this coalition is responsible for the required amount of AS quantity being available throughout the whole product horizon with the required reliability. If an ancillary service is demanded by the system the responsible coalition must deliver the required amount.
In what follows the basic concepts are introduced in a formal way.

**Definition 3.1.4 (Unit)**
A unit $U$ is an technical device that produces or consumes electric power and is further equipped with a control system. The set of all units is denoted by $\bar{U} = \{U | U \text{ unit}\}$.

Since a unit is assumed to be equipped with a control system it can be monitored and controlled by a software agent. The general definition is “An agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through effectors.” [92] and is adopted here.

**Definition 3.1.5 (Agents)**
An agent $\alpha$ is a software component implemented in the control system with which a unit is equipped. An agent perceives information about the unit it represents and the environment via sensors and based on that sets control actions for the unit. The set of all agents is denoted by $\bar{A}$.

Each agent represents exactly one unit and is thus referred to as unit agent. Hence, there is a one-to-one relationship between agents and units. Agents are assumed to have knowledge about the unit they represent such as the current operational state or forecasts. Furthermore, an agent may have knowledge about its environment. The information is processed in order to make decisions about optimal actions for controlling the unit. For an introduction to agent theory refer to [92, 106]. Agents are also responsible for the communication between units and as such for the coalition forming.

**Definition 3.1.6 (Coalition)**
A coalition $C$ is an aggregation of units, i.e. a nonempty subset $C \subseteq \bar{U}$. The set of all possible coalitions is $\bar{C} = 2^{\bar{U}} \setminus \emptyset$.

A coalition is an aggregation of agents or represented units with the same objective, e.g. the provision of ancillary services. The goal of coalition forming is that units are pooled in order to be able to act as a virtual power plant. This way, they are able to overcome market barriers and take part in AS provision. To this end, each unit must know its contribution such that in sum the coalition contribution amounts to the product amount.

**Definition 3.1.7 (Coalition Contribution)**
Let $AS_v = (T^v_{pr}, e^v_{pr}, c^v_{pr})$ be an ancillary service product with respect to $v$. The contribution $cont^v_C(T^v_{pr})$ of a coalition $C$ is the constant amount $e^v_{cont,C}(T^v_{pr})$ of ancillary-service quantity $q_v$ that must be available throughout the whole product horizon $T^v_{pr}$ with corresponding costs $c^v_{cont,C}(T^v_{pr})$, i.e.

$$cont^v_C(T^v_{pr}) = (e^v_{cont,C}(T^v_{pr}), c^v_{cont,C}(T^v_{pr}))$$.
Definition 3.1.8 (Unit Contribution)

Let $A_S v = (T^v, e^v, c^v)$ be an ancillary service product with respect to $v$. Further let $t_{pr,j} \subseteq T^v_{pr}$ be a connected subset of the product horizon. A contribution $cont^v_U$ of unit $U$ to the product $A_S v$ within the time interval $t_{pr,j}$ is defined as the constant amount $e^v_{cont,U}(t_{pr,j})$ of $q_v$ that unit $U$ contributes to $A_S v$ throughout $t_{pr,j}$ with corresponding costs $c^v_{cont,U}$, i.e.

$$\text{cont}^v_U(t_{pr,j}) = (e^v_{cont,U}(t_{pr,j}), c^v_{cont,U}(t_{pr,j})).$$

The interval $t_{pr,j} \subseteq T^v_{pr}$ is a subset of the whole product horizon that can be of arbitrary length. It holds that $T^v_{pr} = \bigcup_{j=1}^k t_{pr,j}$ and $\bigcap_{j=1}^k t_{pr,j} = \emptyset$. Within a time span $t_{pr,j}$ a unit has a contribution which is a pair of a constant amount of the ancillary-service quantity and annotated costs. This means that the contribution of units must not necessarily be the same for the whole product horizon. The reason for this is presented in the subsequent section. The costs of a contribution may be fixed costs, operational cost or marginal costs, for example. The summed contribution then is the contribution of a coalition for the time interval $t_{pr,j}$.

Definition 3.1.9 (Summed Contribution)

Let $C$ be a coalition consisting of units $\{U_1, ..., U_n\}$. The summed contribution of $C$ to an ancillary service product $A_S v = (T^v, e^v, c^v)$ w.r.t. $v$ within the interval $t_{pr,j}$ is defined as the aggregation of the contributions of all units $U_i$, $i \in \{1, ..., n\}$, i.e.

$$\text{cont}^v_C(t_{pr,j}) = (\sum_{i=1}^n e^v_{cont,U_i}(t_{pr,j}), \sum_{i=1}^n c^v_{cont,U_i}(t_{pr,j}))$$

$$=: (e^v_{cont,C}(t_{pr,j}), c^v_{cont,C}(t_{pr,j})).$$

For each time interval $t_{pr,j}$ where $T^v_{pr} = \bigcup_{j=1}^k t_{pr,j}$ and $\bigcap_{j=1}^k t_{pr,j} = \emptyset$ it must hold that the summed contribution equals the coalition contribution, i.e.

$$\text{cont}^v_C(t_{pr,j}) = \text{cont}^v_C(T^v_{pr}) \quad \forall j = 1, ..., k$$

in order to guarantee that the ancillary service product is available to its full extent throughout the whole product horizon.

3.2. Use Case: Primary Control Reserve

In this section, an instance of the formal model for the case of provision of primary control reserves is given in more detail. Subsequently, a concept for the provision of primary control reserves by distributed units based on [72, 59] is presented that serves as use-case for the presented research project.
3.2.1. Mapping to Formal Model

The ancillary service considered throughout this thesis is the provision of primary control reserves (cf. Chapter 2). First, the general description of an ancillary service is mapped to the use case of primary control reserve. Second, a concept for the provision of primary control reserve by distributed RPU is introduced.

The measurable quantity for the quality type of frequency stability is the system frequency of the power system, i.e. \( \nu = f \). The nominal frequency of the European power system is \( f_{\text{nom}} = 50Hz \). The feasible region is \( \pm 200mHz \) around the nominal frequency, thus \( A_f = [49.8Hz, 50.2Hz] \). The safety margin consists of a deadband of \( \pm 10mHz \) and a tolerated measurement error of \( \pm 10mHz \), thus \( D_f = [49.98Hz, 50.02Hz] \).

The ancillary service quantity with respect to system frequency is active power \( q_f = P \).

Summarizing, the system frequency should be kept around its set point of \( 50Hz \) and should not exceed the interval \( A_f = [49.8Hz, 50.2Hz] \). The service of primary control is demanded in case of frequency deviations from the nominal value. The cause for frequency deviations are imbalances between power production and consumption in the power system. The tendering of primary control takes place in a symmetric way, thus each unit providing reserves is obliged to provide both, positive and negative reserves. Hence, units must guarantee a margin where they can ramp up and down production or consumption. The right hand side of Figure 3.2 schematically shows a droop of a generating plant. Negative frequency deviations indicate that not enough power is in the system and thus a positive amount of power must be fed into the system or – leading to the same result – loads must be reduced. The other way around, if frequency increases, too much power is in the system and it must be reduced, thus either generators feed in less or loads increase consumption. Mitigation actions due to imbalances in the system are not be activated as long as \( f \in D_f = [49.98Hz, 50.02Hz] \). If a deviation \( \Delta f \) occurs that exceeds the safety margin \( \pm 20mHz \) then the ancillary service quantity must be activated according to the magnitude of the deviation \( \Delta e_f(\Delta f) \).

When it comes to the requirements of activating and delivering primary control reserves, units in the whole interconnected system are in duty, i.e. all units taking part in providing primary control reserves must activate the reserves no matter how far they are from the event triggering an imbalance in the system, i.e. the vicinity \( \text{vic}_{f} \) refers to the whole synchronous grid. The time of activation for primary control reserve is \( \partial t_f = 30 \) seconds and should cover a duration of activation of up to \( t_{\text{act},f} = 15 \) minutes.

In case of primary control reserve the amount of power that must be activated for a given frequency deviation is given by a \( f/P \)-droop in the unit governors. The amount of reserves that must be activated linearly depends on the frequency deviation according to the \( f/P \)-droop. This is a proportional controller for frequency response reserve that generators need to follow when they are operated in parallel (for details refer to \([71, 95]\)).

The frequency response reserve must be activated automatically by a unit controller.
if a frequency deviation is detected. The activation of the maximum amount of power takes place if the frequency deviation is $\pm 200\text{mHz}$. This maximum deviation must be guaranteed to be activated within the time of activation of 30 seconds.

3.2.2. Provision by Renewable Power Units

In the use case presented, control reserves are provided by RPU, more specifically PV and wind units. Within a planning scheme for primary control reserves of an individual agent, it determines the amount of power its corresponding unit is able to provide within a given product horizon based on forecasts. Since PCR products are tendered in a symmetric way, reserves must be available in both directions, i.e. a unit must provide a positive and a negative margin of power reserve, respectively, for a frequency deviation of $-\text{ or } +200 \text{ mHz}$. The contribution then amounts to twice the margin the unit must activate. This also determines the unit’s individual droop-control according to which the unit must activate the reserves as follows. Let $\text{cont}_U$ be the contribution of generating unit $U$. Then unit $U$ must provide a margin in both directions, i.e. increase or decrease production, up to $\left| \frac{1}{2} \cdot \text{cont}_U \right|$ in case of a frequency deviation of $\pm 200\text{mHz}$. The $f/P$-control is then given as

$$\Delta P(\Delta f) = \frac{\frac{1}{2} \cdot \text{cont}_U}{0.2\text{Hz}} \cdot \Delta f.$$  

(3.1)

As a consequence of the symmetric provision, a generating unit cannot feed-in to its maximum possible extent as long as the reserves are not activated. The set point $P_{\text{set}}$, i.e. the level at which the unit is feeding in must be at least $P_{\text{set}} \geq \frac{1}{2} \cdot \text{cont}_U$ or at most $P_{\text{set}} \leq \text{max feed-in} - \frac{1}{2} \cdot \text{cont}_U$.

Figure 3.2 schematically shows the symmetric provision. On the left hand side, a feed-in curve of a power generating unit is shown as a dotted line. The green-shaded area indicates the range the setpoint $P_{\text{set}}$ must lie in to guarantee that the upper and lower margin (indicated as red-shaded areas) are available at all time. The grey line indicates...
3.2. Use Case: Primary Control Reserve

the constant contribution which is the sum of upper and lower margin. For the exemplary set point (green-dashed line) the droop is shown on the right hand side of Figure 3.2. It specifies the change of power feed-in depending on frequency deviations outside the safety margin.

As the unit an agent represents typically cannot meet the requirements for an ancillary service product on its own, agents communicate with each other in order to form coalitions. This way they are able to make a summed contribution to meet appropriate prequalifications and overcome market entry barriers.

Once a bid of a coalition \( C = \{U_1, \ldots, U_m\} \) has been accepted by the market the coalition is obliged to provide the stipulated reserves. The concept is summarized in the following definition.

**Definition 3.2.1 (Base Coalition)**

A base coalition \( C_B = \{U_1, \ldots, U_m\} \) is a set of units that have agreed to provide an ancillary service product during the whole product horizon \( T_{pr} \).

The bidding of the units \( U_1, \ldots, U_m \) within the base coalition have been planned based on long-term forecasts that inherently are subject to errors. Thus, there is redundancy in the base coalition. Because PV and wind units may be fluctuating and the product horizon may be quite long, the concept of the following definition has been introduced. With that it is being taken advantage of the fact that more accurate short-term forecasts are available during the product horizon.

**Definition 3.2.2 (Core Coalition)**

A core coalition \( C = \{U_1, \ldots, U_n\} \) is a subset of a base coalition \( \emptyset \neq C \subseteq C_B \) that actively provide control reserves, i.e. \( \text{cont}_{U_i} \neq 0 \), for a time horizon \( t_{pr} \subseteq T_{pr} \). The time interval \( t_{pr} \) is referred to as lifespan of the core coalition. It holds \( n \leq m \), however the units do not have to be ordered in the same way.

All units \( U_i, i = 1, \ldots, n \) within the core coalition have a constant, non-zero contribution \( e_{\text{cont},U_i}(t_{pr,j}) \neq 0 \) for the whole interval \( t_{pr,j} \), where \( T_{pr} = \bigcup_{j=1}^{k} t_{pr,j} \) and \( \bigcap_{j=1}^{k} t_{pr,j} = \emptyset \). Consequently, \( e_{\text{cont},U}(t_{pr,j}) = 0 \) for \( U \in C_B \setminus C \). Again, it must hold that the summed contribution equals the base coalition’s contribution, i.e. \( e_{\text{cont},C}(t_{pr,j}) = e_{\text{cont},C_B}(T_{pr}) \) for all time intervals \( t_{pr,j}, j = 1, \ldots, k \).

Before each interval \( t_{pr,j}, j = 1, \ldots, k - 1 \) has past a new core coalition must have formed. Thus, the coalition reconfiguration is triggered in time before the end of the interval \( t_{pr,j} \). Within the reconfiguration phase the agents of the whole base coalition communicate with each other and agree upon the members of the next core coalition and their contributions. This amounts to an optimisation problem of finding a core coalition whose summed contribution amounts to the base coalition’s contribution with the objective of minimizing costs and maximizing the coalition’s lifespan.

The length of \( t_{pr,j} \) may vary. This depends on the prediction uncertainty and the resulting reliability of contributions. It is assumed that a minimum reliability must be fulfilled throughout the whole product horizon. Thus the reliability must be regarded as a constraint during coalition forming. Within the reconfiguration phase one objective (besides
minimising costs) is to find a lifespan for which the minimum reliability can be kept. Figure 3.3 visualizes the concept. In the depicted example there are ten units in the base coalition. For each of the three time intervals $t_{pr,j}$ only a subset of units are responsible for providing reserves, i.e. they have non-zero contributions. The length of the lifespan of the core coalitions depends on the reliability with which the summed contribution can be provided. Before the end of each core coalition’s lifespan a reconfiguration is triggered to form the next core coalition. The time to find a new core coalition is restricted since at the end of one coalition’s lifespan the next coalitions must have formed. Thus, the coalition forming process is subject to real-time constraints.

The process of forming a coalition is visualised in Figure 3.4. It is adapted from the The Foundation for Intelligent Physical Agents (FIPA) iterated contract net interaction protocol that has been chosen for agent communication in the research network Smart Nord and adapted for the provision of AS in [75]. This FIPA interaction protocol defines the process for communication and message exchange between agents. The objective within the presented use case is to provide PCR. Thus, the corresponding AS-product is divided into subproducts for which unit agents can propose. In Figure 3.4 the communication between the initiator and one responder is depicted and the steps are annotated with notes relating to the provision of PCR. Initially (Step 0), the product with a corresponding minimum required reliability and the neighbourhood, i.e. the set of agents to include...
into the coalition forming process, have to be specified to the initiator. In case of core coalition forming, the base coalition constitutes the neighbourhood. Each agent within the neighbourhood is a responder and one of them additionally takes the role of the initiator. In Step 1, the initiator sends a call for proposal (CFP) to all responders including the one it mimes itself. After checking if the CFP can be satisfied with a unit contribution (Step 2) each responder sends a refusal or a proposal to the initiator (Step 3). In Step 4, all proposals are evaluated by the initiator with respect to the coalition’s objectives and constraints. In this step the coalition reliability is assessed. The proposals are accepted or rejected in Step 5. The Steps 1 till 5 can be repeated in case the initiator is not able to find a solution that satisfies the requested product. Then a new call for proposals is sent. To start a new iteration, the initiator sends a new round of CFP to the responders. In the final iteration, the initiator informs all responders about their refused or accepted proposals such that in case of acceptance the responders can set their droop control (Step 6). In Step 7 the responders inform the initiator about the result of processing the proposals. In Step 8 the initiator collects all results and in case it received a sufficient number of responses the initiator can e.g. give bids at energy markets. For a detailed description of the process refer to [75].

The coalition forming process must be finished within a certain amount of time. This restricts the number of iterations. The coalition forming for a base coalition – according to the current German market setting (see Section 2.1.3) – has to be performed days ahead of the product horizon. Hence, depending on when the coalition forming process is initiated, the temporal restriction of finding a coalition is up to five days. In case of core coalition forming however, the coalition forming process is temporally restricted by the lifespan of the previous core coalition. In the use case presented in Figure 3.3, the reconfiguration time is set fixed thus giving a lower bound for a coalition’s lifespan. A common time interval for energy products is 15 minutes. Thus it appears appropriate that a reconfiguration process does not take more than that amount of time.

As mentioned before, an indication for the quality of a coalition and with that the length of its lifespan is the coalition’s reliability, i.e. the reliability with which it can provide an AS product for a given time horizon. A method to assess a coalition’s reliability is proposed in the subsequent chapter. This method can be utilized during the coalition forming process in order to check whether a coalition fulfils the minimum reliability while optimizing an objective function such as minimizing costs or maximizing lifespan (see e.g. [41, 59, 72, 75]). The focus is laid on the assessment of core coalitions where short term forecasts are assumed to be available. Besides the use case presented, the reliability assessment method can also be used to assess base coalitions or in general virtual power plants.

3.3. Summary and Discussion

In this section, a formal model has been presented to describe ancillary services to maintain a feasible system state as well as the corresponding requirements and the concept of AS product. This formal model may serve as a framework to describe different types of
ancillary services – as exemplarily presented for frequency and voltage control ancillary services – and map the services and their properties to each other. Based on that, requirements may be compared and methods regarding the quality of one ancillary service product – as the RelACs-method to assess reliability developed in this project – may be adapted based on that framework.

Furthermore, the concepts of unit, agent and coalition have been formally introduced which form the basis of describing an agent-based coalition forming procedure and its assessment. An agent-based coalition forming process has been presented that has the objective of forming coalitions for providing primary control reserves. This serves as a use-case for this research project. For this reason, the framework for the description of ancillary services has been applied to the case of primary control reserves.

The coalition forming process presented as the use case has the objective of finding an appropriate summed contribution for a coalition while minimizing costs and maximizing the coalition’s lifespan. Furthermore, a minimum reliability must be kept. Hence, reliability is a constraint during coalition forming. For this reason, an appropriate measure for the reliability of a coalition regarding its ability to provide control reserves is needed that can be incorporated during coalition forming under real-time restrictions. A corresponding method is presented in the next chapter. Note that the coalition forming process is not part of this thesis. The proposed assessment method may also be used to assess arbitrary coalitions or DVPPs.
4. Reliability Assessment of Ancillary Service Coalitions – The RelACs-Method

The provision of ancillary services is crucial for maintaining system stability. As pointed out earlier, given a decentralisation of the energy production, the concept of providing ancillary services must change as distributed units substitute conventional power plants and take over their tasks. However, decentralised generators are often highly volatile because of the fluctuating character of primary energy sources such as wind and solar power. Thus, one question is whether those units are able to provide power reserves in a constant manner.

The concept of unit coalitions as introduced in Chapter 3 yields a paradigm and a concept for aggregating units in order to overcome market barriers and act as a virtual power plant. However, the question arises what the implications are regarding reliability of AS supply. In this chapter, a method for the reliability assessment of ancillary service coalitions – the RelACs-method – is introduced to answer this question and the research questions given in Section 1.3 serve as a guideline. The focus is laid on the assessment of core coalitions for the provision of primary control reserves where short term forecasts are assumed to be available.

In Section 4.1 the requirements are derived from the previous chapter. In Section 4.2 the definition of reliability in the context of ancillary service provision by distributed units is given and a hierarchical model for reliability assessment introduced. This points out that a coalition’s reliability is based on the reliability of its member units. Thus, in Section 4.3 the process of reliability assessment for one unit is introduced followed by the method for the whole coalition in Section 4.4. This chapter finishes with a summary and discussion.

4.1. Requirements for Reliability Estimation

In the previous chapter the concept of coalition forming has been introduced that has the goal of aggregating units such that they are able to provide ancillary services. For the presented research project the use case of primary control reserve has been chosen (see Section 3.2). Ancillary service coalitions do not only have the objective of satisfying market entry barriers. As a constraint it should also be taken into account how reliably the coalition provides ancillary service products like primary control reserves. This is crucial as the provision of reserves is necessary for system stability. To this end, a method for assessing the reliability of a coalition is needed that can be incorporated into the coalition forming process.
The following aspects are taken into consideration when developing the reliability-assessment method. They form the background of the research questions introduced in Section 1.3.

**Availability** Each member unit of a coalition must be available throughout the whole product horizon, i.e. be in the state to activate reserves. This holds for both the unit availability and the availability of operational equipment.

**Unit availability** relates to failures of units as a consequence of deterioration, e.g.

**Operational equipment availability** relates to units disconnected from the system as a consequence of failures of operational equipment. Those units are not available for providing their contribution.

Only the time to a failure is important here. In that case, availability is equivalent to reliability (refer Section 2.2.1 for terminology). The consequence of a failure of one unit is that the coalition cannot guarantee the contribution to which it has itself committed. Because of temporal restrictions, the remaining available units cannot renegotiate and compensate for the loss of contribution.

**Forecast uncertainties** Agents plan the amount they are able to contribute to an ancillary service product based on predictions that inherently bear uncertainties. Those uncertainties are taken into account.

**Dependencies** Units being dependent on weather conditions such as PV and wind units show dependencies in their power feed-in given they are spatially close to each other. These might influence the reliability of a coalition and are taken into account.

**Contribution amount** In case an ancillary service is called, the AS coalition must guarantee that the required amount can be delivered. Thus the amount of each unit’s contribution must be available to its full extent during product horizon.

The latter aspect relates to the first two, i.e. if a unit is not available this means that it cannot fulfil its contribution. Uncertainties during planning may also lead to the fact that units are not able to deliver to the full extent of their contribution amount. However, the ability to deliver the amount that is demanded is crucial.

Given the setup of Section 3.2 there are certain requirements that must be fulfilled by the RelACs-method. The requirements are listed in the following distinguishing between conditions that reflect the aspects above, the functional and non-functional requirements as introduced in e.g. [91] or [104]. It is assumed that a coalition has already formed and the contributions of all member units are specified for a specific time horizon – the lifespan of the coalition. Note, that the following formulation holds for a coalition or DVPP in a general sense. Of course, this includes core coalitions as introduced in the previous section.
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Conditions

C1 The RelACs-method should consider the possibility of unit failures.
C2 The RelACs-method should consider the possibility of failures of operational equipment if this results in units being disconnected from the system.
C3 The RelACs-method should consider uncertainties due to prediction errors.
C4 The RelACs-method should consider dependencies between units’ power feed-in behaviour.

Functional requirements

FR1 The reliability metric returned by the RelACs-method must reflect the reliability of a coalition with units of arbitrary technology and installed capacity that are able to follow a droop control.

Non-functional requirements

NFR1 The reliability metric returned by the RelACs-method must be processable by the coalition forming process.
NFR2 The reliability metric returned by the RelACs-method must be returned in real-time as specified by the coalition forming process.
NFR3 The computational time of the RelACs-method must scale with the number of units within the coalition.

There are no particular requirements regarding the interfaces between a coalition forming process and the RelACs-method. Besides the coalition’s lifespan and its units’ contributions the inputs that have to be handed to the RelACs-method have to be specified during the design and development process. The specification is given in Section 5.1.

4.2. Definition of Reliability for RelACs

In this chapter, the model behind the RelACs method is introduced. The model may be regarded as a basic architecture for the RelACs-method. With it, a coalition is evaluated with regard to its quality to provide an ancillary service as specified in Chapter 3. In this context, the term reliability as used throughout this thesis is introduced. The following definition takes into account the requirements given in the previous section and yields a measure for the ability of a coalition to provide an ancillary service product.

**Definition 4.2.1 (Reliability)**
Reliability of a coalition with respect to the provision of an ancillary service product is the probability with which this product can be provided within a product horizon under normal operational conditions.

This definition incorporates both failures of units and prediction errors since they can be estimated by probabilities. Thus it is possible to determine the overall probability with
which all unit contributions are delivered as stipulated. Normal operational conditions are conditions and stresses for which the system has been designed. For the reliability assessment, concepts of reliability theory from Appendix A.2.1 are adapted. For the remainder the term reliability is used in the meaning of Definition 4.2.1. If the term is used in another context such as unit reliability then it is mentioned explicitly.

The reliability of a coalition with regard to the provision of an ancillary service product depends on the reliability of all units that are members of the coalition. For this reason, a hierarchical model for reliability assessment is proposed. The reliability of a unit is influenced by several factors. Those criteria are differentiated into controllable and non-controllable factors. This leads to a hierarchy for the evaluation of a coalition’s reliability, which is shown in Figure 4.1.

![Figure 4.1: Hierarchy for reliability evaluation](image)

The non-controllable factors affect a unit’s reliability but they cannot be manipulated or altered in a way that the reliability of a unit - with regard to its contribution - is changed. They form the basis for determining a unit’s reliability. The non-controllable factors again are divided into behavioural and positional factors. The behavioural factors determine the behaviour of a unit which is either the predicted power feed-in that might be obtained from weather forecasts or the failure probability of a unit. The positional factors correspond the location of a unit. On the one hand, this is the spatial or geographical position as specified by longitude and latitude. The spatial vicinity influences the dependencies between units’ power feed-in. On the other hand, this is the topological position, which coincides with the grid node at which the units are located. Note that the categorisation of non-controllable factors into behaviour and position is not exhaustive. Other factors may also be taken into consideration such as the communication network between agents or the sensor infrastructure that delivers information to the agents. Unfortunately, this is out of the scope of this thesis.

As opposed to non-controllable factors an agent is able to adjust the controllable factors lifespan, contribution, and accepted reliability level in order to plan for its unit and achieve a certain objective (e.g. that of a target reliability). The lifespan reflects the time horizon during which an agent wants to make the contribution. The length of the lifespan
corresponds with the prediction horizon and hence the quality of predictions with which the agent plans its contributions. The reason is that the smaller the prediction horizon the better the correctness of the prediction. In other words, the error of a prediction is assumed to increase with time. The contribution is the provided amount of an ancillary services quantity to the product. Note that in context of primary control reserves the term contribution relates to a margin for providing both positive and negative reserves (for Details refer to Section 3.2.2). For the remainder the term contribution is used including both meanings. The accepted reliability level constitutes the minimum reliability a unit must exhibit regarding its contribution which e.g. is a constraint during coalition forming. Details on the controllable factors are found in Section 4.3.

Note that for the reliability assessment of a coalition, the controllable factors are fixed, because the coalition is assumed to have already formed. In this way, the coalition contribution and hence all unit contributions are fixed and for this configuration the reliability value is calculated. During coalition forming though, the controllable factors of each unit may be adjusted in order to fulfil an agent’s objective or to optimize the coalition’s configuration, i.e. the contributions of its member units. How a unit’s reliability is determined is the topic of the subsequent Section 4.3. The interdependence of the controllable factors on each other and relationships between each other are discussed as well.

The constellation of the set of units that are incorporated in the coalition forming process is also a controllable factor. Thus, the choice of units, for instance, depend on the spatial position or the topological position that have influence on dependencies between units. The calculation of a coalition’s reliability, i.e. a fixed set of units with fixed contributions during a time horizon, is presented in Section 4.4.

4.3. Unit Reliability

In the previous section, it has been stated that the reliability of a unit depends on certain factors – controllable and non-controllable factors. In this section, it is specified how a unit’s reliability is determined by the RelACs-method. Here the reliability of a single unit with respect to the behavioural factors is discussed, i.e. forecasts and failures. Figure 4.2 shows the process for the unit-reliability assessment the steps of which are discussed in the following.

![Figure 4.2: Process for unit reliability assessment](image-url)
1. **Categorise unit behaviour** There are different types of units. Different types imply different properties and behaviours. For this reason, units must be categorised with regard to their type. Naturally, a unit may be categorised by its technology type, i.e. whether it is a solar module or a wind turbine. Moreover, the behaviour of a unit can be categorised, which is the focus of the first step.

For the assessment of unit reliability, the behaviour of a unit is crucial. Here, *supply-dependent units* are considered. The power feed-in of units depends on weather conditions such as wind and solar irradiation. This is discussed in more detail in Section 4.3.1.

2. **Specify unit behaviour** In the context of this thesis, a unit’s behaviour, on the one hand, is the way of how it specifies and plans its power output or consumption. This is specified by a prediction of power feed-in. The forecast of the behaviour naturally is subject to errors that must be taken into account when assessing the reliability. To this end, a model of the errors must be known. Of course, the forecasts and error models vary amongst units. Hence, for each unit the individual behaviour based on forecasts must be specified.

On the other hand, a unit’s behaviour is given by its failure rate. Here as well, each unit has individual values depending on e.g. brand and age. Details are discussed in Section 4.3.2.

3. **Assess reliability** Based on the individual forecasts, corresponding error and failure models, the reliability of each unit are determined given a fixed time horizon and contribution. This is found in Section 4.3.1. The assessment based on failures is discussed in Section 4.3.2.

4. **Determine reliability value** As a final step, the value for the reliability is computed.

Subsequently, for both behavioural types – forecast and failures – the whole process is explained independently.

### 4.3.1. Reliability Based on Forecasts

Supply-dependent units are units that produce power dependent on fluctuating RES such as wind or solar irradiation. Hence, those units are only able to deliver power up to an amount relative to the actual supply of wind or solar irradiation. The technologies considered in this thesis are PV and wind units. However, this method may be applied for other technologies as well the units of which are able to follow a droop control. Not only solar irradiation and wind power but other factors such as ambient temperature for solar panels or the height of a wind turbine’s hub also influence the power production. However, this is not in the scope of this thesis.

It is not possible to accurately predict the weather phenomena mentioned before and thus neither is it possible for the power feed-in of a supply-dependent unit. If an agent wants to plan for a supply-dependent unit, it has to do this based on forecasts. As already
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indicated, forecasts inherently are subject to errors. Those errors have to be taken into account when it comes to reliability assessment and determining a unit’s contribution. In order for an agent to deal with this, it has to know the forecast as well as a quantification of the forecast errors. In what follows the forecast of a supply-dependent unit is defined as well as a corresponding error model. This corresponds to Step 2 in the process for unit reliability assessment.

Specification of Unit Behaviour

The behaviour of a supply-dependent unit is described by a power forecast and – since forecasts cannot be calculated correctly – its corresponding error model. First, a definition of forecast is given as well as of the corresponding temporal framework. These definitions are based on common concepts that can be found for example in [2, 80]. After that, the error model is derived and defined.

Note that the terms forecast and prediction are used synonymously throughout this thesis. Unfortunately, investigations of forecasting methods are beyond the scope of this thesis. Methods used in practice are for example sophisticated forecasts using detailed observations of weather phenomena as input, as for example in [74], or forecasts based on time series using time series analysis tools or even methods from machine learning, see e.g. [100]. Moreover, there are different methods for different time horizons, i.e. for example short term and long term forecast. They are utilised depending on their applications and time scales. For investigations regarding ancillary services and core coalition forming, short term forecasts are of interest that cover a short prediction horizon and have a finer temporal resolution. Here, point forecasts are used (see Section 2.3).

For the development of the reliability assessment, it is assumed that each agent has knowledge about its unit’s forecast regardless of how the forecast was obtained. However, it is assumed, that the forecast method that an agent uses is fixed. If an agent changes its forecast method, the error model must be determined again.

**Definition 4.3.1 (Forecast)**

A forecast or prediction of a unit’s power output (or consumption) is a time series of expected (active) power values \( \{x_{t_0+1, \ldots, t_0+k}\} \) for equidistant time steps \( t_0 + 1 \leq t \leq t_0 + k, k \geq 0 \). The power values have a positive sign if power is produced and negative if power is consumed. If it is obvious from the context that a prediction is considered, it is simply denoted as \( \{x_{t_0+1, \ldots, t_0+k}\} \).

The interval \([t_0 + 1, t_0 + k] \) is called prediction horizon or prediction interval whereas the interval \([t_0 - l, t_0] \) for some \( 1 \leq l \leq t_0 \), which the prediction is based on, is called the support of the forecast or the observation interval.

As the previous definition indicates, the support or observation interval is the period within which the forecast is determined, whereas the prediction horizon or prediction interval is the period for which the forecast is made.

As already mentioned, forecasts are subject to errors and they cannot predict the future behaviour accurately. To this end, the quality of a forecast must be evaluated. This is done
by calculating the difference between the predicted values and the actually measured values. Of course, this is only possible when the prediction horizon has passed since only then the measured values are known. The subsequent definition summarises this.

**Definition 4.3.2 (Forecast Error)**
Let $x_{\text{pred},t}$ be a predicted value at time $t$. Further, let $x_{\text{meas},t}$ be the corresponding measured value. The forecast error or prediction error at time $t$ is defined as the difference $\varepsilon_t = x_{\text{meas},t} - x_{\text{pred},t}$.

If the prediction errors of a unit are observed over a longer period of time, for a fixed prediction method and prediction horizon, an error model for the unit may be derived. Given a sequence of errors $(\varepsilon_{t_1}, ..., \varepsilon_{t_n})$, the frequency of occurrence of errors is determined. Given that, the empirical probability distribution can be estimated and, if possible, approximated by a parametric probability distribution. However, the error distribution varies with different lengths of prediction horizons. The resulting error model forms the basis of the RelACs method. After the following definition, the procedure to obtain an agent’s error model is explained in more detail.

**Definition 4.3.3 (Error Model)**
The error model of an agent and its respective unit consists of two properties.

First, it is defined as a random variable $X$ that describes the deviations from the predicted value, i.e. the occurrence of prediction errors. The corresponding distribution function is denoted $F$.

Second, it is defined as the evolution of the error, i.e. the development of the error distribution over time.

Note, that the error model may also depend on factors other than the prediction horizon. E.g. in case of forecasts of pv power the quality of predictions and with that the prediction error depends on the cloud cover (See e.g. [74]). For different situations different error models may exist.

Each unit has its specific characteristic error model, i.e. error distribution and error evolution. For instance, the distribution might be a normal distribution and the development given as standard deviation might take a logarithmic or a square-rooted slope. Hence, this must be investigated for each unit individually. The specification of the error model consists of the following two procedures.

**Procedure 4.3.4 (Error Distribution)**

1. **Observe occurrence of errors** For data at hand consisting of forecast values and actual measures determine all prediction errors $(\varepsilon_{t_1}, ..., \varepsilon_{t_n})$. For this, the empirical error distribution is determined. That is $\hat{F}_{N+1}(x) = \frac{1}{N+1} \sum_{i=0}^{N} \mathbb{1}[\varepsilon_i \leq x]$, i.e. for a data set of $N + 1$ data points $\varepsilon_i$, the empirical distribution function $\hat{F}$ determines for a real number $x$ the ratio of data points $\varepsilon_i$ that are smaller or equal to $x$. This may be visualised by a normalised cumulative histogram. An impression of the empirical density function can be obtained by a normalised histogram.
2. **Fit parametric model to empirical distribution**  The empirical distribution and respective histograms indicate the distribution of prediction errors for a unit. In case of distributed generation units, errors resemble continuous distributions of parametric families such as normal or beta distributions (see e.g. [56, 74]). In order to estimate the parameters of such a statistical model the methodology of maximum likelihood estimation may be utilized. For details on the the maximum-likelihood method refer to e.g. [20]. Hence, the error distribution is described by a distribution given the individual parameters of the unit’s errors, e.g. mean and standard deviation. It is said that the data has been fitted to a parametric model.

3. **Validate the model**  In order to estimate how good the parametric distribution fits the data, the so called Kolmogorov-Smirnov test may be utilised. This is a statistical hypothesis test that evaluates the empirical distribution against the fitted parametric distribution. With that also the error fitting the data to a statistical model is estimated. For details on statistical hypothesis testing and especially the Kolmogorov-Smirnov test refer to e.g. [20].

The error of a prediction is assumed to increase with time, i.e. with increasing length of prediction horizon. For reliability assessment, one is interested in an algebraic description of this development.

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**Procedure 4.3.5 (Development of Errors over Time)**

1. **Observe the development of moments over time**  Consider the mean and standard deviation (so called first and second moment of a distribution) of the fitted distribution from Step 1 of Procedure 4.3.4. and their development over time. For different lengths of the prediction horizon \( k_1 < k_2 < \ldots < k_m \in \mathbb{N} \geq 0 \) fit the empirical error distribution to a parametric distribution as described in Step 1 of Procedure 4.3.4, obtaining the means \( \mu(k_j) \) and standard deviations \( \sigma(k_j) \) for \( j = 1, \ldots, m \). Note that in order to represent the development of prediction error in this way the family of distributions must be the same for all prediction horizons, e.g. a normal distribution. If the type of distribution changes over time, the representation of the moments’ development must be adapted accordingly.

2. **Fit regression models**  Given the data points \((k_1, \mu_1), \ldots, (k_m, \mu_m)\) and \((k_1, \sigma_1), \ldots, (k_m, \sigma_m)\) a regression is conducted to obtain a description of the error depending on time such as \( \mu = \mu(t) \) or \( \sigma = \sigma(t) \). For details on regression refer to e.g. [20].

3. **Determine best model**  There are different types of functional relationships to fit a regression model, for example linear, logarithmic, square, or root. These models are compared with each other in the sense of how good they fit the relationship between time and \( \mu \) or \( \sigma \), respectively, using the so-called coefficient of determination. The best model is chosen for a description of the error evolution.
Given a supply-dependent unit’s specific behaviour as forecast and error model, Step 3 of the reliability assessment process is conducted.

Assessment of Reliability

As mentioned before, the reliability of a unit depends on the forecast and corresponding error model as well as the controllable factors lifespan, contribution, and reliability level. The controllable factors again depend on each other, i.e. two factors must be given in order to derive the third one.

Subsequently, the controllable factors are introduced in more detail before the method of reliability assessment is elaborated. The relationship between the three factors becomes clearer after the process for reliability assessment has been introduced.

Lifespan The lifespan, i.e. the time horizon for a unit plans its contribution, determines the prediction horizon. The length of the prediction horizon influences the quality of a prediction. For a fixed prediction method a longer prediction horizon leads to higher prediction errors. This is represented by the standard deviation of the error model that is increasing for increasing prediction horizons. The other way around, the smaller the prediction horizon the better the correctness of the forecast. This means that a contribution may be provided with a higher reliability for shorter lifespans and equivalently longer prediction horizons.

Contribution The contribution is the provided amount to an ancillary service product. Smaller contributions can be provided with higher reliability.

Reliability level This is the reliability of a contribution for a given time horizon, i.e. lifespan. It might be for example a level that must be guaranteed during coalition forming. If this level is being relaxed, higher contributions might be accepted.

In the following, the process for unit reliability assessment is explained. It consists of three steps. At first, it is assumed that the time horizon and the contribution of the unit are fixed. Later on in this section, it is pointed out how to deal with the variation of the controllable factors. Let $T$ be the lifespan and consider a unit $U$. Since the unit already made its contribution, the lifespan $T$ is of fixed length. Further, let $X_U$ be the random variable describing the unit’s prediction errors with distribution function $F_U$ the fixed prediction horizon.
Procedure 4.3.6 (Unit Reliability)

1. **Determine minimum of prediction** A requirement for the provision of ancillary services is that the service is available to its full extend throughout the whole time horizon $T$ (see Section 4.1). Particularly, this must be guaranteed for the point in time with lowest value of unit $U$’s forecast. For this reason the minimum value of the prediction $\min_{t \in T} \text{pred}_U(t)$ is determined.

2. **Determine accepted deviation** Given the contribution $e_{\text{cont},U}$ of unit $U$ and the minimum predicted value from the previous step, the accepted deviation $x$ from the prediction is calculated as $x = e_{\text{cont},U} - \min_{t \in T} \text{pred}_U(t)$. This is the deviation that must not be exceeded in order to guarantee the provision of ancillary service for every $t \in T$.

3. **Determine reliability with survival function** Given the error model, the reliability of the unit with regard to its contribution is calculated. This is the probability that the deviation from the predicted value is not beyond the accepted deviation $x$ determined in the previous step, i.e. the probability $\Pr(X_U > x)$.

The probability $\Pr(X_U \leq x)$ is calculated directly from the distribution function as $\Pr(X_U \leq x) = F_U(x)$. The probability of interest is the corresponding complementary probability and thus is calculated as $\Pr(X_U > x) = 1 - F_U(x)$. In case of a continuous distribution it holds $\Pr(X_U \geq x) = 1 - F_U(x)$. In accordance with Appendix A.2.1 on techniques of reliability theory the latter function is called survival function. For completeness, the following definition is given.

**Definition 4.3.7 (Survival Function)**
Let $X$ be a random variable with distribution function $F$. Then the function $1 - F$ is called survival function of $F$.

The following result yields the reliability with respect to forecasts.

**Result 4.3.8 (Unit Reliability)**
In summary, the reliability $\rho_{\text{cont},U}(t)$ for the contribution $e_{\text{cont},U}$ of unit $U$ is calculated using the following equation.

$$\rho_{\text{cont},U}(t) = 1 - F_U\left(\frac{e_{\text{cont},U} - \min_{t \in T} \text{pred}_U(t)}{x}\right). \quad (4.1)$$

Note that the survival function sometimes also is referred to as reliability function. In classical reliability theory the distribution function is defined on the interval $[0, \infty)$ such
that the survival function cannot take negative arguments. The reason for this is, that usually life times are investigated that are non-negative. However, for the RelACs-method, the term survival function is used even if the range is $\mathbb{R}$ as it is done in [84] for example.

In the following example, the computational steps for unit-reliability assessment given in Procedure 4.3.6 are exercised. In Figure 4.3 the above steps are depicted. Figure 4.3a shows the prediction of a unit’s power feed-in for a given time horizon. The contribution of the unit is plotted as a constant grey line since it must be guaranteed that it is possible to activate the contribution, i.e. reserves, throughout the whole time horizon. The accepted deviation $x$ that must not be exceeded for all time steps is plotted at the lowest value of the prediction (Step 1&2). The reliable deviation from the prediction is indicated as grey dotted line.

Figure 4.3b shows the density function of the (fixed) error model for the time horizon as the light blue curve as well as the corresponding survival function as gray line. The accepted deviation $x$ also is given. The probability that the deviation from the forecast does not fall below $x$ is given as the light blue shaded area under the density curve and right of $x$, and – alternatively – the value of the survival function at $x$ respectively. This corresponds to the reliability of the unit’s contribution (Step 3).

So far, the reliability for the worst case has been introduced, i.e. a system frequency deviation of $\pm$ 200 mHz. However, frequency deviations with a smaller absolute value occur more often than frequency with a high absolute value. The system frequency can be measured throughout the whole system. With the data, a model of frequency deviations is inferred similarly as described for prediction errors presented above. This results in an empirical or a fitted probability distribution function describing the probability of frequency deviations. There may be different models suitable for different week days or time intervals depending on system loadings. For the RelACs-method it is taken advantage of the fact that frequency deviations have different frequencies of occurrence. For frequency deviations with a small absolute value the contribution that must be activated
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Figure 4.4: Relationship between absolute value of frequency deviation, unit contribution and reliability

is smaller thus having a higher reliability. Figure 4.4 exemplarily visualizes this. In the example case the system frequency deviations are represented by a normal distribution. To incorporate the presented fact in the RelACs-method the following preparations are made. Subsequently, it can be integrated in the steps given above.

Procedure 4.3.9 (Preparation of Frequency Deviations)

1. Model frequency of occurrence of frequency deviations Since different levels of frequency deviation have different probabilities of occurrence a distribution function $F$ of frequency deviations is determined.

2. Partition of frequency deviations Consider the interval $[-200mHz, 200mHz]$ between the maximum and minimum feasible frequency deviation. This is partitioned into equidistant intervals. Denote the intervals by $I_f = [a_f, b_f]$. 

3. **Determine probability of occurrence**  According to the model obtained by Step 1, for each of the intervals the probability of occurrence is calculated as

\[ \Pr(I_f) = F(b_f) - F(a_f). \]

4. **Determine contribution**  As stated in Chapter 3, in case of primary frequency reserve, the droop control gives the amount of power, i.e. contribution \( e_{\text{cont},U}(I_f) \) that unit \( U \) must provide for a given frequency deviation \( \Delta f \in I_f \). Since here intervals of frequency deviations are considered, the interval boundary with the highest absolute amount of frequency deviation is considered as specified in the following. This leads to an overestimation of the probability of occurrence.

\[
e_{\text{cont},U}(I_f) = \begin{cases} e_{\text{cont},U}(b_f), & \Delta f \geq 0 \\ e_{\text{cont},U}(a_f), & \Delta f < 0 \end{cases} \quad (4.2)
\]

Step 1 of Procedure 4.3.6 gives the minimum predicted value. Then for each interval of frequency deviations \( I_f \) the contribution \( e_{\text{cont},U}(I_f) \) as output of Step 4 of Procedure 4.3.9 is processed to Steps 2 - 3 of Procedure 4.3.6 returning the reliability \( \rho_{\text{cont},U,U}(U) \) of this contribution as

\[
\rho_{\text{cont},U,U}(I_f) = 1 - F_{U,T} \left( e_{\text{cont},U}(I_f) - \min_{t \in T} \text{pred}_U(t) \right). \quad (4.3)
\]

With the previous steps the following result is given.

**Result 4.3.10 (Unit Reliability Including Frequency Deviations)**

Given the partition of the interval \([-200\,mHz, 200\,mHz]\) into equidistant intervals \( I_f \) with corresponding contribution \( e_{\text{cont},U}(I_f) \) of unit \( U \) the reliability considering system frequency deviations is calculated as

\[
\rho_{\text{cont},U,U}(I_f) = \sum_{I_f} \rho_{\text{cont},U,U}(I_f) \cdot \Pr(I_f), \quad (4.4)
\]

where \( \rho_{\text{cont},U,U}(I_f) \) is the reliability of contribution \( e_{\text{cont},U}(I_f) \) and \( \Pr(I_f) \) is the probability of occurrence of interval \( I_f \).

This result gives a weighted sum for reliability values for different probabilities thereby making use of the law of total probability. Thus it is taken into account that contributions with higher reliability occur more often than contributions with lower reliability. Note that with this formulation it is possible to incorporate an arbitrary model for system frequency deviations. This model may be updated, e.g. for different days or seasons. If not stated otherwise, for the remainder the term reliability refers to the reliability as derived in Result 4.3.10, i.e. reliability with consideration of system frequency deviations.
This section finishes with a demonstration of the relationship between the three factors that may be controlled during coalition forming. If an agent is interested in the amount of an ancillary service quantity it is able to contribute within a fixed time horizon $T$ given a minimum reliability level $\rho$ it may proceed similarly: Equation 4.1 can be reformulated as

$$e_{\text{cont},U} = F_{U,T}^{-1} (1 - \rho) + \min_{t \in T} \text{pred}_{U}(t)).$$  \hfill (4.5)

Hence, given a reliability level an agent wants to achieve, it calculates the maximum amount it is able to contribute. This is the minimum of the prediction $\min_{t \in T} \text{pred}_{U}(t))$ (within the fixed time horizon) added by the value of the quantile function\footnote{The quantile function of a random variable with distribution function $F$ is defined as $F^{-1}(p) = \inf \{x \in \mathbb{R} \mid F(x) \geq p\}$ for $0 < p < 1$. If $F$ is invertible, $F^{-1}(p) = x$ with $x$ such that $F(x) = p$. See [20] for details.} at $1 - \rho$. Note that $F_{U,T}^{-1} (1 - \rho)$ may take negative values. This makes sense since the contributions are smaller than the minimum predicted value.

In case an agent wants to determine the length of the prediction horizon in order to fulfil a fixed contribution $e_{\text{cont},U}$ with a fixed reliability level $\rho$, it must consider the error distribution's development for different time horizons given by the error model (see Definition 4.3.3). Assume the data pairs $(k_i, \mu_i)$ and $(k_i, \sigma_i)$, $1 \leq i \leq m$ for different lengths of prediction horizons $k_1 < k_2 < \ldots < k_m$ describe the development of the moments mean and standard deviation, respectively. Then the error distribution functions for different prediction horizons $F_{k_i}$ are also different. This means, that the distribution function must be chosen (if it exists) such that for the fixed contribution, the reliability level is fulfilled, i.e.

$$\argmax_{k_i} \left\{1 - F_{k_i} \left( e_{\text{cont},U} - \min_{t \in T} \text{pred}_{U}(t) \right) \right\} \leq \rho \right\}. \hfill (4.6)$$

As mentioned before, the three controllable factors prediction horizon, contribution, and reliability level depend on each other. Two factors yield the third, as shown above. Figure 4.5 visualises the relationship. The $x$-axis represents the length of the prediction horizon given as relative standard deviation. The $y$-axis gives the contribution relative to the minimum predicted value. The corresponding reliability value is displayed on the $z$-axis.

As one can see, smaller contributions lead to higher reliability and the prediction horizon may be longer. Also for shorter prediction horizons reliability increases and contribution may be higher. The smaller the reliability level is, the higher is the contribution and the longer the prediction horizon can be. Furthermore, a plane is added representing a minimum accepted reliability level. Thus, the combination of prediction horizon and contribution for which the reliability lies above this plane fulfils the requirement.

If an agent has knowledge about this relationship between the controllable factors it may make a decision about the choice of controllable factors in order to achieve a certain goal. If for example the prediction horizon is fixed but an agent wants to fulfil a specific reliability level it may influence the contribution in the required way. This is of particular interest during coalition forming.
In this section the method for determining a unit’s reliability based on forecast incorporating forecast errors has been introduced. However, in case of a failure, a unit being in an ancillary service coalition cannot contribute to the assigned product. The incorporation in the RelACs-method is discussed in the subsequent section.

4.3.2. Reliability Based on Failures

As discussed before, units participating in providing ancillary services as primary control reserves must assure that their contributions are activated to the extend demanded. Thus, not only the deviation from the contribution due to fluctuating weather conditions is crucial. The unit must be functioning during the whole lifespan of the coalition in which the unit is participating. In case of a failure during the lifespan there is no possibility for maintenance. Consequently, the units’ reliability plays an important role, i.e. the probability that the unit performs the function of providing and activating reserves.

For the technologies of RPUs based on solar and wind power there are several studies available investigating the units’ reliability with respect to failure behaviour, e.g. [44, 47, 66, 99]. To incorporate unit failures into reliability assessment is straightforward. The
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reliability of a unit with respect to failures for a given time span is calculated using the survival function as given in Appendix A.2.1 provided the failure rate for the unit is known. A failure may have several causes that are reflected in the failure rate. As the RelACs-estimation is defined for normal operational conditions the failures of units leading to non-functioning are assumed to be independent.

However, this model is not included in the evaluation process for the RelACs-method. The failure rates of units strongly depend on e.g. technology, brand, but also site-specific conditions. Within the scope of this research project only qualitative statements about the unit failures’ influence could be made as no specific units are regarded and thus no specific data is available. The focus here lies on behaviour based on forecasts, the failure behaviour is neglected for calculation. Considering failure behaviour as well could bias results of the feed-in behaviour. For this reason, the unit failure behaviour is only introduced conceptually.

4.3.3. Summary

The ability of a unit being dependent on fluctuating RES to provide AS, in particular PCR, depends on weather conditions and possible failures. The latter would lead to a complete loss of the unit’s contribution to an AS product. Weather conditions and the corresponding power production is based on predictions. Thus, the contribution a unit is able to make to an AS product is subject to uncertainties. This may lead to the fact that the contribution cannot be delivered to the extent demanded. Thus, models have been presented to include these facts by estimating the reliability of a unit’s contribution as part of the RelACs-method. Deviating from a stipulated contribution due to unforeseen weather fluctuation and the event of a failure are regarded as independent. Thus, the reliability of unit $U$ including both is calculated as the product of both probabilities

$$\rho(U) = \rho_{\text{cont}}(U) \cdot \rho_{\text{fail}}(U),$$

(4.7)

where $\rho_{\text{cont}}$ denotes the reliability with respect to a contribution considering the feed-in behaviour and $\rho_{\text{fail}}$ the reliability with respect to unit failures.

As introduced in Chapter 3, in many cases one unit is not able to provide an AS product on its own. Consequently units form coalitions. Especially in case of supply-dependent units there are dependencies between the unit’s power feed-in when they are spatially close to each other. Furthermore especially in case of units being connected to radially operated distribution systems units may be disconnected from the system due to equipment failures. Thus there may also be dependencies considering the units’ topological position. In the subsequent section the RelACs-method is extended to assessing a coalition of units. This is based on unit reliability and includes dependencies between units.

4.4. Coalition Reliability

In the previous section, it has been introduced how to determine the reliability of a unit given an ancillary service contribution. As mentioned in Section 3.2, in many cases a unit
is not able to provide an ancillary service product on its own and thus units aggregate to coalitions. For system stability it is essential, that an ancillary service product is provided with a certain reliability.

As stated in Section 4.2, the reliability of a coalition with respect to the provision of an ancillary service product is the probability with which this product can be provided within a product horizon. In order for the coalition to guarantee that the contribution amount they have themselves committed to may be delivered throughout the whole product horizon the concept of core coalition has been introduced, i.e. a coalition that in this constellation exists only for a certain subset of the product horizon (see Section 3.2 for details). This time intervals is referred to as the lifespan of the coalition. In the following – if not specified otherwise – the term coalition refers to a core coalition that has itself commit to provide an AS product for a certain lifespan. Each member unit of the coalition must be able to provide its contribution to its full extent during the core coalition’s lifespan. It is not possible that units within a coalition compensate for each other because of the temporal restrictions with which the product must be delivered. Furthermore, units of the same technology show dependent behaviour with regard to power feed-in that has influence on the reliability of an ancillary service coalition. The higher the spatial vicinity the higher the dependence may be.

Besides the technical reliability of each member unit as discussed in Section 4.3.2 also the technical reliability of the operational equipment of the power grid the units are located in must be taken into account. Especially in radial systems this is important because if e.g. one line fails all units of the coalition that lie downstream this line are not available for contributing to the ancillary-service product even though the units themselves are functioning.

In this section a method is proposed to assess a coalition’s reliability taking into account spatial as well as topological dependencies between units. Figure 4.6 shows the process for the assessment the steps of which are briefly discussed in the following.

1. **Categorize unit positions** In case of the assessment of the whole coalition different categories are introduced for the positions of units within the coalition. The categories are the spatial and topological position. This categorization reflects the categories in Section 4.3, i.e. the unit behaviour regarding forecasts and failures. In case of spatial position the exact geographic positions do not necessarily have to be known.
Rather the influences and geographical vicinity of units that result in similar behaviour and thus dependencies are of importance. The topological position relates to the grid connection nodes of units.

2. Dependency model The dependency model specifies the dependencies between units. In the hierarchical model for reliability assessment in Section 4.2, the positions of units appear as non-controllable factors. In case of spatial position, the behaviour of a coalition’s member units with regard to forecasts and power feed-in play as well as possible dependencies regarding this behaviour. This is discussed in Section 4.4.1 and a dependency model is presented. The category of topological position on the other hand, has influences on the functioning of units in case of failures of network equipment. Thus units located in the same network section are dependent on the functioning of the same equipment. More details are given in Section 4.4.2.

3. Reliability evaluation Based on the dependency model, the reliability of a coalition is determined. This again, is conducted separately according to the positional categorisation. For the category of spatial position this is done in Section 4.4.1 and for the category of topological position in Section 4.4.2.

4. Reliability value of coalition As a result, the reliability value of the coalition under investigation is calculated.

In order to keep focus on the reasoning, the whole process is explained independently for the spatial position and the topological position.

4.4.1. Spatial Position

In order to determine the reliability of a coalition based on the spatial position of its member units the process introduced in the previous section (see Figure 4.6) must be conducted. However, the spatial position is not considered directly but the dependencies induced by spatial vicinity, i.e. dependencies between power feed-in behaviour between different units. Categorising units according to the geographical regions they are located in and drawing conclusion about their dependencies based on that is problematic since the geographical traits between units may be diverse, e.g. forests that have impact on wind speed or different altitudes that have influence on solar irradiation. Thus an approach based on time series and historical data is used to determine a dependency model.

Dependency Model

With the dependency model, the reliability of the coalition is evaluated. Here, as in case for one unit, the probability of occurrence of system frequency deviations can be incorporated. In the following it is assumed that a coalition consists of at least two units. Otherwise, the method in Section 4.3 is be utilised.

Since sets of units with more than one unit are investigated, concepts from multivariate statistics are needed, which are introduced at first. Subsequently, the concept of survival
4.4. Coalition Reliability

function, that is also important for reliability assessment for coalitions, is extended to the multivariate case. Multivariate distributions are not handled easily, especially when the marginal distributions – in the context at hand the units’ error distributions – are not identical and the random variables show dependencies. However, the methodology of copulas offers a powerful tool to deal with joint distributions and survival functions given arbitrary margins and dependency structure. After stating the main concepts of copulas, their usage with regard to reliability assessment is introduced.

Let \( C = \{U_1, \ldots, U_n\} \), \( n \geq 2 \) be the set of all units within a coalition. Furthermore, the respective error models are supposed to be known, i.e. the random variables \( X_1, \ldots, X_n \) describing the deviation from predicted values, corresponding continuous distribution functions \( F_1, \ldots, F_n \), and a description of how the distribution’s moments evolve with time. Since the coalition \( C \) has already been formed, the lifespan \([t_0 + 1, t_0 + k]\) and with that the prediction horizon is supposed to be fixed. Thus, the error distributions for the lifespan are fixed, too. Additionally, the units in the coalition \( C \) have committed themselves to provide a contribution \( e_{\text{cont.} U_i} \) to an ancillary service product, respectively. For each \( i = 1, \ldots, n \) the maximum accepted deviation from the minimum predicted value \( x_i = e_{\text{cont.} U_i} - \min_{t \in T} (\text{pred}_{U_i}(t)) \) is calculated. As in the case of one unit, one is interested in the probability, that for all \( i \) the deviation from the predicted value is not greater than \( x_i \). The reason for this is that all units must adhere to their contribution during the coalition’s lifespan and it is not possible that the contributions are altered during this time. However, in order to take regard of dependencies all units must be considered at once and the probability of interest is given as \( \Pr(X_1 \geq x_1, \ldots, X_n \geq x_n) \), i.e. the probability that no unit’s feed-in deviates as much from the prediction that it cannot fulfil its contribution. In order to deal with the joint probability, the errors of all units are modelled as a multivariate random vector.

Let \( X = (X_1, \ldots, X_n) \) denote an \( n \)-dimensional random vector and let the corresponding joint distribution function be denoted as \( F_X(x_1, \ldots, x_n) = \Pr(X_1 \leq x_1, \ldots, X_n \leq x_n) \). If the reference is clear, the notation \( F \) is used instead of \( F_{X_1, \ldots, X_n} \) or \( F_X \). Unfortunately, it is not possible to directly infer the probability of interest from the distribution function as in the one-dimensional case, because already for two random variables it holds that \( \Pr(X_1 \geq x_1, X_2 \geq x_2) \neq 1 - \Pr(X_1 \leq x_1, X_2 \leq x_2) \). However, the concept of survival function\(^2\) is extended to the multivariate case.

**Definition 4.4.1 (Joint Survival Function)**

The joint survival function \( \tilde{F}_X \) associated with a random vector \( X = (X_1, \ldots, X_n) \) is defined as

\[
\tilde{F}_X(x_1, \ldots, x_n) = \Pr(X_1 \geq x_1, \ldots, X_n \geq x_n).
\]

As already mentioned, in case of assessing the reliability of a coalition’s contribution the marginals are given as each member unit’s individual error model. The dependencies are modelled using so called Copulas. A short introduction to the concepts of Copula

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\(^2\) As for the one-dimensional case (see Section 4.3.1), the survival function is usually used in terms of random variables with the domain of definition of \( \mathbb{R}_{\geq 0} \) because they represent life times. However here, the term is used for random variables with the domain of definition of \( \mathbb{R} \), as well.
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theory needed is given in the following. More details can be found in the Appendix A.2.3. The theory stated here is based on [24, 84]. The definition of a copula is according to [24].

**Definition 4.4.2 (Copula)**
For every \( n \geq 2 \), a **\( n \)-dimensional copula** (shortly \( n \)-copula) \( C \) is an \( n \)-variate distribution function on \( \mathbb{I}^n = [0, 1]^n \) whose univariate marginals are uniformly distributed on \( \mathbb{I} = [0, 1] \), i.e. \( U_i \sim \mathcal{U}(\mathbb{I}) \).

Basically, the definition states that each \( n \)-copula is associated with an \( n \)-variate random variable \( U = (U_1, \ldots, U_n) \) whose components are uniformly distributed on the identity interval \( \mathbb{I} = [0, 1] \). The other way around, an \( n \)-variate random vector \( U = (U_1, \ldots, U_n) \) of on \( \mathbb{I} \) univariate distributed variables \( U_i \) is distributed according to a copula \( C \).

A very important result is the following theorem that is referred to as “Sklar’s Theorem”. It connects a copula to an arbitrary multivariate distribution. The formulation of the theorem is according to [24].

**Theorem 4.4.3 (Sklar’s Theorem)**
Let \( F \) be an \( n \)-dimensional distribution function with univariate margins \( F_1, \ldots, F_n \). Then there exists a copula \( C \) such that for all \( (x_1, \ldots, x_n) \in \mathbb{R}^n \) (\( \mathbb{R} := \mathbb{R} \cup \{\infty\} \)),

\[
F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)).
\]

Such a copula is uniquely determined on \( F_i(\mathbb{R}) \times \ldots \times F_n(\mathbb{R}) \), where \( F_i(\mathbb{R}) \) is the range of \( F_i \) for \( i = 1, \ldots, n \). Hence, it is unique, when all \( F_1, \ldots, F_n \) are continuous.

Using copulas the joint distribution as well as the dependency structure of random variables are expressed by the marginal distributions and the copula. The marginals may be arbitrary, which offers high flexibility for modelling a coalition’s error structure.

There are different families of copulas. Given the empirical data – the errors of the coalition’s member units – the model can be fitted to a copula type using the Maximum Likelihood method. This method is used to estimate the parameters of a parametric function based on the data at hand. The output parameters are those for which the result of the empirical data is most likely. For details refer to [20].

Furthermore, there are methods available to compare the goodness of fit between different types of copulas for the same data. In order to get an idea of which copula type is suitable, a scatter plot of the empirical copula may be consulted. In order to graphically check the adequacy of a model fit, the empirical data and samples of the fitted model are compared in a scatter plot.

**Assessment of Reliability**

Using a copula-model and given marginal distributions representing units’ error models for a fixed lifespan the dependency structure of a coalition is given. The following
procedure states how the reliability of a coalition is assessed. It is an extension of Procedure 4.3.6 to a set of units incorporating dependencies.

**Procedure 4.4.4 (Spatial Reliability)**
Let $C = \{U_1, ..., U_n\}$, $n \geq 2$ be the set of all units within a coalition $C$ with lifespan $T$ where the distribution function of unit $U_i$ is denoted by $F_i$.

1. **Determine minimum prediction** For each unit $U_i$ the minimum value of the prediction $\min_{t \in T} \text{pred}_{U_i}(t)$ is determined.

2. **Determine accepted deviation** Given the individual contributions $e_{\text{cont},U_i}$ for each unit $U_i$ each accepted deviation from the prediction can be calculated as $x_i = e_{\text{cont},U_i} - \min_{t \in T} \text{pred}_{U_i}(t)$.

3. **Determine reliability with survival function** The joint distribution function is represented by a copula $C$ and the respective marginal distributions $F_1, ..., F_n$ as $F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n))$. Thus, the copula is utilized for reliability assessment. However, for reliability assessment, the joint survival function $\tilde{F} = \text{Pr}(X_1 \geq x_1, ..., X_n \geq x_n)$ is needed. The concept of a copula is adapted to a concept of a survival copula.

**Definition 4.4.5 (Survival Copula)**
Let $X = (X_1, ..., X_n)$ be a random vector with joint survival function $\tilde{F}$ and univariate survival margins $\tilde{F}_1, ..., \tilde{F}_n$. Then for all $(x_1, ..., x_n) \in \mathbb{R}^n$ holds

$$\tilde{F}(x_1, ..., x_n) = \tilde{C}(\tilde{F}_1, ..., \tilde{F}_n)$$

for some copula $\tilde{C}$. This copula is called the **survival copula** of $X$.

*Here, one has to be cautious in order not to confuse the survival copula $\tilde{C}$ with the survival function of a copula $C$ of an $n$-variate uniformly distributed random vector $U = (U_1, ..., U_n)$ i.e. $\tilde{C}(u_1, ..., u_n) = \text{Pr}(U_1 \geq u_1, ..., U_n \geq u_n)$.*

The following result yields the spatial reliability of a coalition.

**Result 4.4.6 (Spatial Reliability)**
The survival copula $\tilde{C}$ can be evaluated at the vector $(x_1, ..., x_n)$ of accepted deviations yielding the survival probability that none of the units deviates more that the accepted value $x_i$ from the prediction, i.e.

$$\rho_{\text{cont},C,T} := \tilde{F}(x_1, ..., x_n) = \tilde{C}(\tilde{F}_1(x_1), ..., \tilde{F}_n(x_n)).$$

(4.8)

As in the one-dimensional case, i.e. the reliability assessment of one unit, so far the reliability for the worst case has been introduced, i.e. a system frequency deviation of
± 50 Hz. Also in case of a coalition consisting of more than one units it is taken advantage of the fact that frequency deviations have different frequencies of occurrence. To incorporate frequency deviations in the RelACs-method for \( n \geq 2 \) the preparations given in Procedure 4.3.9 are made. Subsequently, it is integrated in the steps given above similarly as given in Section 4.3.1 for one unit.

Result 4.4.7 (Spatial Reliability Including Frequency Deviations)

Let \( \mathcal{C} = \{U_1, \ldots, U_n\}, n \geq 2 \) be a coalition with lifespan \( T \) and \( F_1, \ldots, F_n \) the error distribution functions of units \( U_1, \ldots, U_n \) for the lifespan \( T \). Further, the partition of the interval \([-200mHz, 200mHz]\) into equidistant intervals \( I_f \) is given. The corresponding contributions of each unit \( U_i \) are denoted as \( e_{\text{cont,U}_i,f} \) and the corresponding accepted deviations from the minimum prediction as \( x_{i,f} = e_{\text{cont,U}_i,f} - \min_{t \in \mathbb{T}} \text{pred}_{U_i}(t) \). Then the reliability considering system frequency deviations is given as

\[
\rho_{\text{cont,c}}(\mathcal{C}) = \sum_{I_f} \bar{F}(x_{1,f}, \ldots, x_{n,f}) \cdot \Pr(I_f)
\]

(4.9)

This result gives a weighted sum for reliability values for different probabilities thereby making use of the law of total probability. If not stated otherwise, for the remainder the term reliability refers to the reliability as derived in Result 4.4.7, i.e. reliability with consideration of system frequency deviations.

The previous result states how a coalition is assessed with respect to the contributions of all member units to an ancillary-service product taking into account prediction errors of each unit and dependencies between units. The results gives the probability with which the individual contributions are kept for the coalition's lifespan. Note that the dependence model using copulas also covers the case if there are no dependencies between different units.

The focus of the following section lies on the reliability of operational equipment and the assessment of how failures of the equipment influences the reliability of a coalition’s contribution.

4.4.2. Topological Position

In order to assess a coalition with regard to its reliability to provide an ancillary service product, the reliability of the operation equipment the units within the coalition are connected to must be taken into account, as well. This is especially crucial for distribution grids that are (operated as) radial systems that do not fulfil the n-1 principle (see Chapter 2).

To this end, the reliability of the ancillary service product as induced by operational equipment reliability is included in the RelACs-model. It is referred to as topological Reliability. Thus, topological reliability assesses a coalition with regard to the position of its member units in the power system.
Dependency Model

According to Definition 4.2.1 reliability is the probability with which a coalition can provide an ancillary service product within a product horizon under normal operational conditions. Each unit within a coalition has a fixed power contribution for a fixed period of time, i.e. the coalition’s lifespan (this does not necessarily be the length of the product horizon, see Section 3.1.3). If a unit is disconnected from the system as a consequence of a failure of operational equipment, the coalition as a whole is no longer able to provide the ancillary service product to its full extent. This is particularly crucial in case of the provision of control reserves since they must be delivered to the whole system. The topological reliability determines the probability with which all units within the coalition are connected to the system.

As already pointed out, this probability depends on the reliability of operational equipment under normal operational conditions. The operational equipment considered in this research project are lines, cables, and transformers but no busbars or switching elements. Planned failures are not taken into account because the connected units would not negotiate for products due at planned downtimes. The assessment is based on normal operational conditions which means that failures of equipment is assumed to be independent (see Chapter 2).

The dependency between units is reflected by the locations of units relative to each other. In case of a radial system if two units lie on the same feeder they have common operational equipment. These are components that lie upstream of both units. A failure of one of these components results in the disconnection of both units. Thus the connection nodes induce dependencies between units.

In the following, it is stated how topological reliability is assessed. To this end, concepts from graph theory are utilised (see e.g. [9], or [69]). The units in a coalition are connected at the distribution level, i.e. LV or MV level, being a subsystem of the power system. A grid topology of this subsystem is represented as a graph as specified by the following definition. (See e.g. [9].)

**Definition 4.4.8 (Power Grid)**

Let \( E = \{e_1, \ldots, e_m\} \) be the set of components, i.e. lines, cables, and transformers, and \( V = \{v_1, \ldots, v_k\} \) the set of nodes within the subsystem under consideration. The subsystem is defined as a graph, i.e. the ordered tuple \( G = (E, V, \psi) \). \( E \) is also referred to as **set of edges** and \( V \) as **set of vertices**. The mapping \( \psi \) is the **incidence function** that maps an edge to a set of vertices that it connects, i.e. \( \psi(e) = \{v_i, v_j\} \). The edge \( e \) is said to **join** the vertices \( v_i, v_j \) and \( v_i, v_j \) are called the **ends** of \( e \).

With this definition, the topology of a subsystem is naturally represented by means of graph theory. In the following considerations, the terms grid nodes and vertices are used interchangeably as are the terms grid components and edges.

Let \( C = \{U_1, \ldots, U_n\} \) be a coalition that provides an ancillary service product located in a distribution grid \( G = (E, V, \psi) \). In order for a coalition \( C \) to provide the ancillary service product it is obliged to, all of its member units must be connected to the electrical power
4. Reliability Assessment of Ancillary Service Coalitions – The RelACs-Method

Let \( \varphi \) be a map that assigns a unit \( U \) to the node \( v \in V \) it is connected to, i.e. \( \varphi : U \rightarrow v \). In the following, for each unit \( U \in C \) consider the node \( v_U := \varphi(U) \) it is connected to. Furthermore, let the power grid of higher voltage levels be reduced to the node \( v_{grid} \) and suppose the ancillary service is to be delivered to \( v_{grid} \). This may e.g. be a transformer node of a distribution grid. Note that the components summarised in \( v_{grid} \) are not part of neither \( E \) nor \( V \), but all components of \( E \) and \( V \) lie downstream of \( v_{grid} \).

The following concepts are used to identify the connections of vertices (see e.g. [9]). Especially, minimal cut sets, which is a common technique used for system reliability assessment (see Chapter 2), are used to determine topological reliability.

**Definition 4.4.9 (Incidence, Path, Cut Set, Minimal Cut Set)**

Let \( G = (E, V, \psi) \) be a graph. An edge \( e \) with \( \psi(e) = \{v_i, v_j\} \) is said to be **incident** with its ends \( v_i, v_j \), and vice versa.

A **path** \( \pi \) in a graph \( G \) is a finite non-null sequence of alternating vertices and edges \( v_0e_1v_1e_2v_2 \ldots e_kv_k \) such that for \( 1 \leq i \leq k \), the ends of \( e_i \) are \( v_{i-1} \) and \( v_i \), and where the edges are distinct and the vertices are distinct. A path from vertex \( v_0 \) to \( v_k \) is denoted by \( \pi(v_0, v_k) \).

Furthermore, a **cut set** for two vertices \( v, v' \) is defined as a set of edges \( \kappa(v, v') = \{e_1, \ldots, e_n\} \) such that if eliminated, there is no path from \( v \) to \( v' \), i.e. the graph is partitioned.

A **minimal cut set** is a cut set \( \kappa \) such that for each \( e_i \in \kappa \) holds \( \kappa \setminus e_i \) is no cut set.

A minimal cut set is a set of edges that if erased partitions a graph but if the set had one element less the graph would still be connected. In a radial system – as in the case of distribution grids – each edge between two vertices is a minimal cut set. If all components of all units’ minimal cut sets are functioning it is able to contribute to an AS product. Thus it is a suitable instrument for determining a coalition’s topological reliability. A coalition’s minimal cut set is determined using the following definition.

**Definition 4.4.10 (Minimal Cut Set of an Ancillary Service Coalition)**

Let \( C = \{U_1, \ldots, U_n\} \) be a coalition and \( v_{grid} \) the representation of the grid, \( C \) is obliged to deliver an ancillary-service product to.

For \( 1 \leq i \leq n \) define \( K(U_i, v_{grid}) = \{\kappa(U_i, v_{grid})\} \)

the **set of minimal cut sets** of unit \( U_i \) and node \( v_{grid} \).

The **minimal cut set** of coalition \( C \) is defined as

\[
K_C = \bigcup_{i=1}^{n} K(U_i, v_{grid}).
\]

The previous definition states that the minimal cut set of a coalition is the union of minimal cut sets of nodes its member units are located at, and the grid an ancillary service product has to be delivered to. Note that the minimal cut sets of different units are not necessarily disjoint. This is important in the following since the failures of different
4.4. Coalition Reliability

minimal cut sets are not independent. However, if two minimal cut sets of distinct units are equal, it is only taken into account once.

In case of a radial system, a minimal cut set of a unit consists of one edge. Thus, if $v_{U_1} e_1 v_1 \ldots e_k v_{grid}$ is the path from $v_{U}$ to $v_{grid}$ the set of minimal cut sets is $K(v_{U}, v_{grid}) = \{e_1, \ldots , e_k\}$. This way, in the minimal cut set of a coalition the components only occur once during reliability assessment even if two units are connected to the same feeder and their minimal cut sets are not disjoint. This means a failure of a component is only considered once but still all possible component failures are taken into account. As a consequence, the set of minimal cut sets of a coalition describes the dependencies of its member units with respect to their location in the system.

Assessment of Reliability

For the topological reliability, the probability that all units $U \in C$ are connected to the system is of interest. To this end, the above concepts are utilised for the assessment of topological reliability. Again, let $G = (E, V, \psi)$ represent a sub grid of the power grid the units of $C$ are connected to, and $v_{grid}$ the representation of the power grid an ancillary service product has to be delivered to. The failure of a unit and a coalition with respect to the grid topology is defined subsequently.

Definition 4.4.11 (Failure with Respect to Grid Topology)

Let $C = \{U_1, \ldots , U_n\}$ be an ancillary-service coalition that must deliver an ancillary-service product $T_{pr}$ to the grid represented as $v_{grid}$ and let $T$ be the coalition’s lifespan.

The failure $\text{fail}_U(U_i)$ of a unit $U_i$ with respect to grid topology is defined to occur if the node $v_{U_i} = \psi(U_i)$ is disconnected from $v_{grid}$ during $T$. The failure $\text{fail}_C(C)$ of the coalition $C$ with respect to grid topology is defined to occur if for at least one unit $U_i$ a failure with respect to grid topology occurs.

According to the previous definition, the failure of a unit with respect to grid topology occurs if the unit is disconnected from the grid. This means that all components of at least one of its minimal cut sets fail. More precisely, the event of a failure of a unit $U$ is the union of events of failures of all minimal cut sets of $U$. Let $\text{fail}(\kappa)$ denote the event of a failure of the minimal cut set $\kappa = \kappa(v_{U}, v_{grid})$ which is the event that all elements fail during the coalition's lifespan. Let $\text{fail}(e)$ denote the event of a failure of element $e \in E$.

In summary this amounts to

$$\text{fail}_C(U) = \bigcup_{\kappa \in K(v_{U}, v_{grid})} \text{fail}(\kappa) = \bigcup_{\kappa \in K(v_{U}, v_{grid})} \bigcap_{e \in \kappa} \text{fail}(e). \quad (4.11)$$

The failure of a coalition with respect to grid topology occurs if at least one of its member units fail during the product horizon. This is represented by the union of unit failures, i.e.

$$\text{fail}_C(C) = \bigcup_{U \in C} \text{fail}_C(U) \quad (4.12)$$
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or by means of minimal cuts set of $C$, i.e.

$$\text{fail}_g(C) = \bigcup_{\kappa \in K_C} \text{fail}(\kappa). \quad (4.13)$$

A coalition is able to deliver an ancillary-service product if all of its member units are connected to the grid, or the other way around none of its member units fail. The following procedure summarizes the reliability assessment regarding the topological position.

Procedure 4.4.12 (Topological Reliability)
Let $C = \{U_1, \ldots, U_n\}$ be a coalition with lifespan $T$. The topological reliability of $C$ is calculated as follows.

1. **Map units to graph nodes** A distribution grid is represented as a graph where vertices relate to grid nodes and lines, transformer to edges. Units are connected to distinct grid nodes. Correspondingly, unit $U_i$ is mapped to a vertex or node in the graph denoted as $\nu_{U_i}$.

2. **Identify set of coalition’s minimal cuts** Let $\nu_{grid}$ denote the node representing the system coalition $C$ has to deliver an ancillary-service product to. The set of minimal cut sets according to Definition 4.4.10 is

$$K_C = \bigcup_{i=1}^{n} K(\nu_{U_i}, \nu_{grid}).$$

3. **Determine reliability** The following result yields the topological reliability of a coalition.

**Result 4.4.13 (Topological Reliability)**
The reliability of a coalition with regard to the system topology is calculated as

$$\rho_g(C) = 1 - \Pr(\text{fail}_g(C)). \quad (4.14)$$

In case of a radial system, let $\nu_i e_1, v_1 \ldots e_k, v_{grid}$ denote the path from $\nu_i$ to $\nu_{grid}$, and the unit’s set of minimal cut sets $K(\nu_{U_i}, \nu_{grid}) = \{e_1, \ldots, e_k\}$. With that Equation 4.11 simplifies to

$$\text{fail}_g(U) = \bigcup_{j=1}^{k} \text{fail}(e_j), \quad (4.15)$$

and Equation 4.12 is reformulated as

$$\text{fail}_g(C) = \bigcup_{e \in K_C} \text{fail}(e). \quad (4.16)$$

Thus, the following result yields the topological reliability of a coalition in a radial system.
Result 4.4.14 (Topological Reliability in Radial Systems)
The reliability of a coalition with regard to the system topology in case of a radial system can be calculated as

\[ \rho_C(C) = \prod_{e \in K_C} 1 - \Pr(fail(e)). \]  

(4.17)

In summary, the topological reliability in radial systems is the product of survival probabilities of the components in the set of minimal cut sets \( K_C \) of the coalition \( C \).

4.4.3. Summary

The reliability of a coalition \( C \) with respect to the provision of an ancillary service product has been assessed based on the spatial and topological position of the coalition’s member units \( U \). For the case of spatial position the model of unit reliability based on forecasts has been extended by including dependencies between the units with regard to their prediction errors. These dependencies are modelled as joint distribution function using copulas. In case of topological position the dependencies between units relate to their position in the power grid. If units are connected to the same feeder the failure of shared equipment, i.e. components positioned upstream of all units, results in a disconnection of those units from the system. The units’ reliability with regard to the failure of the unit itself must be taken into consideration of coalition reliability, as well. Since unit failures are assumed to be independent of each other the individual survival probabilities are multiplied.

\[ \rho_{fail}(C) = \prod_{u \in U} \rho_{fail}(U). \]  

(4.18)

The integrated reliability of the coalition is given in the following result.

Result 4.4.15 (Coalition Reliability)
The reliability of a coalition \( C \) with lifespan \( T \) is calculated as

\[ \rho(C) = \rho_{cont}(C) \cdot \rho_{fail}(C) \cdot \rho_{top}(C). \]  

(4.19)

In case of topological reliability a simple model of equipment failures is used. The method may be improved by using more sophisticated models as e.g. considering different load situations. This would result in different failure rates due to the fact that components are stressed differently.

For unit reliability regarding forecasts it has been discussed how the controllable factors of lifespan, contribution and reliability level influence each other. Furthermore, it

\[ \Pr(fail_C(C)) = \Pr \left( \bigcup_{e \in K_C} fail(e) \right) = 1 - \Pr \left( \bigcup_{e \in K_C} fail(e) \right) = 1 - \prod_{e \in K_C} \Pr(fail(e)) = 1 - \prod_{e \in K_C} 1 - \Pr(fail(e)) \]

In the above calculation the assumption has been used that the event of equipment failures are considered to be independent.
has been stated how the contribution amount of a unit for fixed lifespan and minimum reliability level are calculated. An interesting extension of the spatial reliability would be a similar problem, i.e. for a fixed lifespan and a minimum level for spatial reliability how are valid combinations of the contributions of a coalition’s units determined. Thus, a possible extension of the presented research project would be to investigate how to use the concept of contour lines to determine combinations of valid contribution that have the same level of reliability (for a fixed time horizon). This would be particularly interesting for coalition forming as it could simplify negotiations between units for finding an optimal configuration of contributions.

4.5. Summary and Discussion

In this chapter, the RelACs-method for reliability assessment of ancillary service coalitions has been introduced that has been designed in order to answer the research question given in Chapter 1:

How can distributed coalitions be assessed with regard to how reliable they can provide ancillary services?

Table 4.1 gives an overview of how the research questions have been approached. At first, the requirements have been derived from the use case given in Chapter 3. After that a definition of reliability in this context has been given. Based on that, a hierarchical model for reliability assessment has been proposed. For the assessment the assumption has been made that no unit is allowed to deviate from its contribution since units cannot compensate for contributions not delivered by other units during the coalition’s lifespan. In accordance with the hierarchical model, a method to assess a single unit with regard to its reliability has been introduced where a distinction has been made between reliability assessment based on unit failures and prediction errors. The first uses models of unit failure rates. For the second, an error model has been used to describe uncertainties resulting from predictions. This model has been extended by incorporating the probability of occurrence of system frequency deviations described by a probability distribution function.

For the assessment of a coalition, first, a model based on the spatial position of units has been introduced that relates to dependencies between units caused by similar influences given by weather conditions. To model these dependencies the statistical method of copulas has been utilised. Second, a model for dependencies between units in terms of their position in the power grid has been proposed based on failures of operational equipment.

The decisions for the development of the method have been presented and discussed. The conditions for design and development as specified in Section 4.1 have been fulfilled, as well. This completes the step of design and development of the design science process presented in Chapter 1.

The definition from reliability theory of technical systems has been adapted and mapped to the use case of AS provision. The implementation of the definition may be strict since
Table 4.1.: Research questions and the approaches made with the RelACs-method

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not a single unit is allowed to deviate from its contribution. However, this reflects the criticality of providing ASs for system stability. It is possible to incorporate arbitrary models for prediction errors into the RelACs-method. The same holds for the copulas and system frequency deviations. These models can be derived from data for example. Unfortunately, studies and analyses for more sophisticated modelling have been outside of the scope of this thesis.

This chapter has given the theoretical framework and presented the models that can be used to implement the RelACs-method. Details about the implementation are given in the subsequent chapter as well as the environment and setup for the evaluation of the RelACs-method.
5. Implementation and Experimental Environment

The objective of the RelACs-method is to assess the reliability of a set of units that as an aggregation provide an AS product. This aggregation can be e.g. a VPP, a DVPP or a coalition. The use case considered in this thesis is the provision of primary control reserves (cf. Chapter 3) and is therefore subject of the investigations here, as well. In particular, the reliability of coalitions is to be assessed during the coalition forming procedure introduced in Section 3.2.2. In that case, a set of units referred to as base coalition have committed themselves to provide control reserves during a product horizon. During the product horizon a coalition forming process has the objective of finding a subset of the base coalition – the core coalition – that exists for a subset of the product horizon – the lifespan. A constraint of this process is the reliability of the core coalition. This is assessed using the RelACs-method which returns an estimate for the reliability of the coalition.

In this chapter, an overview of the prototypical implementation is given. The focus is laid on the interfaces between the RelACs-method and a coalition forming process as well as the process for reliability assessment. After that, the evaluation environment used for experiments is presented along with the modelling choices for the experiments. Subsequently, metrics are introduced to assess the risk resulting from providing AS by RPU for both spatial and topological perspective. These metrics are used for evaluation.

5.1. Prototypical Implementation

The RelACs method as introduced in Chapter 4 with the components of spatial and topological reliability has been prototypically implemented using the programming language python 3.4¹ and methods from the statistical software environment R 3.1.2², as well as methods from the Smart Grid co-simulation framework mosaik³ for implementing the evaluation environment.

In order to assess a coalition (or more generally an aggregation of units) with respect to its reliability, information needs to be exchanged between the coalition as well as additional information about the system with the RelACs-method. For implementation, mostly the paradigm of functional programming has been used. Figure 5.1 shows an entity-relationship model depicting the entities, e.g. data types and objects, used in order to give a qualitative impression of their associations (see e.g. [29]). The attributes specify

¹https://www.python.org/
²www.r-project.org
³https://mosaik.offis.de/
the information needed as input for the RelACs-method in order to determine the reliability of a coalition. The corresponding data must be passed during coalition forming (see Section 3.2).

The entity coalition consists of at least one entity of type unit. The attributes of a coalition are the lifespan that starts at a certain point in time start. A coalition is associated with exactly one dependency model describing the dependencies between the coalition’s units given as a copula model and associated error models.

A unit entity has the following attributes:

id an identifier,

**type** the unit type, i.e. technology,

**peak power** the installed capacity,

**node** an identifier for the grid node the unit is connected to,

**voltage level** the corresponding voltage level,

**prediction** a time series representing the unit’s prediction,

**contribution** the unit’s contribution during lifespan

A unit is associated with exactly one entity of type error model. The error model entity specifies the distribution type of the unit’s prediction errors with the corresponding parameters parameters, e.g. mean and standard deviation, and the temporal resolution to assure the correct mapping between the error model and the trend entity. The trend entity corresponding to one parameter specifies how this parameter evolves with time giving the type (e.g. linear or logarithmic), corresponding intercept, gradient, and resolution. With the trend entity the value for each parameter for a certain point in time is determined thus specifying an instance of the error model. This is in accordance with the error model introduced in Section 4.3.1.

The entity frequency deviation model specifies the distribution of frequency deviations thus giving the type of the distribution as attribute distribution and the corresponding parameters as parameters.

The entity grid model is associated with entities of the types nodes and operational equipment. A grid model consists of at least one node entity. Each operational equipment is associated with two node entities that it connects. The attribute connections of the grid model gives information about the topology, i.e. which nodes are connected to which operational equipment. In order to describe the failure behaviour of operational equipment each of these entities is associated with a failure rate model entity with which the corresponding survival probability can be determined.

Figure 5.2 shows data-flow diagrams (see e.g. [29]) to give an overview of the data exchanged between different entities during the reliability assessment using the RelACs-method for both the spatial and topological case. The diagrams show the external information needed from the system or coalition. In case of spatial reliability this is a model
for the distribution of system frequency. In case of topological reliability it is information about grid topology and failure rates of operational equipment.

The data processing for calculating spatial reliability given in Figure 5.2a reflects Process 4.4.4 and is summarized as follows. With the coalition's lifespan and start the predictions are determined as well as instances of all units' error models. With the error model and copula model instances the dependency model is instantiated. The instance of the dependency model and the units' predictions are passed to the RelACs-method. In case of frequency-dependent reliability calculation, each interval is processed successively in a loop. For each interval the corresponding contributions of all units are calculated. Together with the probability of occurrence a partial reliability is calculated all of which sum up as the total spatial reliability (see Section 4.4.1 for details). The calculation is based on the dependency model and the units' predictions. For the prototypical implementation a pipe between python and R has been used for evaluating the copula-based dependency model.

The data processing for calculating topological reliability given in Figure 5.2b reflects Process 4.4.12 and is summarized as follows. With the grid model, the node of the source of the medium voltage grid and all connections are known. For the combinations of units' nodes and source node the minimal cut sets are calculated. With the minimal cut sets and given the failure rates of operational equipment, the topological reliability is calculated (see Section 4.4.2 for details).

The presented implementation of the RelACs-method is used to evaluate the method itself. In the subsequent section the evaluation environment and setup are presented.

5.2. Evaluation Environment

In what follows, the evaluation environment is introduced that is used to evaluate the RelACs-method. The evaluation is based on simulations. To this end, scenarios have to be specified limiting the set of investigations. Since the RelACs-method should be incorporated into coalition forming the scenarios used determine the set of coalitions that are assessed with respect to their reliability. Thus, the basis for the investigations is a base coalition according to the concepts introduced in Chapter 3 along with a power grid. In other words, the investigations are restricted to a specific set of units and the grid they are connected to. This is referred to as the base scenario and determines the system under investigation. From the base coalition, a subset, i.e. a core coalition, is responsible for providing the power reserve for primary control. The reliability of the core coalition is investigated.

There are different factors that have influence on the constellation of a core coalition and with that on the reliability. These factors are categorized and summed up as follows. A similar classification is found in [98]. They form the framework for the evaluation environment as shown in Figure 5.3.

1. **External scenario** The external scenario comprises factors that cannot be controlled by the system but still have influences on the system, i.e. the constellation of the core
Figure 5.1.: Entity-relationship model of entities relevant as input for the RelACs-method
5. Implementation and Experimental Environment

(a) For calculation of spatial reliability

(b) For calculating of topological reliability

Figure 5.2.: Data flow diagrams of RelACs-method
coalition and with that on the reliability. Here the factor of weather conditions is taken into account. Especially the volatility of weather predictions plays an important role as they influence the possible contributions units are able to make.

2. **Core scenario** The core scenario comprises factors that may be controlled by coalition forming. According to Section 4.2, these are factors regarding the choice of units to incorporate into coalition forming (e.g. by type and position), lifespan and constellation of contributions. For the evaluation setup the factors restrict the set of core coalitions. These are:

- **Ratio of capacity** restricts the combination of units, e.g. number of PV and wind turbines, since it gives the ratio of installed capacity of a core coalition relative to the base coalition;
- **Lifespan** of a coalition that in particular has influence on the quality of predictions;
- **Dependencies** describes the dependencies structure between units which corresponds to their geographic position;
- **Contributions** the constellation of contributions, e.g. if all units have the same share in the product or if there is a high variance among the shares,
- **Distribution in grid** determines if units are connected uniformly among all feeders or if they are located at few feeder, e.g.

3. **Product scenario** The factors of the product scenario reflect the requirements for ancillary service products. They cannot be controlled by coalition forming. In case of primary control reserve this is the amount of power to be provided by a coalition. The product horizon is not considered as a factor since core coalitions are formed to provide reserves for smaller time spans.

The process for experiments is visualised in Figure 5.3 with annotations of the steps. Altogether, the factors define the scenario or experiment setting for evaluation. Each factor can take different values that are referred to as factor levels (see Appendix A.2.4 for details). After specifying the values or levels of the factors (Step 0), this scenario instantiation is loaded within the evaluation environment (Step 1). This is the first step of actually generating the scenario and preparing it for investigations. After that, a sample of valid core coalitions is generated (Step 2), i.e. a set of coalitions based on scenario settings and with that fulfilling product requirements. All the information of a scenario setting and coalition configuration needed for evaluation is saved as an instance of a coalition object. The coalition object is serialized (Step 3) such that it is possible to be loaded by any function or method, in particular the RelACs-method (Step 4). After the RelACs-method has been conducted (Step 5) for all core coalitions the results are saved such that they can be used for evaluation. This choice of design allows the reliability assessment of coalitions not only in the context of the evaluation in this thesis. Moreover, the coalition data may be used for other assessment methods, too.

In the subsequent sections the concepts of base scenario, external scenario, core scenario, and product scenario are defined in more detail and modelling choices are introduced.
5.2.1. Base Scenario

As mentioned before, the base scenario consists of a base coalition and grid model the units within the base coalition are connected to. The base coalition is a set of units that altogether are obliged to provide an ancillary service product – primary control reserve in the use case at hand (cf. Section 3.2). The units are given as models for the specific technology type and size regarding installed power.

The grid structure is given as grid models for different voltage levels. The models are specified by nodes, connecting lines and transformers as well as characteristics of lines and transformers. Furthermore, the coupling points between the different voltage levels are defined.

5.2.2. External Scenario

In the setup of this research project only PV and wind units are investigated. There is a direct relationship between weather conditions (solar irradiation and wind speed) and power feed-in. Thus for each unit, weather conditions are considered by using time series representing predictions of power feed-in (cf. Definition 4.3.1). For this research project, time series for PV and wind units could be used that had been made available in the research cluster Smart Nord. These time series serve as predictions. Different factor levels relating to predictions in the scenario setup are modelled according to the volatility of predictions.

For wind units the ramps of the prediction from one time step to the next one are estimated. The standard deviation of the ramps serves as a measure of variability of the
prediction. In more detail, let \( \text{vol}_{\text{pred}} \) denote the volatility of a prediction. According to the notation of Definition 4.3.1, volatility of a prediction \((x_{t_1}, \ldots, x_{t_k})\) of length \(k\) is defined as \( \text{vol}_{\text{pred}} = \sigma_{\text{ramps}} \) where \( \sigma_{\text{ramps}} \) is the standard deviation of ramps \(((x_{t_1} - x_{t_1+1}), \ldots, (x_{t_k} - x_{t_k+1}))\).

PV units naturally have ramps depending on sunrise and sunset that directly influence the maximum possible amount of power that may be produced. Thus, the volatility of a prediction is measured by the standard deviation of the clearsky index of the prediction. Here, the clearsky index is regarded with respect to power output and is the ratio of the predicted power feed-in and the theoretically maximum possible power output \(k^*_{\text{pv}} = \frac{x_{t_{\text{pv}}}}{\max[p_{t_{\text{pv}}}]}.\) The maximum possible power output depends on location, tilt, and orientation of a PV-panel (see e.g. [74]). The volatility of a PV-prediction is defined as \( \text{vol}_{\text{pred}} = \sigma_{\text{clearsky}} \) where \( \sigma_{\text{clearsky}} \) is the standard deviation of clearsky indices \((k^*_{\text{pv,1}}, \ldots, k^*_{\text{pv,k}})\).

For investigations, four different weather situations have been chosen according to the presented measures. These are high, average, low, and no volatility or fluctuation of predictions. For a detailed reasoning and choice of time series refer to Appendix A.3.1. The case of no fluctuation however is not based on time-series. Artificial data has been generated to obtain a volatility of \( \text{vol}_{\text{pred}} = 0 \) for both cases PV and wind. For reduction of complexity of the investigations it has been assumed that both, PV and wind predictions, are on the same level of volatility.

### 5.2.3. Core Scenario

In what follows, the modelling choices are presented that represent the factors of the core scenario. These factors determine the set of units that are generated during experiments.

**Ratio of Capacity**

The constellation of units within a core coalition is modelled within the scenario setup as ratio of installed capacity of PV and wind units. More precisely, this is the ratio \( \phi_{\text{cap}} \) of summed installed capacity of units in the core coalition relative to the installed capacity of the whole base coalition, i.e.

\[
\phi_{\text{cap}} = \frac{\text{installed capacity of core coalition}}{\text{installed capacity of base coalition}}. \tag{5.1}
\]

**Lifespan**

The lifespan of a core coalition determines the quality of predictions since it reflects the length of prediction horizon. The prediction errors increase with increasing prediction horizon.

The quality of prediction relates to prediction errors given as error models according to Definition 4.3.3. For the investigation of both, PV and wind units, a normal distribution with zero mean is chosen to model the error distribution although e.g. [56] and [43] suggest otherwise. The normal distribution seems to underestimate small prediction errors but overestimates bigger prediction errors. Moreover, the error model is considered
fixed although it varies with different weather conditions and situations (see e.g. [74, 33]). However, sufficient data of predictions of power feed-in of single units and related prediction errors had been lacking for this thesis. Corresponding studies and analyses have been outside of the scope of this thesis. For investigations to be conducted in the next chapter the aim is to get an impression of how reliability changes with different settings where the quality of the errors plays a more important role rather than the actual quantification and choice of distribution. However, the error models may be interchanged without changing the evaluation process. Thus it is possible to adapt the error model once better data is available.

For the choice of values to instantiate the error model literature values are used. Usually, the relative RMSE is used as a measure for the quality of a forecast. In case the error distribution has a mean of zero, the RMSE equals the standard deviation for the forecast. Thus, data of RMSEs for different prediction horizons are used to fit a regression model of the evaluation of the standard deviation of the forecast. The calculations to choose the error-models used for RelACs-experiments is given in Appendix A.3.1. The relative RMSE is the RMSE relative to the mean measured values. Thus, the standard deviation of an error model for a given prediction horizon is the relative standard deviation times the mean of power forecast according to data given by the in external scenario. The lifespan itself is chosen to be a multiple of 15 minutes according to the typical, minimal length of time intervals on energy markets.

Dependencies

The dependencies within a base or core coalition are modelled via copulas given margins according to the units’ error models (cf. Section 4.4.1). For a similar reasoning as before, due to the lack of data, the Gaussian copula is being chosen to model dependencies within a coalition. As in the case of error-model, this yields qualitative information of reliability of a coalition of DER. This model is e.g. presented and demonstrated for the use of predictions in [39]. The dependence structure between units may vary for different weather situations similarly as for the error models.

The different choices for the correlation structure of the Gaussian copula are given in Appendix A.3.1. For investigations the two cases of no correlations or high (according to literature values) correlations are distinguished. Correlations between units of different technology types are assumed to be zero. For reasons of conceptual clarity, all pairwise correlations between units of one technology type are assumed to be the same, respectively. That is why the dependency \( D \) is given as a tuple of correlations \( \text{corr}_{\text{PV}}, \text{corr}_{\text{wind}} \) for PV and wind units, i.e. \( D = (\text{corr}_{\text{PV}}, \text{corr}_{\text{wind}}) \).

Contributions

The constellation of contributions is measured by the uniformity of contributions with respect to the ratio of the whole amount of power to the provided amount of power. Let \( U_1, \ldots, U_n \) be the units within a core coalition \( C \) and let \( \max e_{\text{cont, } U_i} \) be the maximum contribution of unit \( U_i \) which amounts to \( \min \text{pred}_i \) the minimum of predicted
values of unit $U_i$ within the product horizon (since otherwise the reliability would be too low). The share of unit $U_i$ is defined as ratio of actual contribution and maximum contribution $\text{share}_i = \frac{e_{\text{cont},U_i}}{\max e_{\text{cont},U_i}}$.

The uniformity of contribution is defined as $\nu = 1 - \sigma_{\text{shares}}$ where $\sigma_{\text{shares}}$ is the standard deviation of shares $(\text{share}_1, ..., \text{share}_n)$. A perfect uniformity, i.e. a uniformity of one, is given if every unit contributes the same share according to its maximum contribution. This is the case if $\text{share}_i = \frac{P_{\text{tar}}}{\max e_{\text{cont},C}}$ for every $i = 1, ..., n$ where $\max e_{\text{cont},C} = \sum_{i=1}^{n} e_{\text{cont},U_i}$ is the maximum contribution of the coalition and $P_{\text{tar}}$ the target power of the coalition.

In order to obtain different settings for the uniformity of contributions, a noise parameter $\eta \in [0, \sqrt{12}]$ has been implemented when determining the contributions of units within a core coalition. On the share of each unit, noise is added that is drawn uniformly from the interval $[-0.5 \cdot \eta, 0.5 \cdot \eta]$ (while assuring that in sum the coalition contribution is fulfilled). The expected standard deviation of the noise amounts to $\frac{1}{\sqrt{12}} \cdot \eta$ (see e.g. [20]). Thus, a uniformity of

$$\nu = 1 - \frac{1}{\sqrt{12}} \cdot \eta \quad (5.2)$$

is expected. Hence, for a given uniformity, the noise parameter is chosen according to Equation 5.2.

A valid choice for uniformity of contribution $\nu$ lies within $[0, 1]$. The value 0 means that the standard deviation of unit shares is 1. This means that lots of units in the core coalition have contribution zero which contradicts the parameter choice for shares of units.

### Distribution in Grid

For generating scenarios, two values are specified to randomly distribute units to the grid given in the base scenario. The first is the maximum distance of units to the closest transformer relative to the length of the feeder, denoted as $\text{dist}_t$. The second value is the ratio of feeder relative to all feeder of the grid to which the units are distributed, denoted as $\text{dist}_f$ where $\text{dist}_f \in (0, 1]$. With different pairs of $\text{dist}_t$ and $\text{dist}_f$, different unit distributions are achieved. The maximum distance to the closest transformer relative to the feeder length must be chosen between $\text{dist}_t \in (0, 1]$. Similarly, the ratio of feeder lies between $\text{dist}_f \in (0, 1]$.

Both factors imply a different degree of vicinity of units with regard to their position in the model grid. More precisely, the number of shared operation equipment varies, i.e. the number of components that lie upstream of the same units. A core coalition with units distributed on a lower number of feeder, i.e. a low feeder ratio, share more equipment than with a high feeder ratio. This similarly holds for a low distance to the transformer compared to a high distance. Note, that coalitions distributed according to the same values of $\text{dist}_f$ and $\text{dist}_t$ do not necessarily have the same number of shared equipment. Those factors restrict the positioning of units, but still the distribution of units is conducted randomly.
5. Implementation and Experimental Environment

5.2.4. Product Scenario

The product requirements serve as a frame for valid coalition contributions since they specify which requirement according to power must be fulfilled. The amount of power $e_{fr}^r$ a coalition must provide as reserves is given in kW. According to the current German market setting 1 MW is the minimum required power reserve. However, choices of smaller product sizes are made for investigations in order to gain insights on coalition reliability.

5.3. Risk Assessment

As stated in Chapter 3, for system stability, it is crucial that the provision of ancillary services is guaranteed within the required time frames. The amount of power that RPU are able to procure for an ancillary service product is based on forecasts. With the RelACs-method, uncertainties resulting from forecast errors, possible failures of units or operational equipment are incorporated during the process of coalition forming. Still, there is a remaining risk that the required amount of a power reserve cannot be delivered in case of a frequency deviation.

In this section, a method for estimating the risk resulting from a an AS coalition is given, more specifically a coalition providing primary control reserves. This estimation serves as an evaluation metric for assessing reliable coalitions. First, the method itself is introduced in a formal way. After that, its integration in the evaluation environment is discussed.

According to [62], risk is defined as a “combination of the probability of occurrence of harm and the severity of that harm”. This also corresponds with the definition in [83].

In this thesis, three indicators that primary control reserve cannot be provided are considered as already discussed in Chapter 4. These are the failures of each unit in the coalition, the quality with which the power feed-in can be predicted, and the reliability of operational equipment. The first one is not investigated in more detail, here. The second one corresponds to the spatial reliability assessment, the third one to the topological reliability assessment. The latter two are being presented in separate subsections. However, they share the same definition of risk.

**Definition 5.3.1 (Risk)**
The risk resulting from a coalition that provides an ancillary service product is the expected amount of ancillary service quantity that cannot be activated within a product horizon under normal operational conditions.

In case of the provision of PCR, according to this definition and in correspondence to the definitions of [62, 83], the harm to the power system is power that cannot be delivered for primary control reserve which may result in instabilities in the power system. The severity of the harm is measured by the amount of power that cannot be activated. Thus subsequently, risk is calculated as sum of the severity of harmful events, i.e. not delivered amount of power, weighted by their probabilities of occurrence.
5.3.1. Spatial Risk

The risk according to the spacial positions of a coalition’s units relates to uncertainties of predictions (similar to Section 4.4.1). The steps for assessing a coalition’s risk are stated in the following. The process is similar to the process visualized in Figure 5.2a. Instead of the RelACs-assessment the risk assessment comes into place. The Steps 1 - 4 are the same steps used for preparations of reliability assessment of Procedure 4.3.9 in Section 4.3.1. For sake of completeness and readability the steps are given here again.

Procedure 5.3.2 (Spatial Risk Assessment)

1. **Model frequency deviations** Since different levels of frequency deviation have different probabilities of occurrence a distribution function \( F \) of frequency deviations is determined.

2. **Partition frequency deviations** Consider the interval \([-200\, mHz, 200\, mHz]\) between the maximum and minimum feasible frequency deviation. This is being partitioned into equidistant intervals. Denote the intervals by \( I_f = [a_f, b_f] \).

3. **Determine probability of occurrence** According to the model obtained by Step 1, for each of the intervals the probably of occurrence is calculated as

   \[
   \Pr(I_f) = F(b_f) - F(a_f).
   \]

4. **Determine target amount of power** As stated in Chapter 3, in case of primary frequency reserve, the droop control gives the amount of power \( e_{\text{cont},U_i}(I_f) \) that unit \( U_i \) must provide for a given frequency deviation \( \Delta f \in I_f \). This corresponds to the unit’s contribution at the given frequency deviation. Since here intervals of frequency deviation are considered, the interval boundary with the highest absolute amount of frequency deviation is considered as specified in the following:

   \[
   e_{\text{cont},U_i}(I_f) = \begin{cases} 
   e_{\text{cont},U_i}(b_f), & \Delta f \geq 0 \\
   e_{\text{cont},U_i}(a_f), & \Delta f < 0 
   \end{cases}
   \]

   (5.3)

   where \( e_{\text{cont},U_i}(a_f) \) and \( e_{\text{cont},U_i}(b_f) \) are the reserves that according to the droop control must be provided given a frequency deviation of \( a_f \) and \( b_f \), respectively. In sum, the target contribution for the whole coalition is \( e_{\text{cont},c}(I_f) = \sum_{i=1}^{n} e_{\text{cont},U_i}(I_f) \).

5. **Monte-Carlo simulation of power feed-in** Since the target power is determined based on predictions, the actual power fed in may strongly deviate from the target power even if it has been determined in order to keep a minimum reliability.

   The power feed-in of all units at each point in time of the product horizon is simulated using a Monte-Carlo simulation approach (for a general introduction see e.g. [23, 48, 76]). This is done based on the prediction of power feed-in of each unit and
the joint error model of unit predictions given by a copula model. In other words, forecast deviations for all units are sampled at once using the copula model and thus incorporating dependencies. An algorithm for copula simulation from [78] is used.

Let \((y_{t,1}, ..., y_{t,n})\) denote the vector of prediction errors at time \(t\) of units \(U_1, ..., U_n\). The errors then are added to the prediction \(\text{pred}_{U_i}(t)\) of unit \(U_i\) \((i = 1, ..., n)\) at time \(t\), i.e. the simulated power feed-in amounts to \(P_{\text{sim},i} = \text{pred}_{U_i}(t) + y_{t,i}\). Furthermore, it is checked if for all units \(P_{\text{sim},i} \in [0, P_{\text{peak},i}]\) holds where \(P_{\text{peak},i}\) is the peak power of unit \(U_i\).

The simulated power is compared to the contribution and for each unit the available power is calculated as

\[
P_{\text{act},i,t} = \begin{cases} 
(\text{cont},U_i, I_f) & P_{\text{sim},i} \geq P_{\text{tar},i} \\
\text{sim},U_i, & P_{\text{sim},i} < P_{\text{tar},i}
\end{cases} 
\]  

The power that is activated by the whole coalition amounts to \(P_{\text{act},c,t} = \sum_{i=1}^{n} P_{\text{act},i,t}\).

6. Estimate power not deliverable Let \(n_{mc}\) denote the number of simulation steps which is the product of time steps and Monte-Carlo steps. Then with the results of the Monte-Carlo simulation, an estimation of the average amount of not delivered power for a given interval of frequency deviations is given as:

\[
\bar{P}(I_f) = \frac{1}{n_{mc}} \cdot \sum_{\Delta P < 0} \Delta P,
\]

where \(\Delta P = P_{\text{act},c,t} - P_{\text{tar},c}\).

7. Calculate risk The final step gathers the results of the previous steps.

### Result 5.3.3 (Spatial Risk)

The remaining risk is calculated as

\[
\text{risk} = \sum_{I_f} \bar{P}(I_f) \cdot \Pr(I_f).
\]
e.g. [76]) but too time-consuming for the scenarios regarded here since the acceptance rate might be low. The technique of truncated sampling might be another alternative. There is literature proposing sampling methods for truncated sampling of multivariate normal distributions given inequality constraints [73, 107]. These methods should be extended to enable truncated sampling of meta-distributions with an underlying copula model. However, the choice of the sampling as it is implemented so far within the interval regards higher deviations from predictions more often. In case the deviation is positive, this has no influence on the risk. In case the deviation is negative, the risk is overestimated. Hence the risk estimation obtained by Procedure 5.3.2 is an overestimate of spatial risk.

Another information that is obtained by the steps above is the value for simulated reliability, i.e. a measure for the probability that control reserves are provided to the extent demanded. To this end, Equation 5.6 is altered such that the number of cases is counted for which a violation of the contribution occurs. The power that cannot be delivered is neglected.

**Result 5.3.4 (Simulated spatial reliability)**
The simulated reliability is calculated as

$$\rho_{\text{sim,c,T}} = \sum_{I_f} \left( 1 - \frac{1}{n_{mc}} \cdot \sum_{\Delta p < 0} 1 \right) \cdot \Pr(I_f). \quad (5.7)$$

This measure is used as an evaluation metric, as well.

### 5.3.2. Topological Risk

As seen in Section 4.4.2, the topological reliability determines the probability that at least one unit of an AS coalition is disconnected from the system, and therefore cannot deliver the amount of power it is obliged to. Units within a coalition cannot compensate for a failure of one unit during the coalition’s lifespan because of real-time restrictions. This means that the whole coalition cannot fulfil its task. However, the reliability estimation only reflects the fact that the ancillary service cannot be delivered to its full extend rather than the amount of power that cannot be delivered. The latter may strongly depend on the position of units in the grid in relation to the other units’ positions.

Figure 5.4 shows an example of two different core coalitions each consisting of four units in the same grid consisting of four LV-feeders. On the left side of the figure (Case a)), all four units are connected on the same node whereas on the right side (Case b)) the units are evenly connected to one branch each. In Case a) the disconnection of one of the units relates to fewer line failures than in Case b). Thus the reliability of Case a) is higher compared to Case b).

In terms of power however, a failure with respect to grid topology of one unit in Case a) would include the failure of all other three units and with that the coalition could not deliver an AS product. In Case b), only a failure of the transformer or the busbar result in a topological failure of the whole coalition.
Both, Cases a) and b), have a high mean distance to the transformer node. In Case a) units are concentrated, i.e. have a high number of shared operational equipment. In Case b) units have a small number of shared operational equipment since they are uniformly distributed within the grid.

In accordance with Definition 5.3.1, topological risk is the expected amount of power that cannot be delivered. In other words, this is the amount of power not being delivered weighted by its probability of occurrence. This is a similar approach as used for system reliability assessment presented in [12]. There the probabilities of occurrence of different contingencies are used to weight the corresponding impact (see Chapter 2).

For risk assessment with respect to grid topology it does not suffice to only consider minimal cut sets as in the case of topological reliability since the power contribution of a disconnected unit must be regarded, as well. Thus, the path $\pi_i := \pi(v_{source}, v_i)$ from the source node $v_{source}$ (e.g. the node $v_{grid}$ representing the system to which the power reserve has to be delivered) to the node $v_i$ of a unit $U$ is considered (cf. Definition 4.4.9).

If the paths of different units share operational equipment their failures with respect to grid topology are not independent. For example in Case a) of Figure 5.4 it holds $\Pr(\text{fail}_i(U_j) | \text{fail}_i(U_j)) = 1$ for $i, j = 1, \ldots, 4, i \neq j$. In more detail if unit $U_j$ has failed means that one element of its path $\pi_{U_j}$ has failed. Consequently, unit $U_i$ fails as well since both units lie on the same grid node and $\pi_{U_i} = \pi_{U_j}$. In Case b) of Figure 5.4, the probability of failure with respect to grid topology of two different units $U_i, U_j, i \neq j$ depends on the shared operational equipment which in this case are the transformer $e_T$ and bus $v_B$ as the intersection of paths is $\pi(v_{grid}, U_i) \cap \pi(v_{grid}, U_j) = \{e_T, v_B\}$.

It can be seen that the hierarchy given by the position of units in the grid reflects these dependencies. Thus, consider $v_{source}$ as source and all units behind it as downstream of $v_{source}$. Note, that a core coalition may have more than one source if they are e.g. connected to more than one medium voltage grid. The concepts introduced here can then be adapted accordingly.

The level of hierarchy is represented by the downstream level of units. Here, the downstream level of a unit $U_i$ is defined as the number of units $U_j$ lying downstream of $v_{source}$ but upstream of $U_i$ (units lying on the same node are not counted in terms of down-
stream level). The hierarchy can also be represented by a tree-structure (cf. Chapter 2). Figure 5.5 exemplarily shows this. On the left side of the figure, a grid with allocated units of a core coalition are shown. On the right side, a tree structure of the topological dependencies between the units as well as the downstream levels are depicted. Since two units connected to the same node have the same path and downstream level they are summarized as one unit.

The process to determine the topological risk for a core coalition $C = \{U_1, \ldots, U_n\}$ is given in the following. An example of determining the dependency structure is given below and visualized in Figure 5.5.

---

**Procedure 5.3.5 (Topological Risk Assessment)**

0. **Separate Grid according to grid levels** For reasons of computational costs the grid is separated into sub-grids. This is done e.g. according to the voltage level. For each sub-grid let $v_{source}$ be the distinct source node that is connected to the system upstream of the considered sub-grid. From the viewpoint of the system upstream, the sub-grid is considered as one node, as well. The power contribution of the connected units is summed up and considered as one unit connected to that node.

1. **Determine paths and order units** Consider one sub-grid with connected units $U_1, \ldots, U_n$. For each unit $U_i, i = 1, \ldots, n$ determine the path $\pi_{U_i} := \pi(v_{source}, U_i)$. Order the units according to the length of their paths.

2. **Determine the dependencies between units** Introduce a matrix $M$ with

$$M(i, j) = \begin{cases} 
1, & \text{if } i = j \text{ or } U_j \text{ lies downstream of } U_i \\
0, & \text{else}.
\end{cases} \quad (5.8)$$
To this end, initially set $M$ as identity matrix of shape $(n,n)$. For each unit $U_i$ and each unit $U_j$ with $j > i$ (i.e. $\pi_{U_j} \geq \pi_{U_i}$) determine the intersection of paths and distinguish between the following cases:

- if the intersection equals $\pi_{U_j}$ and $\pi_{U_j} = \pi_{U_i}$ consider units as one, add up their power contributions and delete row and column $j$ of matrix $M$;
- if the intersection equals $\pi_{U_i}$ but $\pi_{U_i} \neq \pi_{U_j}$ the unit $U_i$ lies upstream of $U_j$, set $M(i,j) = 1$;
- if the intersection is not empty, add an auxiliary node $\nu_a$ at the end of the intersection and add a row and column in $M$, accordingly.

Let $n'$ denote the number of remaining units, i.e. including summed units and auxiliary units. For convenience let $U_1, ..., U_{n'}$ denote the remaining units in the order given by $M$. Note that the vector $L$ with $L(j) = \sum_{i=1}^{n} M(i,j)$ gives the downstream levels of units and auxiliary units according to the path order.

3. Determine relevant failure combinations of units
Let $\eta \in \{0,1\}^{n'}$ be the vector determining if unit $U_j$ for $j = 1, ..., n'$ (including auxiliary units) has failed with respect to grid topology, i.e. if it is connected to the system or not:

$$\eta(j) = \begin{cases} 1, & \text{if unit } U_j \text{ is connected}, \\ 0, & \text{if unit } U_j \text{ is disconnecte from system}. \end{cases} \quad (5.9)$$

As mentioned before, if a unit $U_i$ that lies upstream of unit $U_j$ has failed, $U_j$ is disconnected from the system, too. Thus, not all combinations $2^{n'}$ are relevant. For this reason, determine all relevant failure combinations as follows: if $\eta(j) = 1$ but $\eta(i) = 0$ for a unit $U_i$ lying upstream of $U_j$ as indicated by the matrix $M$, i.e. $M(i,j) = 1$, erase $\eta$ from the set of relevant combinations.

4. Determine probabilities of failure
For each relevant failure combination $\eta$ determine the probability of occurrence, i.e. the probability that exactly this failure happens. This is done based on reliability values of operational equipment. Let $A(U_i)$ denote the direct ancestor of unit $U_i$ according to the downstream level, i.e. $A(U_i)$ is the first unit lying upstream of $U_i$. Note that the ordering of units by downstream level is not necessarily in accordance with the ordering of $\eta$.

Let $Pr(fail(\pi_{U_i}))$ denote the probability of failure of at least one element of the path $\pi_{U_i}$. Consider a combination $\eta$ of unit failures and set $Pr(\eta) = 1$. Then for each $i = 1, ..., n'$

- if $\eta(i) = 1$, i.e. $U_i$ is connected, then $Pr(\eta) = Pr(\eta) \cdot Pr(fail(\pi_{U_i}) | A(U_i))$. Note that if $U_i$ is connected indicates that $A(U_i)$ is connected, too. Thus, in order not to include failures more than once, only the survival probability of the path from $A(U_i)$ to $U_i$ is considered.
- if $\eta(i) = 0$ and $A(U_i)$ is connected $Pr(\eta) = Pr(\eta) \cdot Pr(fail(\pi_{U_i}) | A(U_i)))$. 

5. Determine amount of power not delivered  For each relevant combination of unit failures with respect to grid topology determine the according amount of power that is not available for activation of power reserves since units are disconnected from the system. Let \((P_1, ..., P_{n'})\) be the vector of contributions of units \(U_1, ..., U_{n'}\) where auxiliary units have contribution zero and summed units the corresponding sum of contributions. The amount of power that cannot be delivered for the failure combination \(\eta\) amounts to

\[
\bar{P}(\eta) = \sum_{j=1}^{n'} (\eta_j + 1) \cdot P_j.
\]

(5.10)

6. Calculate risk  The final step gathers the results of the previous steps.

Result 5.3.6 (Topological Risk)
The remaining risk is calculated as

\[
\text{risk} = \sum_{\eta \text{relevant}} \bar{P}(\eta) \cdot \Pr(\eta).
\]

(5.11)

Note that in case the system has been separated into subsystems as given in Step 0 the probability \(\Pr(\eta)\) has to be weighted by the probability that all elements above the sub-grid under consideration have not failed, this is the probability \(\Pr(\text{fail}(\pi(v_{\text{grid}}, v_{\text{source}})))\).

Subsequently the first steps are explained for the example given in Figure 5.5.

1. Given units \(U_1, ..., U_6\) and transformer \(e_T\) one ordering according to path length is \((e_T, U_1, U_5, U_6, U_2, U_3, U_4)\).

2. Two auxiliary nodes \(v_a\) and \(v_B\) must be included and units \(U_5\) and \(U_6\) are summed up (since they are connected to the same node), denoted by \(U'_5\). Then an ordering of units according to path lengths results in \((e_T, v_B, U_1, U'_5, v_a, U_2, U_3, U_4)\). The matrix given the downstream relations is given as

\[
M = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Thus the vector indicating the downstream level results in

\[
L = (1, 2, 3, 3, 4, 5, 5, 6).
\]
3. The number of all relevant combinations is large. For conceptual clarity only one example is given: \( \eta = (1, 1, 1, 0, 1, 1, 0, 0) \) is a relevant combination of unit failures with respect to grid topology. The combination \((1, 1, 0, 1, 0, 1, 0, 0)\) is not relevant since unit \(U_2\) is indicated as connected although unit \(U_1\) that lies upstream of it is not connected.

5. The vector of power contributions is given as
\[
(0, 0, P_{U_1}, P_{U_2} + P_{U_3}, 0, P_{U_4}, P_{U_5}, P_{U_6}).
\]

The entries with the value zero relate to transformers or auxiliary nodes. The amount not delivered according to \(\eta\) as given before is \(P_{U_1} + P_{U_2}\).

Note that the above process does not include the dependency of the amount of contribution with respect occurrence to system frequency deviations as in the case of spatial risk estimation. This may be incorporated and investigated in future work. However, for an impression of risk according to failures of operational equipment the maximum contribution suffices as a worst case consideration.

### 5.3.3. Risk Evaluation Environment

The risk evaluation is incorporated into the evaluation process as shown in Figure 5.3 by replacing the reliability assessment using the RelACs-method with the risk assessment method. The information given by a core coalition is read in via an interface, the risk (spatial or topological) is estimated and the according results are stored. This makes it possible to investigate the risk in relation with different design values and in comparison to reliability. The risk measure is utilized in order to evaluate whether the reliability assessment is reasonable.

### 5.4. Summary and Discussion

In this chapter, selected facts about the prototypical implementation have been given with the focus on interfaces and data exchanges between the RelACs-method and a coalition evaluation or forming procedure. Furthermore the evaluation environment has been presented including a setup of scenarios used for investigations. In this context, a set of factors has been introduced that are assumed to have influence on the reliability of a coalition. The modelling and implementation of these factors have been presented in order to incorporate them into the evaluation process. The models are mainly based on literature values. The usage of the RelACs-method has been designed such that it is possible to interchange and update the models. The scenarios derived here serve as exemplary studies and constitute an entry point for further investigations and more general studies. The results also depend on the choice of factors and factor levels, thus it has been discussed what sufficient parameter domains are. Nevertheless, these are model assumptions that may only yield qualitative impressions rather than quantitative results.
Furthermore, estimates for risk assessment for both the spatial and topological case have been introduced to evaluate the RelACs-method. The simulation-based risk measure for the case of spatial position is an overestimation of risk. Using the methods of so-called rejection or truncated sampling may yield more accurate results than the sampling step implemented so far. Furthermore, also temporal dependencies may be incorporated in the sampling step. The models currently used for the topological risk estimation are quite simple and should be extended to more sophisticated models, e.g. models of failure rates. Furthermore, different load situations may be incorporated based on predictions available for risk estimation.
6. Evaluation

The objective of the evaluation is to gain insight in the functioning of the RelACs-method. More precisely, the goal is to investigate if the functional and non-functional requirements derived in Chapter 4 are satisfied. According to the process model presented in Chapter 1 this relates to the steps demonstration and evaluation.

For the evaluation of the RelACs-method the following four steps have been conducted:

**Functional validity** Before the RelACs-method is applied for assessing primary control coalitions it must be assured that the method functions as expected. This corresponds to the basic assumptions for tests and the functional requirements for the method.

**Interoperability with coalition forming** In order to use the RelACs-method as a constraint during coalition forming the non-functional requirements must be fulfilled.

**Recommendations for application** With the first two evaluation steps it is shown that the method fulfills its functional and non-functional requirements and that it is possible to incorporate the method into the coalition forming process yielding a viable and processable output. Under these preconditions one is interested in supporting the decision process regarding which units to incorporate into the coalition forming process. This then serves as a guideline or basis for a search heuristic. To this end, different level combinations of the factors that have been identified in the previous chapter are investigated. Using techniques from design of experiments (see Appendix A.2.4) recommendations for choices of a coalition's constellation are given for different conditions.

**Risk estimation of reliable coalitions** The risk metrics introduced in Section 5.3 return the expected amount of power that cannot be activated for both the spatial and topological case.

For the evaluation steps, scenario-based investigations are conducted according to the evaluation and scenario setup in Section 5.2. For the first two steps as well as the fourth step, hypotheses are introduced. In these cases, the outcome of the experiments is used for hypotheses testing. One experiment may yield results relevant for more than one hypothesis. For the third case, the sampled data is used to evaluate the relationship of reliability with the factors introduced in Section 5.2. In order to make clear distinctions, both the spatial and topological case are investigated separately.

Note, that in case of hypotheses testing, only rejecting a hypothesis yields a significant statement, i.e. to some level of significance, the hypothesis does not hold. If a hypothesis
is not rejected this means that the statement of the hypothesis can be confirmed. However, a hypothesis cannot be verified using simulations.

In an early iteration of the loop between design and development and evaluation only a few scenarios are necessary to initially test the hypotheses. Based on that, it can be derived to what extent the RelACs-method should be improved and which properties should be investigated in more detail.

6.1. Functional Validity

The goal of investigations of the first evaluation step is to show that functionality of the RelACs-method is as required and expected. To this end, first two basic assumptions are tested on which all other investigations are based. These are the fact that the RelACs-method can be utilised to assess arbitrary sets of units and that incorporating frequency deviations into the assessment is advantageous. The hypotheses that form the basis for investigations are given in Table 6.1.

The method is tested for arbitrary coalitions regarding the aspects listed below. These choices are made to obtain diverse cases and with that more generality of the results.

- **The constellation of units** regards the types and sizes (with respect to installed capacity) of units within a coalition, i.e. a homogeneous coalition consists of similar units and sizes whereas a heterogeneous coalition is a mixture of different unit types and size.

- **The constellation of contributions** refers to the shares of units’ contributions relative to their predictions. This is tested since all constellations must be assessable.

- **Distribution of units in the grid** refers to the position of units in the power grid and their positions relative to each other in an arbitrary grid.

These circumstances are investigated for testing Hypothesis Hb-g1 to confirm that the RelACs-method is applicable for different aggregations of units. In order to conduct investigations it must hold that a valid coalition has formed, i.e. that the summed contribution of all units equals the product size. Hypothesis Hfv-s1 states that the metric returned by the RelACs-method is a viable measure to reflect spatial reliability.

The assessment method regarding spatial reliability introduced in Chapter 4 incorporates probabilities for system frequency deviations. It is assumed that this benefits the method’s output regarding a better estimation of reliability. Although it is not strictly a functional property, this is tested in this section as Hypothesis Hb-s1, because the reliability assessment of all investigations are based on this assumption.

As pointed out before, the prediction errors increase with increasing time horizon. The same holds for probabilities of operation equipment failures. It must be shown that this behaviour is reflected also for a coalition’s lifespan. This is tested with Hypothesis Hfv-s2 and Hypothesis Hfv-t1. Furthermore, it demonstrates that the uncertainties given by prediction errors and equipment failures are incorporated in the RelACs-method in a reasonable way.
### Table 6.1: Hypotheses for testing basic assumptions (Hb) and functional validity (Hfv)

<table>
<thead>
<tr>
<th>Label</th>
<th>Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hb-g1</td>
<td>The method can be applied for arbitrary coalitions for which a valid coalition contribution has been found.</td>
</tr>
<tr>
<td>Hb-s1</td>
<td>The RelACs-method incorporating system frequency yields a better estimation of a coalition’s reliability.</td>
</tr>
<tr>
<td>Hfv-s1</td>
<td>The spatial reliability calculated by the RelACs-method gives a viable estimate for spatial reliability.</td>
</tr>
<tr>
<td>Hfv-s2</td>
<td>The spatial reliability calculated by the RelACs-method reflects that uncertainties increase with increasing time horizon.</td>
</tr>
<tr>
<td>Hfv-t1</td>
<td>The topological reliability calculated by the RelACs-method reflects that the probability of equipment failures increases with increasing time horizon.</td>
</tr>
</tbody>
</table>

(g – general, s – spatial, t – topological)

### 6.1.1. Choice of Scenario Instances and Experimental Setup

Two different base coalitions have been considered for investigations. The first one has been adapted from [72] where investigations had been conducted for 17 PV units with 10 kW peak installed capacity each. The second base coalition consists of the same number of units but of different types and sizes. The units that have been chosen are given in Table 6.2 together with the grid level to which they are connected. The scenario including the first base coalition is referred to as homogeneous and the second one as heterogeneous scenario. The grid of the base scenario has been chosen to be one low voltage grid connected to a medium voltage grid. For details on the grid models refer to [59].

Depending on the hypotheses that are tested, the settings of the scenario parameters regarding external, core and product scenario are varied. In Table 6.3 an overview is given which factor levels are chosen to form scenarios for hypotheses testing. Furthermore, for each hypothesis, Table 6.3 indicates the factor levels for which investigations are conducted. If more levels are checked for one hypothesis this means that combinations for these level choices have been investigated. Each factor level combination results in different core coalitions. The following factor levels are held fixed for all hypotheses: the grid model of the base scenario and the size of the core coalition relative to the size of the base coalition.

For testing Hypothesis Hb-g1 both, the homogeneous and the heterogeneous base coalitions, are considered. For both base coalitions, all combinations of the remaining factor levels have been tested according to Table 6.3. The goal of testing Hypothesis Hb-g1 is to show that the reliability can be calculated for any given coalition. For this reason, all factor level combinations are investigated.

In case of Hypothesis Hb-s1, the outcome of the RelACs-method is compared when considering the probability of frequency deviations compared to considering the worst case. That is, each unit’s contribution must be provided to its full extent, i.e. as demanded
for a maximum frequency deviation of \( \pm 200 \text{ mHz} \). To this end, different settings of the core scenario are investigated with different combinations of factor levels regarding the product scenario as indicated in Table 6.3 resulting in 16 different factor level combinations. Note that for the experiments conducted the system frequency deviations incorporated in the RelACs-method are modelled to be normally distributed (see Appendix A.3.1). However, the reliability values are likely to behave differently if the frequency deviations follow another distribution, e.g. in case the distribution is heavily tailed this might decrease reliability. In any case, the experiments should be repeated if a different distribution for frequency deviations is given. For all investigations the interval of feasible frequency deviations \( I_f \) is partitioned into 20 intervals of length 20 mHz.

The investigations for testing Hypothesis Hfv-s1 are based on different weather variability. To this end, the four different conditions regarding weather-dependent fluctuations of high, average, low and constant are chosen. The latter refers to a situation where there is no fluctuation in power feed-in but the level is constant.

In case of Hypothesis Hfv-s2 it is tested if the reliability obtained by the RelACs-method decreases with the lifespan. Thus, ten different core coalitions are considered that are valid for the same combination of factor levels regarding the core scenario and product size as well as the lifespan of four hours. The units’ contributions are held fixed and the reliability is calculated for smaller lifespans of one hour and 15 minutes. A similar setting is chosen for testing Hypothesis Hfv-t1. The same coalitions as in case of Hypothesis Hfv-s2 are chosen and the distribution in the grid is kept the same for all experiments.

### 6.1.2. Results

For all factor level combinations given in the column of Hypothesis Hb-g1 in Table 6.3 valid coalitions have been generated. For each of the valid coalitions reliability values have been obtained by the RelACs-method for both spatial and topological reliability

### Table 6.2.: Number of units by type, size, and connected voltage level in heterogeneous base scenario

<table>
<thead>
<tr>
<th>Voltage level</th>
<th>Installed capacity [kW]</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV 10 30 60 200 500 500 2000</td>
<td></td>
</tr>
<tr>
<td>lv*</td>
<td>6 4</td>
<td>10</td>
</tr>
<tr>
<td>lvt*</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>mv*</td>
<td>2 1</td>
<td>2 5</td>
</tr>
<tr>
<td>mvt*</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

*(lv – low voltage level, lvt – transformer node in low voltage level, mv – medium voltage level, mvt – transformer node in medium voltage level)*
### Table 6.3: Choice of factor levels for experiments regarding functional validity

<table>
<thead>
<tr>
<th>Factors</th>
<th>Levels</th>
<th>Hypothesis</th>
<th>Hb-g1</th>
<th>Hb-s1</th>
<th>Hfv-s1</th>
<th>Hfv-s2</th>
<th>Hfv-t1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base scenario</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coalition</td>
<td>homogen.</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>heterogen.</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>grid model</td>
<td>MV + LV grid</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>External scenario</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>weather</td>
<td>constant</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>low</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>average</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core scenario</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ratio</td>
<td>100 %</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Lifespan</td>
<td>1/4 hrs</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>1 hrs</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>4 hrs</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>dependency</td>
<td>(0, 0)</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>(0.67, 0.81)</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>noise</td>
<td>0</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>trafo dist</td>
<td>1</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>ratio feeder</td>
<td>1</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Product scenario</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>10 kW</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td></td>
<td>25 kW</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
value. This confirms Hypothesis Hb-g1 because regardless of the unit constellation, unit contributions and distribution in grid the RelACs-method returns a reliability value.

For the different factor level combinations described in the previous section the results are depicted in Figure 6.1. It can be seen that for all cases of the exemplary factor level combinations the reliability obtained when frequency deviations are considered are higher than for the case when the occurrence of frequency deviations is neglected. This supports Hypothesis Hb-s1. The different magnitudes of differences result from the fact that investigations have been conducted based on different scenarios.

In order to show that the RelACs-method yields viable values for spatial reliability the metric of simulated reliability based on MC-simulation introduced in Section 5.3 is conducted. Figure 6.2 shows the results for the four different weather conditions. According to Table 6.3, for each case there are eight factor level combinations for each of which a valid coalition has been generated. For each coalition an analytical estimate regarding spatial reliability has been computed using the RelACs-method as well as the simulated value. It is obvious that for all cases the analytical value lies below the simulated value. This is because the RelACs-estimation is based on the minimum predicted value. The higher fluctuations are the higher are the deviations from this minimum thus resulting in an underestimation of the reliability. With decreasing fluctuations the differences between reliability for the analytical and simulative case decrease. This interrelation is emphasized by the comparison of analytical and simulated reliability for the case of constant weather variability. In that case, both values are almost identical. This shows that the RelACs estimation returns justified values and Hypothesis Hfv-s1 has been confirmed.

Figure 6.3 shows the results of testing Hypothesis Hfv-s2 and Hypothesis Hfv-t1. It can be seen that for a coalition with a fixed contribution the reliability decreases for longer lifespans for both the spatial and the topological case. This confirms the fact that the uncertainties induced by prediction errors or failures of operational equipment are integrated in a reasonable way in the RelACs-method. This supports Hypothesis Hfv-s2 and Hypothesis Hfv-t1.

Altogether, the functional validity of the RelACs-method has been confirmed with re-
6. Evaluation

Figure 6.2.: Comparison of reliability with simulated reliability for different weather conditions

- (a) Spatial reliability
- (b) Topological reliability

Figure 6.3.: Behaviour of reliability for different lifespans
6.2. Interoperability with Coalition Forming

So far, it has been tested if basic assumptions and functional requirements are fulfilled by the RelACs-method. The reliability of a coalition must be incorporated into the coalition forming process and handled as a constraint during the same. First, the input-output relations as specified must be fulfilled. Thus, on the one hand, the RelACs-method obtains inputs – from the coalition forming or the system – from which the reliability is calculated, and on the other hand returns a value that can be handled by the coalition forming. Furthermore, it must be shown that non-functional requirements are fulfilled, as well. Those refer to the input-output relations, the computational time and the size of a coalition.

The hypotheses of Table 6.4 are tested. Hypotheses Hfv-g1 refers to the fact that it is possible to process the information obtained by the coalition and to return a value that can be interpreted and processed by the coalition forming. The input-output relations are given in Chapter 5.

As pointed out in Chapter 3, the coalition forming to find new core coalitions must yield results within real-time. Since the RelACs-method has to be included as a constraint and must possibly be called several times during coalition forming the method must satisfy real-time requirements, as well. This is tested using Hypothesis Hcf-g2.

Additionally, it is tested – according to Hypothesis Hcf-g3 – if the computational time is independent of the structure of the coalition, i.e. weather it is homogeneous or heterogeneous. This allows a more general statement than obtained by testing Hypothesis Hcf-g2.

Furthermore, the size of a coalition and the distribution of its units in the grid might influence the calculation of reliability. Thus it has been tested if the RelACs-method does not scale more than linearly with bigger coalition sizes and lower vicinity with regard to the units’ positions in the grid (see Hypothesis Hcf-g4).

<table>
<thead>
<tr>
<th>Label</th>
<th>Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hcf-g1</strong></td>
<td>The output can be processed by the coalition forming.</td>
</tr>
<tr>
<td><strong>Hcf-g2</strong></td>
<td>The method fulfils real-time requirements. I.e. the method returns an output within a time limit such that it can be processed by the coalition forming in time.</td>
</tr>
<tr>
<td><strong>Hcf-g3</strong></td>
<td>The computational time is independent of the type and size of units as well as their distribution in grid.</td>
</tr>
<tr>
<td><strong>Hcf-g4</strong></td>
<td>The method scales linearly with the number of units and size of grid topology.</td>
</tr>
</tbody>
</table>

(g – general)
6.2.1. Choice of Scenario Instances and Experimental Setup

For testing Hypotheses Hcf-g1 - Hcf-g4, the base scenarios introduced in the previsions evaluation step is scaled by factors two, four, eight and sixteen. More precisely, for both, the homogeneous and heterogeneous base coalition, the number of units for each type and size are multiplied by two, four, eight and sixteen, respectively. Furthermore, the number of grids considered in the grid scenario are scaled the same way. This results in four scenarios denoted as \textit{base1}, \textit{base2}, \textit{base4}, \textit{base8} and \textit{base16} for both, the homogeneous and the heterogeneous setup. For example, the scenario \textit{base2} for the homogeneous case consists of 34 PV units with installed capacity of 10 kW\textsubscript{peak} and two low voltage grids connected to the same medium voltage grid whereas in the heterogeneous case the numbers in Table 6.2 are multiplied by two each. The grid scenario is the same for the homogeneous and the heterogeneous case.

The choices for factor levels are made as given in the column according to Hypothesis Hb-g1 in Table 6.3 except for the case of \textit{power}, i.e. product size. In that case the factor levels 10 and 25 are scaled according to the factors of the scenario, e.g. for the scenario \textit{base2} (for both, homogeneous and heterogeneous scenario) the factor levels for \textit{power} are 20 and 50. For all factor level combinations and all scenarios the reliability is calculated using the RelACs-method. This amounts to 64 experiments for each base scenario.

6.2.2. Results

The RelACs-method returns the reliability of a coalition given the necessary inputs. Those are the lifespan and the contributions of all units of the coalition, the error models of the units, as well as the dependency model between all units (see Section 5.1). Furthermore, information of the distribution of the system’s frequency must be known. The reliability is by definition (see Definition 4.2.1) a probability and implemented as such (Chapter 4), i.e. a value between zero and one. In [41, 72, 75] it has been demonstrated that the reliability value can be interpreted and handled by a coalition forming process using the value as constraint to find optimal coalitions. Although for the cited references, a simplified version of the RelACs-method has been used, the interpretation and the format of the reliability value has not been changed. Thus, Hypothesis Hfc-g2 has been confirmed.

Figure 6.4 shows boxplots of the computational time of all experiments for computing spatial and topological reliability for both the homogeneous (Figure 6.4a) and the heterogeneous base scenario (Figure 6.4b) as well as the extended scenarios \textit{base2 to base16}. The concrete numbers can be found in Appendix A.3.2, Tables A.8 till A.11. However in the case of spatial reliability, the computational time could only be estimated since during reliability calculation a pipe between R and python has been used. For this reason, the response time for calling R via the pipe and loading libraries has been measured for each experiment. Subsequently, the minimum value has been subtracted from the measured time for reliability calculation. The experiments have been running on a machine with a 2.4 GHz six-core AMD Opteron processor under Ubuntu 14.04 LTS.

For both \textit{base1} cases the time does not exceed 15 seconds for spatial reliability and
0.001 seconds for topological reliability. For the scaled cases \textit{base6} the computational time hardly exceeds 170 seconds for spatial reliability. In case of topological reliability the computational time lies even beyond 0.1 seconds.

As pointed out in Section 3.2, a coalition forming strategy for finding a core coalition should find a valid and reliable core coalition at most within 15 minutes, the minimum time span for energy products. Thus, computational times of up to 2.5 minutes seem quite high considering that the assessment might have to be repeated several times in case an iterative coalition forming process is chosen. However, the estimation of computational time is a lower bound. Moreover, the implementation of the RelACs-method is a prototype and has potential for optimisation. In case of base coalition forming, the temporal constraint is not as strict as the time for finding a coalition may be up to five days. This leads to the conclusion that Hypothesis Hcf-g2 has been confirmed.

Comparing the values for the scaled base scenarios between the homogeneous and heterogeneous case it is obvious that their maximum values do not differ much. However, the spans for calculation time for the heterogeneous cases are higher, i.e. there are coalitions for which the time for assessing the reliability is shorter. Since the maximum values correspond the conclusion is valid that Hypothesis Hcf-s3 can be confirmed, i.e. the computational time does not differ with different unit types and sizes in the sense that an upper bound holds for both cases. However, there are some cases, i.e. coalition constellations, that exhibit lower computational times than others. Further investigations might show if this is the case for certain conditions.

Figure 6.4 suggests a quadratic relationship between the scaling of base scenario and computational times. But note that the x-axis is logarithmic to the base two whereas the y-axis has linear scale. Since the relationship is not obvious, a linear and quadratic regression have been executed on the average computational times with respect to the scaling factors. In all cases, i.e. \textit{homogeneous}, \textit{heterogeneous}, \textit{spatial}, and \textit{topological}, the $R^2$-values of regression for both linear and quadratic regression were high. For all cases of the spatial case the quadratic regression resulted in a slightly better fit. The concrete values are annotated in Figure 6.4. Statistics the computational times are found in Tables A.8, A.10, A.9, A.11 in Appendix A.3.2. However, as the linear fit is well, too, Hypothesis Hcf-g4 cannot be rejected.

In principle, the RelACs-method can used as an additional constraint during coalition forming. However, as mentioned before, optimisation of the implementation should be executed, at least for calculating spatial reliability since the usage of two different tools is not sound for the purpose of coalition forming fulfilling real-time requirements. Choosing a faster machine for computations may reduce the time, as well. Another suggestion to reduce computational time is – if applicable for the coalition forming strategy used – to calculate different coalition’s reliability values in parallel which may be done easily for the RelACs method. Furthermore, for computation of spatial reliability the acceptable deviations of system frequency are separated into equidistant intervals (see Section 4.3) for each of which the reliability assessment is conducted. Thus, computational time may be reduced by considering less intervals under the opportunity cost of reducing the reliability estimate.

The RelACs-method’s computational time scales with the number of units rather than
6. Evaluation

(a) Homogeneous base coalition

(b) Heterogeneous base coalition

Figure 6.4.: Boxplots and mean values of computational time for base scenarios for both the homogeneous and the heterogeneous case together with regression lines for linear and quadratic regression.

the types and sizes of units. This indicates that it might be advantageous to incorporate larger units with respect to installed capacity than smaller ones to fulfil product requirements. This impression is emphasised by Figure 6.5 showing how the reliability scales with the number of units in a coalition for the combinations of the cases heterogeneous, homogeneous and spatial, topological reliability. The figure shows the average and standard deviation of reliability values for the different scaling factors. In all cases the reliability decreases with the number of units. The installed capacity that is considerably higher in the heterogeneous case does not appear to have influence.

6.3. Recommendations for Coalition Forming

The previous results already indicate that for different factor level combinations the reliability varies differently. Thus it seems likely that there is a systematic relationship
6.3. Recommendations for Coalition Forming

(a) Homogeneous scenario, spatial reliability

(b) Heterogeneous scenario, spatial reliability

(c) Homogeneous scenario, topological reliability

(d) Heterogeneous scenario, topological reliability

Figure 6.5.: Scaled reliability: mean values and standard deviation for different scaling factors
between ‘good’ and ‘bad’ choices for factor level combinations, i.e. combinations that per se yield more reliable coalitions or the other way around.

The following experiments are conducted to investigate this relationship. The findings may yield valuable input for the coalition forming strategy giving hints under what external and market conditions which units to incorporate into coalition forming and how the structure of a coalition should be chosen, i.e. what are good choices for the constellation of the coalition.

6.3.1. Choice of Scenario Instances and Experimental Setup

The objective is to estimate the main effects on reliability of different factors or interaction effects between different factors. A main effect is the change of a quality measure (here reliability) if the level of one factor is changed to another level. An interaction effect evaluates the change in a quality measure if the levels of two or more factors are changed together (details are given in Appendix A.2.4). If a (main or interaction) effect is classified as significant then it has high influence on reliability. This knowledge yields insight on optimal choices for coalition forming given different situations. First, an overview is given of the procedure conducted for evaluation of spatial reliability. The steps are discussed in more detail subsequently.

1. Choice of scenario instances
   For all factors set the factor levels.

2. Choice of sample size
   For each factor level combination, i.e. experiment, set the sample size $n$, i.e. the number of repetitions for the experiment.

3. Conduct experiments
   Run simulations according to Figure 5.3, i.e. for each factor level combination, $n$ valid core coalitions are generated. A valid core coalition fulfils the product requirement. It is possible that not for each factor level combination a valid core coalitions can be generated. In all other cases, the RelACs-method is applied.

4. Preprocessing of results
   For all factor level combinations check if it is possible that valid coalitions are found. Otherwise the factor levels must be chosen differently.

5. Check prerequisites for effect estimation
   In order to be able to make statements about the significance of effects it must be made sure that certain requirements are satisfied (see also Appendix A.2.4).
   1. The samples are representative for their group
   2. The samples of each group are normally distributed
   3. The standard deviations of all groups are equal

6. Evaluate effects
   The effects are calculated according to Appendix A.2.4 as well as their significance.
6.3. Recommendations for Coalition Forming

Table 6.5: Number of units by type, size, and connected voltage level in heterogeneous base scenario

<table>
<thead>
<tr>
<th>Voltage Level</th>
<th>Installed Capacity [kW]</th>
<th>Scenario north</th>
<th>Σ</th>
<th>Scenario south</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV</td>
<td>Wind</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lv</td>
<td>10</td>
<td>30</td>
<td>60</td>
<td>500</td>
<td>2000</td>
</tr>
<tr>
<td>lvt</td>
<td>13</td>
<td></td>
<td></td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>mv</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>mvt</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cap.</td>
<td>990</td>
<td>960</td>
<td>780</td>
<td>2500</td>
<td>4000</td>
</tr>
<tr>
<td></td>
<td>1920</td>
<td>2200</td>
<td>1740</td>
<td>840</td>
<td>500</td>
</tr>
</tbody>
</table>

*(vl – voltage level, cap. – installed capacity, lv – low voltage level, lvt – transformer node in low voltage level, mv – medium voltage level, mvt – transformer node in medium voltage level)*

After evaluation of the spatial reliability, the topological reliability is investigated. Since the topological reliability depends on the position of units within the system rather than the constellation of the coalition, investigations are conducted only for combinations of factor levels related to the degree of vicinity and the factor lifespan. The other factor levels are kept fixed to enable comparability.

Choice of Scenario Instances

For the instance of the base scenarios, models for grids and units are used that have been available in the research network Smart Nord. For scenario design the methodology given in [42, 59] has been conducted. The scenarios used for evaluating the RelACs-method are referred to as RelACs-scenario. Two different scenarios are considered that are derived based on the process of scenario design given in [42, 59]. The first RelACs-scenario has been chosen to reflect a unit penetration of the north of Germany. Thus, the RelACs-scenario north is based on data of the German federal state of Lower Saxony. The second scenario reflects the unit penetration of the south of Germany. Thus, the RelACs-scenario south is based on data of the German federal state of Bavaria. For both RelACs-scenarios, ten low voltage grids are selected such that for the scenario north the installed capacity of PV and wind units together amounts to approximately ten times the minimum power contribution in the current market design. For reasons of comparability, for the scenario south the same setup regarding the choice of grids is used. An overview of the grid models is given in Appendix A.3.1. The number of units within this model grid are listed in Table 6.5 according to installed power and voltage level for both north and south scenarios.

Thus, due to topographical traits and environmental conditions in the south there is a higher penetration of PV-units whereas in the north there is a higher penetration of wind-
Table 6.6.: Choice of factor levels for experiments regarding spatial reliability

<table>
<thead>
<tr>
<th>Factor</th>
<th>Identifier symbol</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(-)</td>
</tr>
<tr>
<td><strong>External scenario</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weather volatility</td>
<td>weather</td>
<td>vol(_\text{pred})</td>
</tr>
<tr>
<td><strong>Core scenario</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio of capacity</td>
<td>ratio</td>
<td>(\phi_{\text{cap}})</td>
</tr>
<tr>
<td>Lifespan</td>
<td>lifespan</td>
<td>(t_{\text{pr.}})</td>
</tr>
<tr>
<td>Dependencies (PV, Wind)</td>
<td>dependency</td>
<td>(D)</td>
</tr>
<tr>
<td>Contributions</td>
<td>noise</td>
<td>(\eta)</td>
</tr>
<tr>
<td>Distribution in grid</td>
<td>transformer distance</td>
<td>(\text{dist}_x)</td>
</tr>
<tr>
<td></td>
<td>feeder ratio</td>
<td>(\text{dist}_f)</td>
</tr>
<tr>
<td><strong>Product scenario</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power reserve</td>
<td>power</td>
<td>(e_{\text{pr}})</td>
</tr>
</tbody>
</table>

units. This is reflected by the numbers given in Table 6.5. The fact that in the RelACs-scenario south there is no wind unit results from the fact than only a small section of the power grid has been chosen as a model. It does not indicate that there are no wind units in the south of Germany.

A summary of the factor level choices, i.e. the instances, for external, core and product scenario is given in Table 6.6 that hold for both north and south scenario. For the choice of measures used refer to Section 5.2. Here, for each factor two levels are chosen. For details on modelling choices refer to Section 5.2 and Appendix A.3.1. The levels are indicated by (-) and (+) referring to whether the factor is on a low or a high level.

Choice of Sample Size

According to Equation A.10, the total number of individual tests should be chosen as

\[ N = 60 \cdot \left( \frac{\sigma}{\Delta \mu} \right)^2, \]

where \( N = m \cdot n \) is the product of factor level combinations \( m \) and sample size \( n \) (see Appendix A.2.4 for details). Reformulating Equation A.10 yields

\[ n = \frac{1}{m} \cdot 60 \cdot \left( \frac{\sigma}{\Delta \mu} \right)^2, \]

the sample size needed to recognize a difference in magnitude of means \( \Delta \mu \) to a certain degree of confidence. Hence, with different choices of \( n \) a different precision for recognizing
Δμ is achieved. σ is the standard deviation of all experiment results, i.e. reliability values. This has been estimated with exemplary pre-experiments and is about 0.225. The number of factor level combinations for investigations regarding spatial reliability amounts to \( m = 2^6 \). The theoretical span of reliability values lies between 0 and 1. Thus, a difference \( Δμ = 0.05 \) is considered as sufficient. This results in a sample size of \( n = 19 \).

Experiments

For the investigations on spatial reliability all factors are relevant except the factor of distribution in grid. For each of the resulting \( m = 2^6 \) factor level combinations the spatial reliability is calculated using the RelACs-method for the sample of \( n = 19 \) coalitions. This yields a sample of corresponding reliability values for an experiment, i.e. factor level combination.

Preprocessing

Not for all factor level combinations with high product sizes and high fluctuating weather conditions valid coalitions could be found, i.e. because of too low predictions there have been cases where no constellation of contributions in the core coalition could be found fulfilling the product requirement. That is why smaller product sizes are chosen as given in Table 6.6.

Check Prerequisites

The prerequisites have been given in the previous section. The findings for the experimental setup given in Table 6.6 are discussed in the following.

Samples are representative for their group The coalitions to assess the reliability for have been generated based on a random order of units in the base coalition, for each coalition/realization using another random seed. Thus, the coalition constellation is considered representative for the corresponding factor level combination.

The samples of each group are normally distributed For each factor level combination, i.e. for each group, it must be assured that reliability values are normally distributed. This is tested using normal probability plots (see [20] for details). Basically, the experiment results are transformed such that they can be graphically compared with the quantiles of a theoretic normal distribution. If the values are normally distributed they approximately lie on a straight line. Deviations from a straight line indicate, e.g. that the values are not normally distributed or that there are outliers. Furthermore, the coefficient of determination \( R^2 \) of fit of the data points to the straight line is given as a measure of how good the data fits the line. The value \( R^2 \) should be close to one. Additionally, a plot showing a bar of \( R^2 \) for each group is plotted as well as histogram of all \( R^2 \)-values. This visualizes the cases in which the requirement might not be satisfied.
Table 6.7.: Choice of factor levels for experiments regarding topological reliability

<table>
<thead>
<tr>
<th>Factors</th>
<th>levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>weather</td>
<td>low</td>
</tr>
<tr>
<td>ratio</td>
<td>0.75</td>
</tr>
<tr>
<td>lifespan</td>
<td>1 hrs, 4 hrs</td>
</tr>
<tr>
<td>dependency</td>
<td>(0.67, 0.81)</td>
</tr>
<tr>
<td>noise</td>
<td>0</td>
</tr>
<tr>
<td>power</td>
<td>250 kW</td>
</tr>
<tr>
<td>transformer distance</td>
<td>0.5, 1</td>
</tr>
<tr>
<td>feeder ratio</td>
<td>0.5, 1</td>
</tr>
</tbody>
</table>

The standard deviations of all groups are equal The standard deviations of all groups must be the same. In order to check if this requirement is fulfilled, for each group the standard deviation of values has been calculated and plotted as well as a histogram of all values.

Evaluate Effects

The effects are calculated according to Equation A.7 in Appendix A.2.4. An effect is the difference of mean values of two different sets. In case of main effects (i.e. effect of just one factor and no interaction effect), this is the mean of all values where the factor is at level ‘+’ and the mean of all values where the factor is at level ‘-’. The significance of effects are calculated as pointed out in Appendix A.2.4. If a factor is significant or not may be visualized as follows. For each effect or interaction effect, a bar with corresponding magnitude is plotted as well as the margins indicating different confidence levels. The confidence levels are computed based on statistical estimates inferred from the data (see Appendix A.2.4). If a bar exceeds a confidence level, it is interpreted as significant to the corresponding level of confidence. This is presented in more detail in Appendix A.2.4 together with an exemplary visualization in Figure A.2.

Setup for Topological Experiments

For the topological reliability introduced in Section 4.4, the position of units in the grid are relevant as well as the coalition’s lifespan. Thus, only the factors transformer distance, feeder ratio and lifespan play a role for investigating topological reliability. The level of the remaining factors are kept fixed to the levels given in Table 6.7. The factor levels for topological experiments are set after spatial experiments. However, the factor level combination of spatial reliability that are kept constant do not have influence on topological reliability except that they determine the coalitions that are considered. The factor level setting plays a role for topological risk evaluation since there the unit contributions are relevant, as well.
Table 6.8.: Statistics for spatial reliability as a result of experiments according to Table 6.6

<table>
<thead>
<tr>
<th>Description</th>
<th>Values scenario north</th>
<th>Values scenario south</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all samples</td>
<td>sample with $D = (0.67, 0.81)$</td>
</tr>
<tr>
<td>mean</td>
<td>0.1809</td>
<td>0.3543</td>
</tr>
<tr>
<td>std</td>
<td>0.2248</td>
<td>0.2017</td>
</tr>
<tr>
<td>min</td>
<td>$6.11 \cdot 10^{-18}$</td>
<td>0.0593</td>
</tr>
<tr>
<td>max</td>
<td>0.6837</td>
<td>0.6834</td>
</tr>
</tbody>
</table>

6.3.2. Results

Spatial Reliability

An overview of the statistics of the reliability values of experiments according to Table 6.6 is given in Table 6.8. As one can see, the values vary strongly and the maximum value does not lie above 0.7. In addition, a description is given for the case where the factor of dependency is at the level $(0.67, 0.81)$. The reason for this becomes clear subsequently.

As pointed out before, in a first step the prerequisites must be checked. In Figure A.6, Appendix A.3.2 the normal plots for all 64 groups are depicted. Furthermore in the appendix, a plot showing a bar of $R^2$ for each group as well as a histogram of all values are given in Figure A.6. It can be seen that most of the $R^2$-values are bigger than 0.9 and there seem to be only a few outliers. Hence, the values are assumed to be normally distributed. The standard deviations of all groups and the histogram of all values is shown in Figure A.7b of Appendix A.3.2. It can be seen that the span of standard deviations varies quite strongly from almost 0 to 0.025. There is no correlation between $R^2$-values and standard deviations (refer to Figure A.7c in Appendix A.3.2). Thus, the occurrence of outliers does not appear to happen systematically. For the first half of all factor level combinations the standard deviations are quite small. These values relate to a dependency of $(0,0)$. It should be mentioned, that for these combinations the overall reliability is small, too, thus resulting in small standard deviations.

With the relatively high variation of standard deviation the third pre-requisite cannot be assumed fulfilled. Thus the evaluation of effects cannot yield quantitatively reliable results. With an underestimation of standard deviations, the confidence intervals determined with Equation A.9 in Appendix A.2.4 are underestimated, too, because their boundaries depend on the average standard deviation of all groups. This might lead to an incorrect classification of effects as significant. On the left hand side of Figure 6.6, the effects of all single factors as well as the two-fold interaction effects are shown. Furthermore, different confidence levels are indicated. The right hand side of the figure shows a zoomed vision of the left side. As one can see the confidence levels are quite narrow. An effect of about 0.015 is classified as highly significant (i.e. covered by the 0.999 confidence interval). This is quite high compared to the value of 0.05 for which the sample size has been dimensioned (see previous section).
However, qualitative findings can be derived considering the effects shown in Figure 6.6. Obviously, dependency has the strongest effect followed by lifespan and weather. The effect of dependency is positive with respect to the change of levels from '-' to '+' which in case of dependency is a change from low dependencies between units’ prediction errors and high dependencies. On the contrary, the factors lifespan and weather have a negative effect. In case of lifespan this means that with higher lifespan, reliability decreases. In case of weather this means that the more volatile the weather conditions are the lower is the reliability of a core coalition. Additionally, the interaction effects between dependency and lifespan as well as dependency and weather are quite high indicating that these factors should be considered together.

The effects of noise, power, and ratio are classified as significant, as well. The boxplots given in Figure 6.7b visualize the distribution of reliability values for both levels of all factors, respectively. The mean values of each factor level are indicated as blue dot. As one can see, the mean values of noise, power, and ratio do not differ strongly. They lie below the value 0.05. This contradicts the finding of significance. The significance of effects of dependency, lifespan, and weather however, are confirmed visually. Figure 6.7a shows the changes of reliability values for all groups with different level combinations of the factors dependency, lifespan and weather. The three factors considered individually already have a high influence on reliability. As one can see, the interaction effect of all three factors may be considered as highly significant, as well. The boxes in Figure 6.7a do not even intersect. This is a high indication that these factors should be regarded together.

Furthermore, for low dependencies the reliability is very low. The factor level combinations with highest reliability are those with high dependency, short lifespan and low weather fluctuation. Still, for high weather fluctuations (a non-controllable) factor, a combination with high dependency and short lifespan yields relatively high reliability values.

As pointed out before, the preconditions have not been satisfied for all factor level combinations such that results could only be used for qualitative interpretation. Since the
factor dependency has a very high influence on reliability it is considered significant. However, for low dependencies the reliability values are low as well as the standard deviation of groups with dependency at the low level. For this reason, the focus is laid on high dependencies in the following and the factor dependency is kept fixed at the high level (0.67, 0.81).

The findings of the investigations are summed up in the following. Figures A.8, A.7 in Appendix A.3.2 show results from checking the prerequisites. The majority of $R^2$-values is greater than 0.92. The variety of standard deviation has decreased, the values lie within 0.05 to 0.025, the majority of values within 0.05 and 0.01. The high values for standard deviation however appear to be outliers which results in an underestimations of effects. Thus, it is concluded that prerequisites are satisfied and the evaluation of effects is reasonable. Figure 6.8a shows the main and two-fold interaction effects of all factors where dependency is at the high level. It confirms the significance of the factors lifespan and weather. If dependency is considered fixed there is no interaction effect between lifespan and weather. Figure 6.8b shows the main, two- and three-fold interaction effects of the factors noise, power and ratio. These are clearly not significant.

Investigations for the RelACs-scenario south show similar results. A summary is given in Table 6.8. The overall reliability of experiments for scenario south is lower than in the case of scenario north. In Section 6.1 it has been shown that reliability decreases with the number of units in a coalitions. As in scenario south there are no wind units but more smaller (in terms of installed capacity) PV-units (see Table 6.5), more units must contribute to providing control reserves resulting in a reduced reliability.

The influence of the factor dependency is even higher than in the case of scenario north. In case the factor dependency is at the high level (0.67, 0.81) the prerequisites are fulfilled for testing the effects for significance (see Appendix A.3.2). Figure 6.9b shows the main and two-fold interaction effects of all other factors but dependency. The effects of weather and lifespan again are highly significant. Figure 6.9b visualises the values of spatial reli-
6. Evaluation

(a) Main and two-fold interaction effects

(b) Main, two- and threefold interaction effects between the factors noise, power, size

Figure 6.8.: Effects on spatial reliability with factor dependency at the high level

(a) Main and two-fold interaction effects

(b) Boxplots

Figure 6.9.: Effects on spatial reliability with factor dependency at the high level for scenario south

ability as boxplots for the significant factors at both levels, respectively, as well as their interaction effects. One can see that the interaction effect between lifespan and weather is not significant (given dependency at its high level) as the differences in the mean values in case of combination of the factors does not differ significantly to the main effects.

A special characteristic of the scenario south is that there are only PV units. As a consequence of the modelling choice (Section 5.2) each unit in the coalition is correlated to each other unit. Thus the effect of the corresponding factor is higher than in the case of scenario north where there were wind units that have been assumed to independent of PV units.

Altogether, the experiments confirm that the factors dependency, lifespan and weather have highly significant effects on spatial reliability. With the knowledge about dependencies between units there is more information available about the units behaviour regard-
6.3. Recommendations for Coalition Forming

Incorporating the information about dependencies results in higher reliability estimates for providing control reserves. The factor lifespan reflects the quality of predictions. With a longer lifespan the prediction error increases. This results in less reliable statements about the ability of providing control reserves for the given time horizon. High volatile weather conditions indicate that the probability of not providing a stipulated contribution is higher. As the minimum predicted value is regarded for reliability assessment, in general this is lower than for low volatility. Thus, contributions of the same magnitude are provided with a lower reliability in case the factor weather is at the high level.

Beyond that, these three factors have a highly significant interaction effect implying that decisions about which units to consider for coalition forming should incorporate all three factors at once. The combination of high dependencies, a short lifespan and low fluctuating weather conditions is most advantageous. However the factor weather cannot be controlled. Still, for the combination of high dependencies and short lifespan under both weather conditions yields best results.

The three factors contribution, power, and ratio do not have a significant effect on spatial reliability. This indicates that the constellation of contributions may be chosen arbitrarily providing degrees of freedom for a coalition forming strategy. The magnitude of the control reserves to be provided may also be arbitrary as long as the units in sum are able to contribute to the amount demanded. How reliable this contribution can be provided depends on the significant factors. Furthermore, the installed capacity in the core coalition relative to the base coalition does not have significant influence. The reasoning is similar to the case of power. As long as the units in sum make a contribution meeting the product requirements in term of magnitude of the reserves the reliability of the contributions depend on the significant factors.

Even if the installed capacity of the core coalition is not relevant for the spatial reliability of a coalition, the number of units within the coalition is. This has become obvious during investigations presented in Section 6.2. However, including the number of units as a factor for investigations presented in this section has not been possible for the following reasons. Given a number of units does not assure that the capacity of units is enough to provide the amount of power demanded. Coalitions consisting of the same number of units can have significant differences in aggregated installed capacity because this does not only depend on the number of units but also on the installed capacity of all individual units. Hence, in order to find a valid coalition, i.e. one whose sum contribution amounts to the product size, consisting of given number of units would already require an optimisation or search strategy. Unfortunately, this has not been in the scope of this research project.

One could also argue why the minimum predictions have not been used as a criteria for the investigations presented in this section as the reliability assessment is based on those values. Indeed, the minimum prediction is used as an restriction to unit contributions during the coalition forming process. However, this is a varying measure such that no general statements can be made about which units to include in the coalition forming process.
6. Evaluation

Figure 6.10: Results of evaluation of topological reliability

Topological Reliability

The results of investigations regarding topological reliability are given in Table 6.9. It shows that the topological reliability is very high and it does not vary strongly as it has a difference of about 0.0015. The check for pre-conditions obtained that there are outliers regarding the coefficient of determination and standard deviation. The corresponding normality plots are found in Appendix A.3.2. Note that the value of transformer distance and feeder ratio cannot be interpreted as absolute values. They are used to restrict the positioning of units in the model grid to generate different samples for unit distribution. Thus, the following results can only be interpreted in a qualitative way.

Figure 6.10 shows the value for topological reliability for all three factors on their two levels, respectively. This gives an impression about the effects of the factors. The effect of the factor lifespan appears to be the highest. The topological reliability decreases because with longer lifespans the probability of equipment failures increases. The effect of feeder ratio and transformer distance are comparably small. However it gives the impression that the closer units are with regard to shared operational equipment the higher their topological reliability is as less failures of grid components result in a disconnection of the units. The relatively high range of reliability values for the factors feeder ratio and transformer distance may be due to the fact that the levels give a restriction for distributing the units but is not a measure for the actual distribution. The interaction effects do not appear to be significant (cf. Figure A.16 in Appendix A.3.2) since the differences of mean values for different factor level combinations do not vary strongly from the differences of the main effects.

A choice of units lying close to each other regarding shared equipment for coalition formation results in a high reliability with respect to topology. This effect however is low compared to the effect of the coalition’s lifespan. Moreover, this does not reflect the power that cannot be delivered if grid equipment fails. To this end, risk assessment is conducted in the subsequent section.
Table 6.9: Statistics for topological reliability as a result of experiments according to Table 6.7

<table>
<thead>
<tr>
<th>description</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.998963</td>
</tr>
<tr>
<td>std</td>
<td>0.000637</td>
</tr>
<tr>
<td>min</td>
<td>0.998022</td>
</tr>
<tr>
<td>max</td>
<td>0.999645</td>
</tr>
</tbody>
</table>

Table 6.10: Hypotheses for spatial and topological risk evaluation

<table>
<thead>
<tr>
<th>Label</th>
<th>Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hr-s1</td>
<td>The spatial risk increases with decreasing spatial reliability.</td>
</tr>
<tr>
<td>Hr-t1</td>
<td>The topological risk increases with decreasing topological reliability.</td>
</tr>
</tbody>
</table>

(s – spatial, t – topological)

6.4. Risk Estimation of Reliable Coalitions

In order to evaluate if the reliability estimates calculated by the RelACs-method introduced in Chapter 4 for both spatial and topological reliability give reasonable results, corresponding risk estimates have been introduced in Section 5.3. With them, the remaining risk for reliable coalitions is calculated, i.e. the amount of power that cannot be activated. Altogether, the objective of the investigations is to gain qualitative impressions about risk and the connection between reliability and risk. The hypotheses listed in Table 6.10 form the basis for investigations.

Spatial Risk

At first, the relationship between the spatial reliability and risk is investigated in order to test Hypothesis Hr-s1. Since similar findings have been made for different scenarios here only the RelACs-scenario north is consulted.

As in the case of reliability estimation, first the effects of the six factors are investigated. Note that reliability cannot be considered as factor since the reliability values have been sampled and thus is a continuous measure that cannot be used to establish two factor levels. In Figure 6.11a, boxplots are shown where each factor is at both the low and the high level, respectively. Furthermore, the mean values are indicated as blue dots. The risk does not exceed a value of 3.12 kW which amounts to less than 1.25 % of the reserves that should be provided in total as given by the relative risk. The statistics of all values for risk and relative risk are given in Table 6.11.

As one can see, the factors of power, lifespan and weather appear to have significant differences in the mean values of both levels. For all factors that do not appear to be significant there are outliers of high risk values. As becomes clear later, this relates to the significant factors that in combination yield a high risk. Furthermore, risk strongly
depends on the magnitude of the reserves, i.e. the factor power. If higher reserves should be provided, violations of the contributions occur more often and thus the absolute risk is higher. Thus, in the following the relative risk is considered instead. In case of relative risk the factor power does not have any significant influences because the risk values are normed by the amount of reserves that must be provided in total which equals the value of power. The effect of the factors lifespan and weather on relative risk are opposed to the effect these factors have on reliability which is a first indicator for the support of Hypothesis Hr-s1.

A remarkable finding is that spatial risk (absolute and relative) does not appear to be sensitive to the factor dependency which would indicate that Hypothesis Hr-s1 must be rejected. However, the reason for this is the following. The risk is estimated based on a MC-simulation to sample the power feed-in of all units during their coalition's lifespan. Those values are restricted to the domain of \( \times_{i=1}^{n} [0, P_{\text{max}}] \) such that the feed-in of each unit lies between its minimum and maximum possible feed-in. This may result in an over-estimation of risk. These feed-in values are compared with the contributions the units have to make (see Section 5.3). Figure 6.12 visualises this concept in case of coalition of two PV-units with 10 kW installed capacity each for different lifespans and contributions indicated by vertical and horizontal lines. The grey area frames the domain between 0 and \( P_{\text{max}} \), the blue area indicates the feed-in values that do not lead to a violation of both units’ contributions. The dotted and the starred points represent the sampled feed-in vectors for both units in case of high dependency and low dependency, respectively. The samples based on low dependency appear as a circle whereas the sample based on high dependency appear to be aligned along a diagonal. This is most obvious in case of short lifespans since in that case the prediction errors are smaller and the effect is higher. This shows that the probability that the feed-in of both units lies above the contributions is higher with higher dependencies since more elements lie within the blue rectangle. This reflects the reliability. The risk sums up the corresponding magnitude of power below the contributions which appears to be approximately the same for both cases with low and high dependencies resulting in the fact that dependency does not have influence on risk as it has on reliability. Hence, the effects of risk for dependency at the low level appear to be the same as for dependency at the high level. The reason for this may be that the sampling as implemented so far does not take into account temporal dependencies between feed-in values. To support this, further investigations must be conducted.

Nevertheless, the following investigations take into account dependency at the high level for risk evaluation since the samples drawn based on consideration of dependencies represent the fact that deviations from predictions of units are interlinked. Furthermore, the behaviour of spatial risk appeared to be similar for different levels of dependency. Thus, if the level of dependency is fixed for investigations of the relationship of risk to the remaining factor levels yields viable results. Moreover, in the previous section it has become clear that for the factor dependency at the high level no reliable coalitions were found. Here, the focus lies on investigations for reliable coalitions. Figure 6.11b shows the boxplots for relative risk of all factors on both levels in case the factor dependency is at the high level. There are no more outliers due to normalization with the values of power levels. The upper bound for risk that seems to appear in Figures 6.11a and Figure 6.11b
Table 6.11: Statistics for spatial risk

<table>
<thead>
<tr>
<th>Description</th>
<th>Risk [kW]</th>
<th>Relative Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.812059</td>
<td>0.004633</td>
</tr>
<tr>
<td>std</td>
<td>0.873149</td>
<td>0.004170</td>
</tr>
<tr>
<td>min</td>
<td>0.069125</td>
<td>0.000691</td>
</tr>
<tr>
<td>max</td>
<td>3.112131</td>
<td>0.012449</td>
</tr>
</tbody>
</table>

relates to the combination of the factors lifespan on the level 4 hrs and weather at the level high as is obvious with Figure 6.11c. The interaction appears to be significant and high volatile weather conditions and a long lifespan present the highest risk.

For the relation between reliability and relative risk refer to Figure 6.11d that shows bars for mean values of reliability and relative risk for the combinations of the significant factors lifespan and weather in case dependency is at the high level. It can be seen that the average risk for coalitions that are reliable is low whereas the risk for not reliable coalitions is comparably high. However, for the experiments conducted the effect does not appear to be linear. This may be due to the fact that risk is an overestimation whereas reliability is an underestimation and thus the values cannot be directly mapped to each other. The correlation between risk and relative reliability for all experiments with dependency at the high level is -0.825. This result supports the opposite effect of reliability and risk.

For the case of dependency at the high level the findings support the fact that Hypothesis Hr-s1 cannot be rejected. However, more experiments on different factor levels are necessary to gain more support of the hypothesis. Further investigations on this are necessary to identify the reason why risk is not sensitive against dependencies. The results might be used to improve the risk assessment. Possibly, the results can be incorporated in the RelACs-method, as well.

**Topological Risk**

In order to assess the topological risk, experiments according to Table 6.12 are conducted. The levels of factors dependency, weather, noise and ratio are kept constant. For the factors lifespan, power, feeder ratio and transformer distance two levels are chosen. This is the same setup as for experiments for topological reliability except that power is considered as a factor as well. The reason for this is that for the computation of topological risk the contributions of units play a role and contributions may be higher the higher the power reserves are.

The results for the topological risk and relative risk (normalised by the level of factors power) are given in Table 6.13. The overall risk is very low. The maximum expected power that cannot be activated is around 0.04 %.

The check of prerequisites for topological risk can be found in Appendix A.3.2, Figure A.19. The $R^2$-values are sufficiently high and the standard deviations vary from about ten to twenty percent of the overall standard deviations which is assumed to be accept-
6. Evaluation

(a) Boxplots of spatial risk for factors at both levels, respectively

(b) Boxplots of relative spatial risk for factors at both levels, respectively, in case of dependency is at the high level

(c) Boxplots of significant factors for relative spatial risk at both levels and their interaction in case of dependency is at the high level

(d) Comparison of mean values of spatial reliability and relative risk for level combinations of significant factors in case of dependency is at the high level

Figure 6.11.: Results of evaluation of spatial risk
Figure 6.12.: Visualisation of sampling of power feed-in combinations
Table 6.12: Choice of factor levels for experiments regarding topological risk

<table>
<thead>
<tr>
<th>factor</th>
<th>levels case 1</th>
<th>levels case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>dependency</td>
<td>(0.67, 0.81)</td>
<td></td>
</tr>
<tr>
<td>lifespan</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>weather</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>noise</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>power</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>ratio</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>feeder ratio</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>transformer distance</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

able. The effects plotted in Figure 6.13c give an impression of which factors or interactions have a significant effect on topological risk. In Figures 6.13a, the boxplots of the four factors on both levels are shown. It can be seen that the factors regarding the distribution of units in the grid are very low and thus not significant. The factors lifespan and power however have a significant effect as does their interaction. The interaction is visualised in Figure 6.13b. The factor level combination with the highest risk is power at 250 kW and lifespan at 4 hours.

In order to evaluate the relationship between topological reliability and risk the factor power is disregarded because it has not been relevant for reliability. The setting for experiments is as in the previous section given in Table 6.12. In Figure 6.14, bars for the mean values of reliability and risk are given for the different groups of factor level combinations. It can be seen that the average topological risk is higher for groups with a low average reliability. This supports Hypothesis Hr-t1. The highest changes appear for changes of the levels of factor lifespan. In comparison to that, the effect of distribution in grid is negligible. Additionally, the topological risk is relatively low and the reliability relatively high, at least for the grid and time horizons considered in the scenario setup. However there are indications that risk is associated to contributions of units with high absolute contributions resulting in the presumption that it might be advantageous to distribute the contributions to more units. However this contradicts the findings for spatial reliability indicating that coalitions should consist of fewer units. Thus further investigations might be valuable to investigate if the assessment of topological reliability can be disregarded during coalition forming.

6.5. Summary and Discussion

In this chapter, the RelACs-method has been evaluated in four steps. First and second the functional and non-function requirements have been checked. To this end, hypotheses have been derived and tested. For both of these evaluation steps, artificial scenarios have been used, i.e. they did not have the purpose of reflecting a realistic unit penetration.
Table 6.13.: Statistics for topological risk as a result of experiments according to Table 6.12

<table>
<thead>
<tr>
<th>Description</th>
<th>risk [kW]</th>
<th>relative risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.036533</td>
<td>0.000208</td>
</tr>
<tr>
<td>std</td>
<td>0.028868</td>
<td>0.000125</td>
</tr>
<tr>
<td>min</td>
<td>0.007192</td>
<td>0.000072</td>
</tr>
<tr>
<td>max</td>
<td>0.097700</td>
<td>0.000409</td>
</tr>
</tbody>
</table>

(a) Boxplots of topological risk for factors at both levels, respectively
(b) Boxplots of topological risk for the interaction of factors lifespan and power
(c) Main and two-fold interaction effects on topological risk

Figure 6.13.: Results of evaluation of topological risk
The hypotheses regarding functional requirements could be confirmed thus supporting the statement that the RelACs-method yields a viable reliability estimate for arbitrary coalitions that can be processed by a coalition forming procedure. The spatial reliability is relatively low. This is due to the fact that the reliability estimate is an underestimation of reliability. An improvement might be achieved by considering the course of the prediction rather than its minimum to take account of the fluctuating character. Furthermore, the interpretation of reliability, i.e. no unit in the coalition is allowed to contribute less then the stipulated amount, might be too restricting. A relaxation has been shown to be achieved by incorporating probabilities of deviations of system frequencies for spatial assessment. This might be introduced for topological assessment, as well.

The hypotheses with regard to the non-functional requirements could also be confirmed. The computational time approximately scales with the number of units. However, the computational time should possibly be optimised by using only one tool for computations, use more efficient code or possibly conduct parallel computations. The reliability of a coalition appears to be declining with higher numbers of units. This leads to the impression that it is beneficial to include fewer units into coalition forming.

In a third step, the influence has been investigated for the different factors and their impact on reliability. With the results recommendations for coalition forming can be given with respect to which units to incorporate under different circumstances. This information can be used as a starting point for heuristics to pre-select units for coalition forming. Thus, for different factor level combinations the coalitions have been generated / sampled and their reliability estimated. In case of spatial reliability, two different scenarios have been used for experiments reflecting a typical unit penetration of the north and of the south of Germany, i.e. more wind and less PV and the other way around. The factors noise, power, ratio have no significant effect on spatial reliability. This means it does not
matter how units split their contributions, how high the amount of reserves is and how much installed capacity there is in the coalitions as long as valid coalitions can be found. It only is relevant if units show dependencies, for how long they provide reserves and what weather conditions prevail. An increasing lifespan and increasing weather conditions have a negative effect on a coalition’s reliability whereas increasing dependencies have a positive effect. This influence is higher the more units show dependent behaviour. Thus the information about dependencies between units is beneficial for coalition forming since it indicates that units of the same technology type that are spatially close to each other should be incorporated. This information may be beneficial for coalition forming as presented in Section 3.2 where initially a neighbourhood of units must be given. Hence, information about the spatial position may obtain sufficient neighbourhoods. The factor weather is non-controllable. However, for different weather conditions the factor combination of high dependencies and small lifespans are a good choice with respect to reliability when choosing units to incorporate into coalition forming. Of course, further investigations on different and bigger scenario should be conducted to confirm these results.

In case of topological reliability, the effect of the location of units with respect to grid topology appeared to be negligible. A significant, negative effect has been identified for the lifespan of a coalition. The overall topological reliability is relatively high. The significant factor lifespan has a similar effect on spatial reliability and is therefore already be taken into account during coalition forming. Thus, considering topological reliability might be disregarded as it is indirectly influenced. Note that topological reliability as estimated so far is more of a qualitative than a quantitative estimation since quite simple models have been used.

In order to assess the RelACs-method, risk measures have been executed to determine the expected amount of power that a coalition cannot activate as control reserve. The risk measure for spatial risk should be improved because it is not sensitive to dependencies. To achieve that e.g. temporal dependencies between successive prediction errors might be taken into account. In case of high dependencies the risk of reliable coalitions is high whereas for decreasing reliability the risk appears to increase thus showing reasonable behaviour. Similar results have been demonstrated for the case of topological risk.
7. Conclusion and Outlook

With the decentralisation of the power production, conventional power plants are substituted and renewable power units take over their tasks. The concept of aggregating RPU to coalitions provides a solution to enable distributed units to provide power products as a (dynamic) virtual power plant. This concept can also be assigned to form coalitions for the provision of ancillary services. These are crucial for maintaining the quality of service in electricity supply. Since renewable power units are often subject to fluctuating weather conditions, their contributions to ancillary service products are subject to uncertainties. Thus, coalitions should be assessed with regard to how reliable their contributions for ancillary services are. In the presented research project, the RelACs-method has been developed as an artefact to enable this assessment for the use case of providing primary control reserves.

In this chapter, a summary of this thesis is provided outlining the development of the RelACs-method and giving first evaluation results together with a discussion of the quality of the research results. Subsequently, possible extensions and improvements are proposed. This chapter concludes with an outlook to future demand for research.

7.1. Summary and Discussion

The objective of the presented research project has been to develop a method to assess a coalition of RPU or (dynamic) virtual power plants with respect to how reliable they are able to provide ancillary services deployed to maintain a feasible system state. As a use-case the provision of primary control reserves for frequency control has been chosen. More precisely, a method to incorporate as a constraint into a coalition forming process has been developed that may be used under real-time restrictions.

The procedure for the research project has been according to the design-science process [40] where an objective centred solution has been sufficient, i.e. the objectives of the solution have been inferred from the problem definition and served as a starting point for design and development of the solution. In Chapter 2, the necessary background for this project has been introduced and relevant related work has been presented and discussed.

In Chapter 3, a formal framework has been proposed for the description of ancillary services to maintain a feasible system state as well as the requirements for those services. Additionally, a formal understanding has been derived for the provision of AS products by coalitions of RPU that are represented by software-agents. Furthermore, the use-case of providing PCR has been presented in more detail as well as an approach for the provision of PCR by RPU. With this the problem has been identified and the need for an
assess ment method regarding the reliability of AS-products by distributed coalitions has been motivated, thus, forming the objective of the presented research project.

An approach to fulfil the need for reliability assessment has been developed in Chapter 4 – the RelACs-method for the reliability assessment of ancillary-service coalitions. First, the functional and non-functional requirements for a solution have been specified and a definition of reliability in the context of AS-provision by RPU has been derived. The following list gives the design choices in order to take into account the aspects relevant for reliability assessment.

**Prediction uncertainties** The uncertainties of predictions are represented by error models that specify the probability distribution of prediction errors at a certain point in time. For different time horizons different parameters for the distributions are used to reflect the increase in uncertainty with time. The error model is obtained by fitting a parametric model.

**Dependencies** The dependency structure of a coalition with respect to the power feed-in of the units is modelled using so-called copulas together with the units’ error models. This results in a joint distribution function for joint deviations of predictions of all units. The copula model is obtained by fitting parametric models or using non-parametric models.

**Unit failures** The failure behaviour of units is described by failure rate models commonly used to describe the reliability of technical systems.

**Grid reliability** The fact that units are disconnected from the system for the reason of failures of operational equipment is assessed based on models of failure rates of operational equipment. The events relating to disconnection of units are obtained by using methods of graph theory.

The aspects of uncertainties and unit failures relate to single units. The first influences the dependencies between units. The second is assessed separately. The aspects dependencies and grid reliability are also considered separately since the first one reflects the dependencies based on spatial vicinity whereas the latter refers to relationships due to the topological position of units, i.e. their grid connection nodes and linking lines.

In Chapter 5, implementation details of the RelACs-method have been given with the focus on data exchange. The evaluation environment has been presented that enables scenario-based experiments. The choices of factors to incorporate for investigations has been discussed and relevant parameter domains have been given. Furthermore, risk measures have been introduced to estimate the expected amount of reserves that cannot be provided in relation to reliability. The design and implementation of the evaluation environment enables that results for different scenarios and parameter choices can be generated and that it is possible to reproduce evaluation results.

The demonstration and evaluation of the RelACs-method has been presented in Chapter 6. To this end, scenarios have been defined as a frame for investigations. The evaluation has been separated into four steps. A brief overview of the corresponding results
is given in the following. The use case of the presented research project has been the provision of primary control reserves by coalitions of RPU with the objective to integrate the RelACs-method as a constraint into a coalition forming process.

**Functional validity** In the first evaluation step, it could be confirmed that for the given setup the functional requirements are fulfilled by the RelACs-method. The method returns a viable measure for a lower bound of the reliability of arbitrary coalitions. Furthermore, the basic assumption that this measure can be processed and incorporated by a coalition forming process has been confirmed. It could be shown that the RelACs-method can be utilised to assess arbitrary aggregations of PV and wind units.

**Interoperability with coalition forming** In the second evaluation step, it could be confirmed that for the given setup the non-functional requirements are fulfilled by the RelACs-method. The computational time scales with the number of a coalition’s member units and up to a certain number of units, real-time requirements are fulfilled.

**Recommendations for application** Given the fact that it is possible to integrate the RelACs-method into the coalition forming process, the third evaluation step has been to derive recommendations for coalition forming. The reliability of coalitions that have been formed under influences of different factors has been investigated. With the results, conclusions can be drawn about the settings that are advantageous in the sense that under those circumstances reliable coalitions may be obtained. Thus, two different scenarios reflecting rural sites for both north and south Germany have been investigated in order to reflect different penetrations of RPU.

The constellation of the coalition and the constellation of contributions has not shown significant influence. In more detail, the installed capacity of the coalition and the product size do not play a role as long as it is possible to find a coalition. However, there have been indications that smaller numbers of units are more advantageous. Furthermore, the way units split their contributions, i.e. if units have a similar share in the whole contribution or not, is not a significant factor. The most significant factor having a positive effect on reliability is the dependency between units with regard to power feed-in. This implies that units lying spatially close to each other provide more reliable products. The position of units in the power grid, i.e. the grid nodes to which units are connected, does not have a significant influence on reliability. The effect of the coalition’s lifespan is more relevant. The lifespan of a coalition is the duration for which a coalition provides reserves. It has a negative effect on reliability indicating that shorter lifespans yield more reliable results. The best cases for provision of primary control reserves by RPU are low volatile weather conditions. However, in case of high fluctuations, the combination of spatially close units with short lifespans still constitute the best choice for PCR-coalitions.

These results can be interpreted as recommendations of how to form reliable coalitions, i.e. aggregate spatially close units for short time spans. This may serve as an entry point for developing a heuristic to form reliable coalitions.
Risk estimation of reliable coalitions  The above-mentioned metrics have been utilised in order to investigate the relationship between reliability and risk. For topological reliability, the expected behaviour that risk increases with decreasing reliability could be confirmed. The risk assessment metric in case of spatial reliability has returned the expected behaviour for the case that units within a coalition are highly dependent.

These results are based on artificial and regionally-specific scenarios and model assumptions to obtain insights of the behaviour and quality of the RelACs-method. It is possible to generalise the results in the sense that scenarios, factor level choices and models used can be easily substituted and altered within the evaluation environment. Experiments should be repeated and extended using more mature models based on extensive data and analyses. This would be subject to another iteration of the cycle between the steps of design and development, demonstration and evaluation of the design science process. During the evaluation process several properties of the RelACs-method have been identified that may be improved. The findings are presented subsequently.

7.2. Possible Improvements to the RelACs-Method

Based on the evaluation results, the following improvements and adaptations to the RelACs-method have been encountered to be beneficial. Furthermore, possible extensions of the methods are outlined. The improvements are listed separately for the spatial and the topological assessment of the RelACs-method.

Spatial Position  In case of assessment of reliability regarding the spatial position of units the following improvements are possible.

Account for fluctuations  So far, the spatial reliability obtained by the RelACs-method is an underestimation as it gives a lower bound for the reliability since the assessment is based on the lowest predicted value. However, taking into account the fluctuating character of predictions, the assessment might be more precise. Evaluating the reliability at different points in time throughout the lifespan or integrating probabilistic forecasts instead of point forecast might be an entry point to this extension. Proper extensions and methods for modelling the dependencies have to be made.

Temporal dependencies of errors  Given a prediction error at some point in time it is likely that an error occurs in the subsequent point in time, too. Taking into account these temporal dependencies might yield better estimates regarding the reliability of ancillary services. With the knowledge of temporal dependencies between forecast errors uncertainties may be reduced. For this possible extension, the usage of probabilistic forecasts as a basis to predict trajectories of uncertainties might be useful. This approach might be beneficial to adapt the risk assessment method, as well.

Relaxation of assessment  The RelACs-method as implemented so far encounters a violation of a coalition's contribution if at least one member unit cannot provide the
required amount at any point in time of the considered time horizon. A relaxation of this strict interpretation should be considered. This might be achieved e.g. by reducing the temporal restriction, i.e. violations are allowed for a certain time span, or by reducing restrictions regarding the contributions, e.g. a certain percentage that cannot be activated is accepted. An investigating to which extend a relaxations is uncritical to system stability should be conducted, e.g. by assessing the resulting risk.

**Topological Assessment** The following propositions might be used to improve the assessment of reliability related to the topological position of units.

**Improvement of grid model** For the assessment implemented so far constant failure rates for operational equipment are used. These models might be extended to predict different load situations and adapt the failure rates accordingly since failures may occur with different probabilities depending on their loads. Additional equipment might be introduced into the assessment. The existing method is based on radial systems. This may be extended to network systems.

**Simulative approach** The analytical approach used in the RelACs-method may be substituted by a simulative approach, e.g. using Monte-Carlo methods. This might be particularly beneficial for investigations of network systems. However, the computation time might be a constraining factor for this approach.

**Relaxation of assessment** As in case of spatial considerations for the topological case a relaxation of the assessment might be sufficient, too. To this end, the probability of occurrence of system frequency deviations may be incorporated similarly to the spatial case. Moreover, partial failures of units might be considered, as well.

**Expansions** The RelACs-method has been evaluated for the case of providing primary control reserves. The method may be adapted for other ancillary serves such as the provision of secondary and tertiary control reserves or voltage control. To this end, the framework for the formal description of ancillary services may be utilized to identify common properties and requirements and to identify adaptations that must be made due to deviating requirements. In accordance to this, the RelACs-method may be adapted.

### 7.3. Further Research

During the evaluation process, especially with regard to the application of coalition forming, properties have been encountered that might be extended based on the existing RelACs-method. The following items might be considered for further research.

**Schedule-based assessment** The provision of ancillary services are often subject to real-time restrictions and a continuous activation of power reserves must be guaranteed, i.e. an activation at any point in time. In case of energy products, a schedule of
stipulated amounts of energy for certain time intervals has to be fulfilled. This means that the power output of a unit does not necessarily have to be constant and in case of a virtual power plant units may compensate for discrepancies in other units’ schedules. Hence, the requirements differ from those for ancillary services. Further research might show how the methodology of the RelACs-method may be adapted for schedule-based energy products.

**Heuristics for coalition forming** As mentioned before, the evaluation results give indications for choices during coalition forming regarding unit properties and choices for composing units that result in reliable coalitions. The process used for evaluation may be extended to a finer resolution of factor levels and a meta-model for reliability depending of the factor levels may be derived. Then for a given system state (e.g. load or weather conditions) the choice of units to include for negotiations may be narrowed down. Based on that a search heuristic may be developed to obtain reliable coalitions.

**Reliable contributions from the dependency model** An alternative to the previous suggestion may be an analytical approach. A future research project might be to investigate if it is possible to use the concept of so-called contour lines to find reliable coalitions. A procedure for the one-dimensional case has been discussed in Section 4.3. The copula model used to assess reliability with regard to the spatial position determines a reliability for accepted deviations from prediction errors. These deviations are related to the units’ contributions. For a fixed set of units there are different constellations of the unit’s contributions with the same reliability. These constellations may be described as contour lines, i.e. the set of all constellations of contributions that result in the same reliability. The other way around, for a given level of reliability, future research might investigate if a set of contributions can be found fulfilling this reliability. The outcome would be a set of coalitions with a reliable contribution.

**Value at risk** For further research based on the evaluation setup used in this research project, a value-at-risk approach might be developed. To this end, a meta-model for risk depending on different factor level combinations may be derived. The methodology of value at risk may be applicable, i.e. for a given level of accepted risk the factor level combinations lying below this level may be determined. This information may be incorporated into the decision-making process of operators of VPP or to give recommendations for market constraints. To this end, the risk measure might be annotated with monetary costs, as well.
A. Appendix

A.1. Preliminary Work

In this section, an overview is given of preliminary work related to the presented research project. Below, the corresponding contributions are listed along with the parts touched of the RelAC-method. The contributions have been presented at appropriate conferences or published in journals in order to cover an audience of the related fields such as computer science and electrical engineering.

• A Concept for Reliability Assessment for the Provision of Ancillary Services [7]
  – Definition of reliability
  – Concept of reliability assessment

• Assessing Reliability of Distributed Units with Respect to the Provision of Ancillary Services [6]
  – Formal model of ancillary services
  – Hierarchical model for reliability assessment
  – Unit reliability assessment
  – Relationship of controllable factors

• Correlations in Reliability Assessment of Agent-based Ancillary-Service Coalitions [8]
  – Dependency model using copulas
  – Short introduction to usage of copulas
  – Coalition reliability assessment
  – First properties of the RelACs-method

• Efficient Provision of Ancillary Services by Decentralized, Volatile Generating Units [87]
  – Concept of risk assessment
  – Usage of risk assessment to evaluate the RelACs-method

• Distributed Coalitions for Reliable and Stable Provision of Frequency Response Reserve – An Agent-based Approach for Smart Distribution Grids [72]
- Integration of RelACs-method into coalition formation
- Agentenbasierte Vorhaltung und Erbringung von Primärregelleistung durch koordinierte Verbünde dezentraler prognoseunsicherer Anlagen [41]
- Assessment of coalitions using the RelACs-method along with stability considerations
- Regionally-Specific Scenarios for Smart Grid Simulations [42]
- Process of scenario design

A.2. Theory

A.2.1. System Reliability Theory – Formal Concepts

In this chapter an overview of basic concepts (mainly from [89] and [81]) for quantitative approaches for the assessment of system reliability needed for this thesis is given. This section is an extension of Section 2.2.1. Here, formal concepts of basics for reliability assessment are given.

The reliability assessment follows an actuarial approach, i.e. probabilistic methods are used and the information about an item is described by and derived from a probability density function of the time to failure. The advantage of this approach is that items do not have to be modelled explicitly.

In order to describe the state of an item at time \( t \), a state variable \( X \) is introduced which is interpreted as a random variable. It is defined as follows:

\[
X(t) = \begin{cases} 
1, & \text{item is functioning at time } t \\
0, & \text{item is in a failed state at time } t.
\end{cases}
\] (A.1)

The time to failure \( T \) is the time from when an item was put into operation until the first time a failure occurs. \( T \) as well is considered as a random variable. The starting time is assumed to be \( t = 0 \). The state variable is directly connected with the time to failure. Hence, Equation A.1 can be restated as

\[
X(t) = \begin{cases} 
1 & \text{if } t < T \\
0 & \text{if } t \geq T.
\end{cases}
\] (A.2)

Note that in this thesis \( t \) is measured in time. However, there are other indirect time measures such as operational steps, too.

The probability of failure at time \( t \) is the probability with which a failure occurs until \( t \), i.e.

\[
F(t) = Pr(T \leq t),
\] (A.3)

where \( F \) is a probability function. Let \( f \) denote the according failure density function which is assumed to be continuous. Then

\[
F(t) = \begin{cases} 
\int_0^t f(u)du & \text{for } t > 0 \\
F(t) = 0 & \text{for } t \leq 0.
\end{cases}
\] (A.4)
It holds that \( \lim_{t \to \infty} F(t) = 1 \) which means that the item will be failed at infinite time.

If one is interested in the probability with which an item is still functioning at time \( t \), i.e. no failure occurred within the interval \((0, t]\), the so called survivor function or reliability function is used. It is defined as

\[
R(t) = \begin{cases} 
1 - F(t) & \text{for } t > 0 \\
1 & \text{for } t \leq 0.
\end{cases}
\]  
(A.5)

This means \( R(t) = 1 - \Pr(T \leq t) = \Pr(T > t) \). At starting time, the item is functioning which is reflected by the fact that \( R(0) = 1 \). Furthermore, \( \lim_{t \to 0} R(t) = 0 \). On the one hand, the probability of a failure at time \( t \) is the value \( F(t) \) which corresponds to the area under the failure density curve above the interval \((-\infty, t]\). On the other hand, the survival probability is the value of \( R(t) \) corresponding to the area under \( f \) above the interval \((t, \infty)\).

The probability that an item that survived until \( t \) fails during the interval \((t, t + \Delta t]\) for \( t > 0 \) is expressed by the conditional probability

\[
\Pr(t < T \leq t + \Delta t | T > t).
\]  
(A.6)

This can expressed as follows

\[
\Pr(t < T \leq t + \Delta t | T > t) = \frac{\Pr((t < T \leq t + \Delta t) \cap T > t)}{\Pr(T > t)}
\]

since \( (t < T \leq t + \Delta t) \subseteq (T > t) \)

\[
= \frac{\Pr(t < T \leq t + \Delta t)}{\Pr(T > t)}
\]

\[
= \frac{F(t + \Delta t) - F(t)}{R(t)}.
\]

From this, the failure rate function \( z \) can be derived as follows:

\[
z(t) = \lim_{\Delta t \to 0} \frac{\Pr(t < T \leq t + \Delta t | T > t)}{\Delta t}
\]

\[
= \lim_{\Delta t \to 0} \frac{F(t + \Delta t) - F(t)}{\Delta t} \cdot \frac{1}{R(t)}
\]

\[
= f(t) \cdot \frac{1}{R(t)}.
\]

Table A.1 shows the relationship between the reliability indicators. \( R(t) \) and \( F(t) \) are uniquely determined by the failure rate \( z(t) \). For detailed derivation see, e.g., [89]. Note that there are similar concepts for repairable systems, see, e.g. [89].

The failure rate function typically has the shape of a 'bathtub curve'. The first phase of an item's life is called the infant mortality phase or burn-in phase because during this time failures often occur due to e.g. weak materials or variations in the quality in production. Thus, the failure rate is high at the beginning but strongly decreasing in the first phase. The second phase is the so called stable phase or usual life phase. During this time the failure rate function is constant or slightly increasing. A third phase is the item’s wear-out...
### A.2. Theory

<table>
<thead>
<tr>
<th>probability of failure $F(t)$</th>
<th>reliability $R(t)$</th>
<th>failure density $f(t)$</th>
<th>failure rate $z(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - R(t)$</td>
<td>$\int_0^t f(\tau) d\tau$</td>
<td>$1 - e^{-\int_0^t z(\tau) d\tau}$</td>
<td></td>
</tr>
<tr>
<td>$1 - F(t)$</td>
<td>$\int_0^{\infty} f(\tau) d\tau$</td>
<td>$e^{-\int_0^t z(\tau) d\tau}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{dF(t)}{dt}$</td>
<td>$-\frac{dR(t)}{dt}$</td>
<td>$z(t) \cdot e^{-\int_0^t z(\tau) d\tau}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{1-F(t)} \cdot \frac{dF(t)}{dt}$</td>
<td>$-\frac{1}{R(t)} \cdot \frac{dR(t)}{dt}$</td>
<td>$\frac{f(t)}{\int_0^t f(\tau) d\tau}$</td>
<td></td>
</tr>
</tbody>
</table>

**Table A.1.**: Relation of formulas between reliability characteristics for repairable systems according to [81]

period. The failure rate function is strongly increasing because of deterioration, fatigue and so on. Of course, the duration of each phase depends on the item under consideration and therefore varies strongly.

There are different types of failure rate models (details can be found in [81]). Given a failure rate function, the distribution and survivor function can be estimated (see Table A.1). One common failure rate model is a constant failure rate $z(t) = \lambda = \text{const}$. This can, e.g., be used to model electrical components that have a long life cycle. Then the failure density function results in $f(t) = \lambda e^{-\lambda t}$ and the survivor function in $R(t) = e^{-\lambda t}$.

### A.2.2. Risk Assessment Theory

In this section a brief overview is given of basic terminology concerning risk assessment. According to [62] risk is defined as the “combination of the probability of occurrence of harm and the severity of that harm”. In classical risk assessment a harm is “physical injury or damage to persons, property, and livestock” [62]. The process of risk assessment comprises risk analysis and risk evaluation. **Risk analysis** is defined as “systematic use of available information to identify hazards and to estimate the risk” [62] and **risk evaluation** as the “procedure based on the risk analysis to determine whether the tolerable risk has been achieved” [62].

In [83] risk is defined in a similar way as function of frequency and consequence of undesirable events. The frequency of occurrence may be given as number, rate or probability. The consequence must be determined with sufficient indicators. The process of risk assessment is presented in a more detailed way as indicated in Figure A.1. First preparation for the analysis are being made. This step mainly is concerned with the defining
expectations for the results and the complexity. In the second step, the system under investigation is defined comprising definition of the object of analysis, a delineation of items belonging to the system and a definition of system boundaries. As a third preliminary step, possible events harming the system must be identified and unambiguously defined. The corresponding consequences must be identified as well. The step of risk estimation comprises the frequency and consequence analysis. Appropriate methods with respect to the goals of the analysis must be conducted. The consequence analysis corresponds to the events identified and should incorporate short-term as well as long-term consequences. After risk estimation the results is presented in an appropriate way such that it is possible to estimate the risk with regard to external criteria determining whether the risk is acceptable or not. According to [62] a tolerable risk is a “risk which is accepted in a given context based on the current values of society”. In case the criteria are fulfilled, a reduction of risk can follow the process of risk assessment. In case the criteria are not fulfilled, actions must be taken in order to optimise the system. The success of the optimisation must be investigated by another iteration of the risk assessment process.

A.2.3. Copula Theory

This section is an extension of the concepts presented in Section 4.4.1. In order to keep the argumentation clear, some facts stated before are repeated here.

The theory stated here is based on [84], and [24]. The definition¹ of a copula is according to [24].

¹There are other definitions that are rather technical that can be found in the given literature.
**Definition A.2.1 (Copula)**

For every \( n \geq 2 \), a **\( n \)-dimensional copula** (shortly \( n \)-copula) \( \mathcal{C} \) is an \( n \)-variate distribution function on \( \mathbb{I}^n = [0,1]^n \) whose univariate marginals are uniformly distributed on \( \mathbb{I} = [0,1] \), i.e. \( U_i \sim \mathcal{U}(\mathbb{I}) \).

Basically, the definition states that each \( n \)-copula can be associated with an \( n \)-variate random variable \( U = (U_1, \ldots, U_n) \) whose components are uniformly distributed on the identity interval \( \mathbb{I} = [0,1] \). The other way around, an \( n \)-variate random vector \( U = (U_1, \ldots, U_n) \) of on \( \mathbb{I} \) univariate distributed variables \( U_i \) is distributed according to a copula \( \mathcal{C} \).

A very important result is the following theorem that is referred to as “Sklar’s Theorem”. It connects a copula to an arbitrary multivariate distribution. The formulation of the theorem is according to [24].

**Theorem A.2.2 (Sklar’s Theorem)**

Let \( F \) be an \( n \)-dimensional distribution function with univariate margins \( F_1, \ldots, F_n \). Then there exists a copula \( \mathcal{C} \) such that for all \( (x_1, \ldots, x_n) \in \mathbb{R}^n \) (\( \mathbb{R} := \mathbb{R} \cup \{\infty\} \)),

\[
F(x_1, \ldots, x_n) = \mathcal{C}(F_1(x_1), \ldots, F_n(x_n)).
\]

Such a copula is uniquely determined on \( F_i(\mathbb{R}) \times \cdots \times F_n(\mathbb{R}) \), where \( F_i(\mathbb{R}) \) is the range of \( F_i \) for \( i = 1, \ldots, n \). Hence, it is unique, when all \( F_1, \ldots, F_n \) are continuous.

Sklar’s Theorem has the following important result.

**Theorem A.2.3 (Result of Sklar’s Theorem)**

Given univariate distribution functions \( F_1, \ldots, F_n \) and any \( n \)-copula \( \mathcal{C} \), the function \( F : \mathbb{R}^n \to \mathbb{I} \) with \( F(x_1, \ldots, x_n) = \mathcal{C}(F_1(x_1), \ldots, F_n(x_n)) \) is an \( n \)-dimensional distribution function with margins \( F_1, \ldots, F_n \).

As a result, a copula \( \mathcal{C} \) can be derived from a multivariate distribution function by applying Sklar’s Theorem. If the distribution functions \( F_1, \ldots, F_n \) are continuous distribution functions, and \( F_1^{-1}, \ldots, F_n^{-1} \) are the corresponding pseudo inverses\(^2\) \( \mathcal{C} \) is given as

\[
\mathcal{C}(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)),
\]

where \( u = (u_1, \ldots, u_n) \in \mathbb{I}^n \). Furthermore, the marginals can be transferred to each other via \( U_i = F_i(X_i) \) for \( i = 1, \ldots, n \).

The concept of a copula can be adapted to a concept of a survival copula.

---

\(^2\) For a distribution function \( F \) the pseudo inverse or quantile function is defined as \( F^{-1}(s) = \inf \{ t \mid F(t) \geq s \} \).
**Definition A.2.4 (Survival Copula)**

Let $\mathbf{X} = (X_1, ..., X_n)$ be a random vector with joint survival function $\bar{F}$ and univariate survival margins $\bar{F}_1, ..., \bar{F}_n$. Then for all $(x_1, ..., x_n) \in \mathbb{R}^n$ holds

$$\bar{F}(x_1, ..., x_n) = \bar{C}(\bar{F}_1, ..., \bar{F}_n)$$

for some copula $\bar{C}$. This copula is called the **survival copula** of $\mathbf{X}$.

Here, one has to be cautious in order not to confuse the survival copula $\bar{C}$ with the survival function of a copula $C$ of an $n$-variate uniformly distributed random vector $\mathbf{U} = (U_1, ..., U_n)$ i.e. $\bar{C}(u_1, ..., u_n) = \Pr(U_1 \geq u_1, ..., U_n \geq u_n)$. However, there is a connection between $\bar{C}$ and $C$.

Let $C$ be the copula of $\mathbf{X}$ and $\mathbf{U} = (U_1, ..., U_n)$ with $\mathbf{U} \sim C$, i.e. $C$ is the distribution function of $\mathbf{U}$, then it holds

$$\bar{C}(u_1, ..., u_n) = C(1 - u_1, ..., 1 - u_n).$$

The survival function associated with $C$ can be given explicitly as

$$\bar{C}(u_1, ..., u_n) = 1 + \sum_{k=1}^{n} \sum_{1 \leq i_1 < i_2 < ... < i_k \leq n} \Pr(A_{i_1} \cap ... \cap A_{i_k}) \bigg( -1 \bigg)^k,$$

where $A_{i_1} \cap ... \cap A_{i_k}$ are the marginals related to $(i_1, ..., i_k)$.

The advantage of using copulas is that the joint distribution as well as the dependence structure of random variables can be expressed by the marginal distributions and the

---

3 This follows from [78]

$$\bar{C}(u_1, ..., u_n) = \Pr(U_1 > u_1, ..., U_n > u_n)$$

$$= \Pr(1 - U_1 \leq 1 - u_1, ..., 1 - U_n \leq 1 - u_n)$$

$$= \bar{C}(1 - u_1, ..., 1 - u_n).$$

4 This results from probability theory (see e.g. [54]): for $n$ events $A_1, ..., A_n$ the joint event is given as $\bigcap_{k=1}^{n} A_k$. Then for the complementary event holds:

$$\Pr\left(\bigcap_{k=1}^{n} \bar{A}_k\right) = \sum_{k=1}^{n} \left( -1 \right)^{k+1} \sum_{i_1, ..., i_k \in \{1, ..., n\}} \Pr(A_{i_1} \cap ... \cap A_{i_k})$$

with $A_i$ being the event that $U_i > u_i$ it follows

$$\bar{C}(u_1, ..., u_n) = 1 - \Pr\left(\bigcap_{k=1}^{n} \bar{A}_k\right)$$

$$= 1 + \sum_{k=1}^{n} \left( -1 \right)^k \sum_{i_1, ..., i_k \in \{1, ..., n\}} \Pr(A_{i_1} \cap ... \cap A_{i_k}),$$
copula. The marginals can be arbitrary, which offers high flexibility for modelling a coalition’s error structure.

There are different families of copulas. Given the empirical data — the errors of the coalition’s member units — the model can be fitted to a copula type using the Maximum Likelihood method. This method is used to estimate the parameters of a parametric function based on the data at hand. The output parameters are those for which the result of the empirical data is most likely. Subsequently, this is stated in more detail.

Furthermore, there are methods available to compare the goodness of fit between different types of copulas for the same data. In order to get an idea of which copula type is suitable, a scatter plot of the empirical copula can be consulted. In order to graphically check the adequacy of a model fit, the empirical data and samples of the fitted model can compared in a scatter plot.

As mentioned earlier, a joint distribution can be expressed by a copula $C$ and the respective marginal distributions $F_1, \ldots, F_d$ as $F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d))$. Thus, the copula can be utilized for reliability assessment. However, for reliability assessment, the joint survival function $\overline{F} = \Pr(X_1 \geq x_1, \ldots, X_d \geq x_d)$ is needed that can be expressed by a survival copula $\overline{C}$ (see Definition A.2.4). In order to obtain the coalition’s reliability, the reliability values of all regions can be multiplied since they are assumed to behave independently.

A.2.4. Experimental Design

In this section a brief overview about techniques from experimental design are given that have been used in this thesis. This background is mainly based on the textbooks [68] and [98]. For an introduction in English refer to [93], for example.

The aim of experimental design is to gain knowledge about a system’s input-output behaviour given different initial conditions or settings. This behaviour can then be modelled using meta-modelling techniques.

Each system performs a specific task or function. How well the function is fulfilled by a system can be measured by certain quality measures, a system output. The quality measure or feature must be a continuous measurement and it can be used in order to assess the system’s behaviour. To this end, experiments are executed and with that samples from the quality features drawn.

In order to conduct experiments, the system and its boundaries must be well defined and the system’s input parameters specified. The parameters are inputs relevant to the system. They can be classified into controllable or non-controllable inputs. The latter may account for noise or unknown behaviour. So called factors are a subset of the system’s parameters that are regarded within experiments. The choice of factors determines the experimental design. The chosen settings for those factors are called levels. For each factor there must be at least two levels. All levels are set fix for investigations.

To investigate the system with respect to the quality measure, different techniques can be used. These are the mean of the quality feature given a factor level, the effect of a factor as comparison of means as well as interaction effects of two or more different factors. First,
some notations and the evaluation measures are introduced. Second, the significance of measures is discussed and after that the necessary number of experiments is specified.

Evaluation Measures

Let $k$ be the number of factors considered for experiments and $l$ the number of levels for each factor\(^5\). Further, let $m$ denote the number of combinations of factor levels, i.e. the number of repetitions of one experiments, and $N = n \cdot m$ the number of measurements. In a full-factorial design, experiments for all $m = l^k$ combinations of factor levels are considered. In a two-level case this amounts to $m = 2^k$ experiments. In that case, the two levels can be coded as + and −, where $+$ indicates the higher value or a preferred value and $-$ the respective opposite, for example. Subsequently, full factorial designs with two levels are considered. To this end, let $y_i$ denote the output, i.e. quality measure, of experiment $i$.

Mean

The quality measure can be evaluated investigating the mean of the samples drawn during experiments. An estimator for the true mean $\mu$ is given as

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i.$$  

Effect

The effect of a factor on the system’s output can be measured by the difference in the output for two different factor levels. Consider a factor denoted by $A$ with the two levels $+$ and $−$. Denote $\bar{y}_{A^+}$ the mean of sample values where $A$ is at the level $+$, and $\bar{y}_{A^-}$ the mean of sample values where $A$ is at the level $−$. The effect of factor $A$ then is given as

$$E_A = \bar{y}_{A^+} - \bar{y}_{A^-}.$$  

Interaction Effect

The interaction effect of two or more factors, where each factor is assumed to have two different levels $+$ and $−$, measures how much the effect of one factor depends on the other factors. If the dependence is significant those factors must be considered together. In case of two factors $A$ and $B$, the interaction effect is given by the difference of the mean effect of $A$ for all samples where $B$ is at the level $+$ and the mean effect of $A$ where $B$ is at the level $−$, i.e.

$$E_{A(B)} = E_{A(B^+)} - E_{A(B^-)} = \frac{1}{2} \left( \bar{y}_{A+(B^+)} - \bar{y}_{A^-(B^+)} \right) - \frac{1}{2} \left( \bar{y}_{A+(B^-)} - \bar{y}_{A^-(B^-)} \right),$$

where $\bar{y}_{A+(B^+)}$ denotes the mean of values where factors $A$ and $B$ both are at the level $+$ and $\bar{y}_{A+(B^-)}$ the mean of values where $B$ is at the level $−$ but $A$ is at the level $+$, etc. The effect of $A$ on $B$ amounts to the same.

\(^5\)Here, the number of levels for each factor is assumed to be equal. However, general experimental design is not restricted to that choice.
Table A.2.: Signs of factor combinations to determine effects of factors $A$, $B$, $C$ and interaction effects $AB$, $AC$, $BC$, and $ABC$

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>AB</th>
<th>AC</th>
<th>BC</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

In case of $k$ factors, there are $2^k - 1$ effects that can be investigated since the mean of all sample values is not considered to give valuable information. The effect is calculated as

$$E = \frac{2}{m} \cdot \sum_{i=1}^{m} (\text{sgn} \cdot \bar{y}_i),$$

(A.7)

where $\bar{y}_i$ is the mean of sample values for factor combination $i$ and sgn the corresponding sign. Table A.2 exemplary gives the $2^3 - 1$ sign columns of all $2^3$ factor combinations for all effects of three factors $A$, $B$, $C$ each having two levels + and −.

Significance of Effects

The basis to investigate the significance of effects are confidence intervals. The concept briefly is introduced for the mean of a sample set. Since an effect or interaction effect is the difference of two values, the significance of this difference is evaluated in order to draw conclusions on the significance of the effect.

Confidence of Mean The value $\bar{y}$ given above is an estimator for the true mean $\mu$ and its value depends on the values of the sample set. Variations of $\bar{y}$ for different sample sets occur due to randomness. A confidence interval is an interval that covers the true value $\mu$ with a certain probability that must be specified beforehand. The confidence interval for $\mu$ is given as

$$[\bar{y} - t \cdot s_y, \bar{y} + t \cdot s_y],$$

where $s_y = \frac{s}{\sqrt{n}}$ is an estimator for the standard deviation of the mean value, $s$ is an estimator for the standard deviation of all sample values, and $t$ is given by the $t$-distribution depending on the confidence level, i.e. the predefined probability to cover $\mu$, and a number of degrees of freedom $f$ which in this case are $f = N - 1$. The concept of confidence interval can be adapted to evaluate other measures, as well, such as differences between means.
Significance of Effects  An effect is the difference of the mean of sample values for a factor being at the level + and the mean of sample values of the same factor at level −. For an interaction effect the difference of the effect of one factor for different levels of other factors is estimated. In order to assess the significance of (interaction) effects, it is investigated if the respective difference significantly differs from zero. To this end, the confidence interval of the difference is determined that – to a certain level of confidence – covers the true difference. If the value zero is contained in the interval it means that the difference only occurred due to randomness or it is too small to detect it with significance. The level of significance depends on the confidence level.

In order to conduct these investigations, the following prerequisites must hold for each group, i.e. each combination of factor levels:

- the single values of the two groups whose difference is assessed must be representative,
- the single values of each group must be normally distributed, and
- the standard deviation for both groups must be the same.

Given these preconditions, the following steps can be conducted in case two means are being compared. Consider the estimator for the difference \( \hat{d} = \bar{y}_I - \bar{y}_{II} \) of the mean values \( \bar{y}_I \) and \( \bar{y}_{II} \) of two groups I and II. Determine the confidence interval for \( \hat{d} \) for a certain confidence level as

\[
[\hat{d} - t \cdot s_{\hat{d}}, \hat{d} + t \cdot s_{\hat{d}}],
\]

where \( s_{\hat{d}} = \sqrt{\frac{1}{N} \cdot s} \) is the standard error, \( s^2 = \frac{s_I^2 + s_{II}^2}{2} \) is the mean of variances of the groups. The value \( t \) depends on the degrees of freedom \( f = N - 2 \) and the confidence level, according to a \( t \)-distribution.

It is common to calculate the 95%, 99% and 99.9% confidence intervals, where the level of significance is given as indifferent if the effect lies in between the 95%- and 99%-interval, significant if it lies between the 99%- and 99.9%-interval, and highly significant if it lies within the 99.99%-interval.

This can be generalized to the case where the interaction effects of more than two factors are investigated. As mentioned before, there are \( m - 1 = 2^k - 1 \) effects of interest. The confidence interval of an effect is given as

\[
[\hat{d} - t \cdot s_{\hat{d}}, \hat{d} + t \cdot s_{\hat{d}}],
\]

where \( s_{\hat{d}} = \sqrt{\frac{1}{N} \cdot s^2} \) is the standard deviation of the effect and \( s^2 = \frac{1}{m} \cdot \sum_{i=1}^{m} s_i^2 \) the mean of variances of the factor level combination that is an estimator for the variance. \( t \) is a value according to the \( t \)-distribution for the given confidence level and \( f = N - m \) the degrees of freedom.

The significance of effects can be visualized using a representation as given in Figure A.2. There, the confidence levels for a \( 2^2 \) factorial design are plotted symmetrically around the zero-axis. The (interaction) effects are given as bars. If a bar exceeds one of the lines, the
effect is significant to the according level of confidence. For factor \( A \) the bar exceeds the 99.9%-interval and therefore is highly significant. The bar representing the effect of \( B \) can be considered significant since the bar exceeds the 99%-interval. The interaction effect of \( AB \) together can be considered as indifferent and more data should be collected.

Choice of Number of Experiments

As a first approximation, the number of investigations that should be conducted does not depend on the number of factors. It is suggested to be approximately

\[
N = 60 \cdot \left( \frac{\sigma}{\Delta \mu} \right)^2 ,
\]

because with that in 90% of all cases the 99% confidence interval does not contain 0 if the true value is \( \Delta \mu \). Thus the significance of an effect can be recognized with high probability. \( \Delta \mu \) gives the correctness one wishes to achieve, i.e. the magnitude of an effect that should be recognized. \( \sigma \) is the variance of sample values. If \( \sigma \) is not known beforehand, experiments should be conducted with small values of \( n \). If the experiments already yield satisfying results w.r.t. to the quality of results no further investigations are necessary. Otherwise, an estimator for \( \sigma \) can be determined and with that \( N \) and \( n \sim N/m \).

A.3. Additions to Experiments

A.3.1. Experimental Setup

Model for Frequency Deviations

For the modelling of frequency deviations to incorporate into reliability and risk assessment conducted in Chapter 6 data has been made available from the Technical University of Dortmund. The data comprises almost 160,000 system frequency measures in a temporal resolution of one second for January 27th till 29th in 2013. Frequency deviations may be modelled for different days, seasons or during certain events, e.g. However, for
the investigations in the presented research project a model for the given time horizon has been sufficient.

For modelling the deviations of system frequency from the nominal value of 50 Hz have been calculated and the resulting data has been evaluated with respect to its distribution. Figure A.3 shows plots of a histogram and a cumulated histogram of the data. The data has been fitted to a normal distribution with mean value of 0.0001911 and standard deviation 0.019671549 as indicated as a red line for both cases probability density and distribution function.

Base Scenario

In the course of the research network Smart Nord evaluation scenarios have been developed that are suitable for investigations concerning different research questions [59, 42]. The outcome of the Smart Nord scenario design is a regionally specific setting of grid structure and unit penetration. For this thesis, the distribution grid, i.e. low and medium voltage levels, have been of interest. In Smart Nord, rural grid structures and a unit penetration typical for Lower Saxony have been chosen. Eight model grids for the low voltage level have been defined according to realistic grid data. As a representative of the medium voltage level a benchmark grid has been adapted to the project’s needs. The low voltage grids have been uniformly assigned to nodes of the medium voltage grid according to project-specific requirements. Based on available data, for PV and wind units representative unit sizes regarding installed power as well as number of units have been derived for each voltage level in the distribution grid. For details see [42, 59]

The assumptions that have been made have been adapted for the evaluations of this thesis. However, only a subset of the Smart Nord scenario have been chosen as base scenario for the evaluation set-up of this thesis. This base scenario then has been assumed to be fix for all investigations in order to guarantee comparability of the results. The according choice of LV-grids for the RelACs-scenarios is given in Table A.3 as well as corresponding information about the grids.
Table A.3.: Information about low voltage grids in RelACs-scenarios

<table>
<thead>
<tr>
<th>low voltage grid</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of nodes</td>
<td>41</td>
<td>139</td>
<td>67</td>
<td>57</td>
<td>169</td>
<td>299</td>
<td>66</td>
<td>103</td>
</tr>
<tr>
<td>estimated inhabitants</td>
<td>99</td>
<td>529</td>
<td>259</td>
<td>177</td>
<td>589</td>
<td>1099</td>
<td>247</td>
<td>332</td>
</tr>
<tr>
<td>number of occurrence in RelACs-scenarios</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) PV low volatility  
(b) PV average volatility  
(c) PV high volatility  
(d) Wind low volatility  
(e) Wind average volatility  
(f) Wind high volatility

Figure A.4.: Volatility of power feed-in

External Scenario

For evaluation, periods of time have been identified where different volatility of power feed-in occur as outlined in the following. In Smart Nord data for power feed-in of PV and wind units have been made available that are used to serve as predictions for the RelACs-scenario. For the scenario instantiation, for both and PV and wind, time segments have been chosen reflecting high, average or low volatility. In Figure A.4 the chosen time segments are indicated. Table A.4 gives the corresponding values. The maximum and minimum of power feed-in are given in percent relative to the installed power since normalized time series have been available. The standard deviation relates to the clearskey index in case of PV-units and to ramps in power feed-in in case of wind units.
### Table A.4.: Information about external setup in RelACs-scenario

<table>
<thead>
<tr>
<th>Volatility</th>
<th>PV units</th>
<th>Wind units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>feature</strong></td>
<td><strong>low</strong></td>
<td><strong>average</strong></td>
</tr>
<tr>
<td>day</td>
<td>8/05</td>
<td>16/09</td>
</tr>
<tr>
<td>max feed-in (%)</td>
<td>81.16</td>
<td>80.89</td>
</tr>
<tr>
<td>min feed-in (%)</td>
<td>64.01</td>
<td>48.89</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.0288</td>
<td>0.1017</td>
</tr>
</tbody>
</table>

### Table A.5.: Parameter choice for error models of PV and wind forecasts
(temporal resolution in minutes)

<table>
<thead>
<tr>
<th>technology</th>
<th>trend</th>
<th>intercept</th>
<th>gradient</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>logarithmic</td>
<td>-0.3006581</td>
<td>0.148431</td>
</tr>
<tr>
<td>wind</td>
<td>logarithmic</td>
<td>-0.048155</td>
<td>0.030512</td>
</tr>
</tbody>
</table>

Core Scenario – Error Model

For the choice of values for the error model [74] and [56] serve as orientation for PV and wind, respectively. In both cases the relative root mean square error (RMSE) for persistence forecast is given. The persistence forecast has been chosen in order to reflect a worst case forecast and thus allows a lower bound estimation.

The values given in [74] and [56] were used as data to fit a regression model of the evaluation of the standard deviation of the forecast error over time. The regression results given in Table A.5 yielded the best results with regard to the r-value of regression. Given a logarithmic trend and a length of prediction horizon $x$ in temporal resolution of minutes then the according relative standard deviation is calculated as $std_{rel} = intercept + gradient \cdot \log(x)$.

To model prediction errors the normal distribution with zero mean has been chosen (see Chapter 5). The standard deviation is increases over time. For choosing the trend of the error model’s standard deviation different regression models have been tested in order to choose the best fit with respect to the regression coefficient $R^2$. Table A.6 gives the values derived from [74] and [56] that served as the basis for the investigations. The results with corresponding values for $R^2$ are given in Figure A.5. In both cases, Wind and PV, for the logarithmic regression model the best result has been achieved.

Core Scenario – Dependencies

According to Section 5.2.3 a Gauss-Copula has been chose to model dependencies between units’ prediction errors. The dependencies however vary in the choice of parameter values.
A.3. Additions to Experiments

Figure A.5.: Regression results for the choice of standard deviation’s trend over time

Table A.6.: Value pairs of standard deviation’s trend as basis for regression

<table>
<thead>
<tr>
<th>time [min]</th>
<th>relative RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>0.3</td>
</tr>
<tr>
<td>120</td>
<td>0.42</td>
</tr>
<tr>
<td>180</td>
<td>0.48</td>
</tr>
<tr>
<td>240</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(a) PV

<table>
<thead>
<tr>
<th>time [min]</th>
<th>relative RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.04</td>
</tr>
<tr>
<td>60</td>
<td>0.08</td>
</tr>
<tr>
<td>180</td>
<td>0.12</td>
</tr>
</tbody>
</table>

(b) Wind
Table A.8.: Statistics for results for scalability of computational time in case of spatial reliability for homogeneous coalitions

<table>
<thead>
<tr>
<th>property</th>
<th>base1</th>
<th>base2</th>
<th>base4</th>
<th>base8</th>
<th>base16</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>6.8553</td>
<td>11.9034</td>
<td>24.9277</td>
<td>59.6924</td>
<td>158.2288</td>
</tr>
<tr>
<td>std</td>
<td>1.6956</td>
<td>2.5708</td>
<td>3.4287</td>
<td>6.4881</td>
<td>8.3015</td>
</tr>
<tr>
<td>min</td>
<td>4.0861</td>
<td>7.1210</td>
<td>15.8640</td>
<td>46.5732</td>
<td>146.3654</td>
</tr>
<tr>
<td>25%</td>
<td>5.3687</td>
<td>9.9247</td>
<td>23.4542</td>
<td>52.8377</td>
<td>150.4416</td>
</tr>
<tr>
<td>50%</td>
<td>6.8006</td>
<td>11.5251</td>
<td>25.0750</td>
<td>61.9852</td>
<td>158.6772</td>
</tr>
<tr>
<td>75%</td>
<td>8.1047</td>
<td>13.7295</td>
<td>26.7572</td>
<td>63.2648</td>
<td>165.0253</td>
</tr>
<tr>
<td>max</td>
<td>9.6892</td>
<td>16.3733</td>
<td>30.1710</td>
<td>71.0290</td>
<td>171.3638</td>
</tr>
</tbody>
</table>

Since a Gaussian copula model is used, the dependencies are described by pairwise correlations between all units. For the sake of comparability, the correlations are assumed to be the same for the same technology. The valid domain of correlations is [0, 1]. The value 0 has been chosen as low value (-) because it represents the case that there are no dependencies but also the case where dependencies are ignored. In order to choose the value for a high value of correlations different studies have been consulted ([58, 57, 97, 82]). However, these investigations have been made for power output or power output variability, i.e. changes in power output within different time spans. It is assumed here that the volatility of power production is a cause for forecast errors. Furthermore, the power plants considered have been bigger with respect to capacity than the units considered in the RelACs setup and investigations have been made for different sides than the one considered here. However, because of lacking data the findings of [57] and [97] have been chosen to serve as guidelines for the RelACs parameter choices for correlations used in the Copula model. The correlation increases with longer time intervals since variability decreases. In order not to choose too optimistic values and to be in accordance with the temporal resolution of the simulation the time resolution of one minute has been chosen.

PV and wind units are assumed to be uncorrelated. For PV units the cross-correlations are 0.67. In case of wind the correlations of power output changes have been found to be very small. However, it has been argued that turbines close to each other have a higher correlation. Since in the RelACs setup few units within a small grid section are considered, a higher value for wind correlations has been chosen as 0.81. Thus the valid parameter domain for PV correlations is [0, 0.67] and for wind correlations [0, 0.81].

A.3.2. Evaluation Results

Integratability in Coalition Formation

Statistics for scalability of computational time (Section 6.2)

- spatial reliability calculation and homogeneous scenarios: Table A.8
- spatial reliability calculation and heterogeneous scenarios: Table A.9
## Table A.9: Statistics for results for scalability of computational time in case of spatial reliability for heterogeneous coalitions

<table>
<thead>
<tr>
<th>property</th>
<th>base1</th>
<th>base2</th>
<th>base4</th>
<th>base8</th>
<th>base16</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>7.38262</td>
<td>11.725617</td>
<td>22.22025</td>
<td>44.32108</td>
<td>109.764267</td>
</tr>
<tr>
<td>std</td>
<td>3.574706</td>
<td>4.106760</td>
<td>5.665089</td>
<td>12.259812</td>
<td>38.219513</td>
</tr>
<tr>
<td>min</td>
<td>3.327608</td>
<td>4.09756</td>
<td>10.819763</td>
<td>20.876746</td>
<td>63.023537</td>
</tr>
<tr>
<td>25%</td>
<td>4.811361</td>
<td>8.977504</td>
<td>18.906885</td>
<td>35.656977</td>
<td>76.120666</td>
</tr>
<tr>
<td>50%</td>
<td>6.180234</td>
<td>11.428617</td>
<td>22.762177</td>
<td>44.324048</td>
<td>107.345805</td>
</tr>
<tr>
<td>75%</td>
<td>9.35203</td>
<td>14.51958</td>
<td>25.97524</td>
<td>55.300284</td>
<td>144.877286</td>
</tr>
<tr>
<td>max</td>
<td>14.396944</td>
<td>19.026805</td>
<td>32.707376</td>
<td>62.227416</td>
<td>167.263997</td>
</tr>
</tbody>
</table>

## Table A.10: Statistics for results for scalability of computational time in case of topological reliability for homogeneous coalitions

<table>
<thead>
<tr>
<th>property</th>
<th>base1</th>
<th>base2</th>
<th>base4</th>
<th>base8</th>
<th>base16</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.002632</td>
<td>0.008121</td>
<td>0.013133</td>
<td>0.033866</td>
<td>0.082406</td>
</tr>
<tr>
<td>std</td>
<td>0.000902</td>
<td>0.013662</td>
<td>0.001263</td>
<td>0.005098</td>
<td>0.006285</td>
</tr>
<tr>
<td>min</td>
<td>0.001620</td>
<td>0.004950</td>
<td>0.011088</td>
<td>0.028389</td>
<td>0.075476</td>
</tr>
<tr>
<td>25%</td>
<td>0.002222</td>
<td>0.005867</td>
<td>0.011990</td>
<td>0.031436</td>
<td>0.077761</td>
</tr>
<tr>
<td>50%</td>
<td>0.002712</td>
<td>0.006114</td>
<td>0.013005</td>
<td>0.033319</td>
<td>0.079277</td>
</tr>
<tr>
<td>75%</td>
<td>0.002971</td>
<td>0.006832</td>
<td>0.014215</td>
<td>0.036610</td>
<td>0.088744</td>
</tr>
<tr>
<td>max</td>
<td>0.008081</td>
<td>0.114977</td>
<td>0.015901</td>
<td>0.067742</td>
<td>0.10189</td>
</tr>
</tbody>
</table>

## Table A.11: Statistics for results for scalability of computational time in case of topological reliability for heterogeneous coalitions

<table>
<thead>
<tr>
<th>property</th>
<th>base1</th>
<th>base2</th>
<th>base4</th>
<th>base8</th>
<th>base16</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.001838</td>
<td>0.003245</td>
<td>0.007255</td>
<td>0.020749</td>
<td>0.056713</td>
</tr>
<tr>
<td>std</td>
<td>0.000721</td>
<td>0.001053</td>
<td>0.002386</td>
<td>0.006448</td>
<td>0.017750</td>
</tr>
<tr>
<td>min</td>
<td>0.000781</td>
<td>0.001352</td>
<td>0.003563</td>
<td>0.01287</td>
<td>0.030938</td>
</tr>
<tr>
<td>25%</td>
<td>0.001345</td>
<td>0.002222</td>
<td>0.005009</td>
<td>0.014787</td>
<td>0.039675</td>
</tr>
<tr>
<td>50%</td>
<td>0.001688</td>
<td>0.003414</td>
<td>0.007580</td>
<td>0.022273</td>
<td>0.057913</td>
</tr>
<tr>
<td>75%</td>
<td>0.002460</td>
<td>0.004170</td>
<td>0.009864</td>
<td>0.025925</td>
<td>0.073110</td>
</tr>
<tr>
<td>max</td>
<td>0.004769</td>
<td>0.005399</td>
<td>0.010437</td>
<td>0.031256</td>
<td>0.082286</td>
</tr>
</tbody>
</table>
• topological reliability calculation and homogeneous scenarios: Table A.10
• topological reliability calculation and heterogeneous scenarios: Table A.11

Recommendations for Coalition Formation – Spatial Reliability

Checking prerequisites for RelACs-scenario north (Section 6.3)
• normality plot: Figure A.6
• spectrum and histogram of $R^2$-values: Figure A.7a
• spectrum and histogram of standard deviations: Figure A.7b
• relationship $R^2$-values and standard deviations: Figure A.7c

Checking prerequisites for RelACs-scenario north, factor dependency at high level (Section 6.3)
• normality plot: Figure A.8
• spectrum and histogram of $R^2$-values: Figure A.9a
• spectrum and histogram of standard deviations: Figure A.9b

Spatial reliability RelACs-scenario south (Section 6.3)
• normality plot: Figure A.10
• spectrum and histogram of $R^2$-values: Figures A.11a
• spectrum and histogram of standard deviations: Figure A.11b
• Boxplots of factors on both levels: Figure A.12a
• Boxplots of factor level combinations of factors dependency, lifespan, weather: Figure A.12b

Spatial reliability RelACs-scenario south, factor dependency at high level (Section 6.3)
• normality plot: Figure A.13
• spectrum and histogram of $R^2$-values: Figure A.14a
• spectrum and histogram of standard deviations: Figure A.14b

Recommendations for Coalition Formation – Topological Reliability

Topological reliability
• normality plot: Figure A.15a
• spectrum and histogram of $R^2$-values: Figure A.15b
• spectrum and histogram of standard deviations: Figure A.15c
• two-fold interaction effects of factors lifespan, transformer distance, feeder ratio: Figure A.16
Risk Estimation

**Spatial risk** relative risk with factor *dependency* at the high level

- normality plot: Figure A.17
- spectrum and histogram of $R^2$-values: Figure A.18a
- spectrum and histogram of standard deviations: Figure A.18b

**Topological risk**

- normality plot: Figure A.19a
- spectrum and histogram of $R^2$-values: Figure A.19b
- spectrum and histogram of standard deviations: Figure A.19c
Figure A.6.: Normal probability plot for spatial reliability of all factor level combinations, scenario *north*
A.3. Additions to Experiments

(a) Spectrum and histogram for $R^2$-values

(b) Spectrum and histogram for standard deviations

(c) Investigation of relationship between $R^2$-values and standard deviations $\sigma$

Figure A.7.: Normality check for spatial reliability, scenario north
Figure A.8.: Normal probability plot for spatial reliability of all factor level combinations, scenario *north* with factor *dependency* at the high level
Figure A.9.: Normality check for spatial reliability, scenario *north* with factor *dependency* at the high level.
Figure A.10: Normal probability plot for spatial reliability of all factor level combinations, scenario south
A.3. Additions to Experiments

(a) Spectrum and histogram for $R^2$-values

(b) Spectrum and histogram for standard deviations

Figure A.11.: Normality check for spatial reliability, scenario *south*
Figure A.12.: Boxplots for spatial reliability, scenario south
Figure A.13: Normal probability plot for spatial reliability of all factor level combinations, scenario south with factor dependency at the level high.
Figure A.14.: Normality check for spatial reliability, scenario *south* with the factor *dependency* at the high level
A.3. Additions to Experiments

Figure A.15.: Normality check for topological reliability, scenario *north*
Figure A.16.: Interaction effects on topological reliability, scenario *north*
Figure A.17: Normal probability plot for relative spatial risk of all factor level combinations, scenario north with factor dependency at the high level.
Figure A.18.: Normality check for spatial risk, scenario *north* with factor *dependency* at the high level.
A.3. Additions to Experiments

Figure A.19.: Normality check for topological risk, scenario north
## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS</td>
<td>ancillary service</td>
</tr>
<tr>
<td>BDEW</td>
<td>Federal Association of the Energy and Water Industry</td>
</tr>
<tr>
<td>CAIDI</td>
<td>Customer Average Interruption Duration Index</td>
</tr>
<tr>
<td>CEER</td>
<td>Council of European Energy Regulators</td>
</tr>
<tr>
<td>CFP</td>
<td>call for proposal</td>
</tr>
<tr>
<td>DER</td>
<td>distributed energy resources</td>
</tr>
<tr>
<td>DSO</td>
<td>distribution system operator</td>
</tr>
<tr>
<td>DVPP</td>
<td>dynamic virtual power plant</td>
</tr>
<tr>
<td>EEG</td>
<td>German renewable energy act</td>
</tr>
<tr>
<td>EHV</td>
<td>extra-high voltage</td>
</tr>
<tr>
<td>EURELECTRIC</td>
<td>Union of the Electricity Industry</td>
</tr>
<tr>
<td>FIPA</td>
<td>The Foundation for Intelligent Physical Agents</td>
</tr>
<tr>
<td>FNN</td>
<td>Network Technology / Network Operation Forum</td>
</tr>
<tr>
<td>HV</td>
<td>high voltage</td>
</tr>
<tr>
<td>IEC</td>
<td>International Electrotechnical Commission</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>ISO</td>
<td>International Organization for Standardization</td>
</tr>
<tr>
<td>LV</td>
<td>low voltage</td>
</tr>
<tr>
<td>MAS</td>
<td>multi-agent system</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>MV</td>
<td>medium voltage</td>
</tr>
<tr>
<td>PCR</td>
<td>Primary control reserves</td>
</tr>
</tbody>
</table>
PV  photovoltaic
RES  Renewable Energy Source
RPU  Renewable Power Unit
ReIACs  Reliability Assessment of Ancillary-Service Coalitions
RMSE  root mean square error
SAIFI  System Average Interruption Frequency Index
SAIDI  System Average Interruption Duration Index
SCR  secondary control reserves
TCR  tertiary control reserves
TSO  transmission system operator
UCTE  Union for the Co-ordination of Transmission of Electricity
UNIPEDE  Association of the European Electricity Industry
VDE  Association for Electrical, Electronic & Information Technologies
VPP  virtual power plant
List of Symbols

General symbols

\( P \)  
Active Power

\( Q \)  
Reactive Power

\( f \)  
Frequency

\( U \)  
Voltage

\( \Pr \)  
Probability

Ancillary Service Provision (Chapter 3)

\( \nu \)  
Measured Quantity

\( \star_{nom} \)  
Setpoint of \( \star \)

\( A_\star \)  
Feasible Region According to \( \star \) (e.g. \( \nu \))

\( D_\star \)  
Safety Margin According to \( \star \) (e.g. \( \nu \))

\( q_\star \)  
Ancillary-Service Quantity w.r.t. \( \star \) (e.g. \( \nu \))

\( \text{AS}_\star \)  
Ancillary-service Product According to \( \star \) (e.g. \( \nu \))

\( T_{pr} \)  
Product Horizon

\( e_{pr}^\star \)  
Product Amount According to \( \star \) (e.g. \( \nu \))

\( c_{pr}^\star \)  
Cost for \( \star \)

\( e^\star \)  
Amount of Ancillary-Service Quantity in General

\( \text{vic}_\star \)  
Vicinity to an event that triggered the ancillary service demand according to \( \star \)

\( \partial t_\star \)  
Time of activation within the ancillary service according to \( \star \) must be provided

\( t_{act,\star} \)  
Duration of activation of ancillary service w.r.t. \( \star \)

\( U \)  
Unit

\( \tilde{U} \)  
Set of All Units

\( \mathcal{C} \)  
Coalition
a Agent
\(\tilde{A}\) Set of All Agents
\(\text{cont}^{**}\) Contribution of Unit \(*\) w.r.t. \(**\)
\(e_{\text{cont},*}^{**}\) Amount of Ancillary-Service Quantity \(q_v\) for Contribution of Unit \(*\) w.r.t. \(**\)
\(c_{*,**}\) Cost for Contribution of \(*\) w.r.t. \(**\)
\(t_{pr,*}\) \(*\)-th Lifespan of a Coalition

RelACs-method (Chapter 4)

\(\text{pred}_v\) Prediction Function with respect to \(*\)
\(F_U\) Distribution Function of Unit Error Model
\(\rho_{\text{cont},*}\) Reliability of Contribution of \(*\) w.r.t. \(**\)
\(\rho\) Reliability Level
\(\mathcal{C}\) Copula
\(\tilde{\mathcal{C}}\) Survival Copula
\(G\) Graph of a Grid
\(e\) Element of Operational Equipment
\(E\) Set of Elements
\(v\) Node in a Grid
\(V\) Set of Nodes in a Grid
\(v_{\text{grid}}\) Node Representing a Subgrid
\(\pi\) Path in a Graph
\(\kappa\) Minimal Cut Set
\(K\) Set of Minimal Cut Sets
\(\text{fail}(\cdot)\) Failure of Component \(*\)
\(\text{fail}_c(\cdot)\) Failure of Unit or Coalition \(*\) w.r.t. Grid Topology.

Implementation (Chapter 5)

\(\text{vol}_{\text{pred}}\) Volatility of a Prediction
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{p,*}$</td>
<td>Clearsky Index at Time *</td>
</tr>
<tr>
<td>share</td>
<td>Share of Unit Contributions</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Uniformity of Contributions</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Noise to Determine Uniformity of Contribution</td>
</tr>
<tr>
<td>$\phi_{cap}$</td>
<td>Share of Installed Capacity</td>
</tr>
<tr>
<td>$\mathcal{D}$</td>
<td>Dependency</td>
</tr>
<tr>
<td>$\text{dist}_t$</td>
<td>Distance to Transformer for Unit Distribution</td>
</tr>
<tr>
<td>$\text{dist}_f$</td>
<td>Ratio of Number of Feeder</td>
</tr>
</tbody>
</table>
The following glossary describes the most important terms used throughout this thesis. Often, a more comprehensive description can be found in the introducing chapters. The given definitions are based on established glossaries as well as literature referenced in the according sections and have been discussed and synchronised with colleagues of the research cluster Smart Nord. While the symbol ~ refers to the term at hand, the symbol ↑ references another term within the glossary.

**Active Power** Contrarily to ↑reactive power, ~ is the part of ↑apparent power that can be used by consumers, i.e. it can be transformed to other energy forms.

**Agent** An ~ is an autonomous computer system located in an environment within which it is able to perceive information through sensors and act upon it by means of actuators in order to fulfil a given objective. It is termed intelligent if it is capable of pro- and reactive behaviour and has the ability to interact.

**Ancillary Service Product** A ~ is a ↑product for tendering ↑ancillary services on a ↑market.

**Ancillary Services** The ~ are services procured from system users in order to support the ↑system operators to provide ↑system services.

**Apparent Power** ~ is the power, that is supplied to electrical consumers. It consists of ↑active power and ↑reactive power.

**Balancing Power Product** A ~ is a market-based implementation for ↑frequency control by ↑frequency control reserves. Tenders receive a price for both power and energy in order to remunerate the provided reserves and the activated ↑balancing energy, separately.

**Balancing Energy** The ~ is the amount of energy that has been retrieved after that activation of ↑frequency control reserves.

**Biomass** The ~ comprises organic, non-fossil materials that can be used to generate energy.

**Coalition** A ~ is an organisational aggregation of ↑unit agents with the objective to provide a ↑product and with that to gain a benefit.

**Commercial Quality** The ~ covers the relationship between energy suppliers and customers an comprises e.g. metering, billing, and emergency services.
Continuity of Supply defines the capability of an electrical power system to guarantee power supply to the end users under pre-defined conditions within a given time horizon. It is also referred to as system reliability.

Control Area is a geographical area within which a transmission system operator is responsible for the control of the corresponding transmission grid.

Controller is a physical unit that combines a mode selector, an adjuster for manual control of an actuator, and, if necessary, a reference-variable adjuster. This may be complemented by a display unit for variables.

Dispatch assigns a schedule to each unit within a control area or coalition for a given time horizon.

Distribution System Operator is a system operator of a particular area within the distribution system.

Distribution Grid comprises the low and medium voltage level of the electrical power grid with the purpose of local distribution of energy.

Electrical Power Grid is an electrical power system.

Electrical Power System is a network of nodes being interconnected by lines on different voltage levels. Its purpose is to supply customers with electrical energy to the end customer.

Electrical Power is an instantaneous value physically defined as the product of current and voltage. While instantaneous values generally refer to a specified point in time, in the context of the power industry also mean power values are used referring to defined time intervals. In this case $P$ is defined as the quotient of work $W$ done in a time interval and the time interval $T$ itself, i.e. $P = W/T$.

Energy is the work stored within a system and describes the capability of for doing work. It may exist in different forms like electric, thermal, or kinetic. Electric energy is the integral of electrical power over time.

Frequency is the electrical frequency of the electrical grid that can be measured in all areas of the synchronous areas.

Frequency Control Reserves comprises primary control reserves, secondary control reserves, and tertiary control reserves for balancing system frequency. In case of an under supply, positive control reserves are activated, i.e. power generation is increased or power consumption is reduced. In case of an over supply, negative control reserves are activated, i.e. power generation is reduced or power consumption is increased.
**Frequency Control** The ~ relates to maintaining the frequency within given margins in case of frequency deviations resulting from imbalances in generation and consumption of active power. In the European synchronous grid this is achieved by providing the ancillary services primary, secondary and tertiary control reserves.

**Interoperability** ~ is the ability of different technical systems to cooperate with each other. This cooperation comprises correct syntactic and semantic exchange of communication via communication.

**Market** A ~ is an organisational form possibly connected via information and communication technology enabling trading between different stakeholders. Depending on the concrete design of the ~, a coordinator might be deployed in order to mediate demand and supply and appropriately match them.

**Meshed System** network system.

**Multi-Agent System** A ~ is a system consisting of several agents that interact in a coordinated way in order to fulfil their (possibly varying) objectives.

**N-1 Principle** n-1 security.

**N-1 Security** The ~ assures the security of supply in case of failures of operational equipment of the electrical power grid. This term is interchangeably used with the term n-1 principle.

**Network System** The ~ describes a configuration of a grid topology. In a ~ all nodes are connected by more than one path and some lines form loops within the system.

**Photovoltaic** ~ refers to the process to transfer solar radiant power to electrical power by using solar / photovoltaic cells.

**Plant** A ~ is an electrotechnical unit feeding electrical power into an electrical grid and thus providing electrical energy.

**Power Plant Dispatch** dispatch.

**Prequalification** A ~ describes the process during which a unit is reviewed to assess if it is able to fulfil technical standards for the provision of control reserves.

**Primary Control Reserves** The provision of ~ is a frequency control ancillary service in order to counteract deviations of system frequency. The ~ must be automatically activated within 30 seconds after a disturbance occurred.

**Primary Energy Carrier** A ~ is a medium whose potential energy is transferred to effective energy, e.g. thermal or electrical energy.
**Product**  In the context of electricity industry, a ∼ represents products for energy and ancillary service products.

**Product Horizon**  A ∼ is the time horizon within which a ↑product must be provided to the stipulated conditions.

**Product Tendering Horizon**  A ∼ is the time horizon within which a ↑product is tendered.

**PV Unit**  A ∼ is a ↑unit based on ↑PV technology.

**Quality of Supply**  The ∼ is a term used to assess the quality of energy supply that comprises the terms ↑commercial quality, ↑continuity of supply, and ↑voltage quality.

**Radial System**  The ∼ describes a configuration of a grid topology. In a ∼ all feeders branch from the source to nodes.

**Reactive Power**  ∼ is the portion of ↑apparent power which is used by electric network elements to create magnetic or electric fields and can thus not be used to do work. Capacitive elements, i.e. elements creating electric fields (like capacitors), cause the voltage to lag behind the current and are thus said to generate ∼ , while inductive elements, i.e. elements creating magnetic fields (like transformers), cause the current to lag behind the voltage and are thus said to consume ∼ . Within an ↑electrical grid, ∼ can have a strong impact on the voltage.

**Redispatch**  In order to conduct a ∼ system operators control the ↑units within their ↑control area with regard to their ↑schedules.

**Renewable Energy Source**  A ∼ is an ↑energy carrier that is directly available in unlimited quantities (e.g. solar irradiation, wind, geothermal energy) or that can be made available through biological processes (e.g. biomass).

**Scalability**  ∼ is the capability of a system to increase its services proportional to the increase of resources. In that case the system is said to scale with the resources.

**Schedule**  A ∼ sets the average ↑power to be generated or consumed by a ↑unit within a time horizon separated into equidistant time intervals.

**Secondary Control Reserve**  The provision of ∼ is a frequency control ↑ancillary service in order to replace ↑primary control reserves and to restore system ↑frequency.

**Smart Grid**  A ∼ represents an ↑electrical power system whose components are interconnected via information and communication technology in order to assure a energy-and cost-efficient as well as secure and reliable power supply.

**Supply-Demand Matching**  ∼ is an approach to coordinate decentralised ↑units in order to achieve an optimal, local balance of generation and consumption of electrical energy.
Synchronous Grid  A ~ is an interconnection of ↑electrical power grids of different regions that is operated at a synchronized frequency.

System Services  The ~ are services provided by ↑system operators in order to ensure required power quality and the stability of the ↑electrical grid.

System Operator  A ~ is a responsible for secure and reliably operation, maintenance and if possible development of an ↑electrical grid within a particular area. According to different voltage levels there are ↑transmission system operators and ↑distribution system operators.

System Control  The ~ is a service by ↑system operators with the objective of the coordination and operation of the system.

System Restoration  A ~ refers to the restoration of the system after a fault.

Tertiary Control Reserves  This provision of ~ is a frequency control ↑ancillary service in order to replace ↑secondary control reserves. The ~ must be activated within 15 minutes after activation by the ↑transmission system operator.

Transmission System Operator  A ~ is a ↑system operator of a particular area within the ↑transmission system.

Transmission Grid  A ~ comprises the high and extra-high voltage levels of the ↑electrical power grid with the purpose to transmit energy over long distances.

Unit  A ~ represents an electrotechnical component which is connected via a node to an ↑electrical grid and can influence its electrical current and voltage. If the ~ is equipped with appropriate information and communications technology, it can be represented by an ↑agent.

Unit Agent  A ~ is an ↑agent which may either reside directly on an embedded system or on a separate computer connected to a ↑unit. It can communicate with other ~ via appropriate information and communication technology.

Virtual Power Plant  A ~ is an organisational aggregation of (primarily small, decentralised) ↑units interconnected via information and communication technology.

Voltage Quality  The ~ is a term that comprises quality measures regarding the voltage in the electrical power system such as frequency or voltage magnitude. It is also referred to as power quality.

Voltage Level  Transmission and distribution of electrical energy is conducted via different ~ . For transmission of electrical energy over long distances higher voltages are used in order to reduce power losses. For distribution of electrical energy to end users lower voltages are used.
**Voltage Control**  The ∼ has the objective to locally maintain voltage within pre-defined boundaries (referred to as voltage bands). Voltage can be controlled via control of active and reactive power feed-in.

**Wind Power Unit**  A ∼ is a unit that transforms the kinetic energy of wind into electrical energy.
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