COMPARISON OF LEAST-SQUARES AND EIGENFILTER TECHNIQUES FOR BROADBAND BEAMFORMING

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ABSTRACT

In this paper two eigenfilter techniques are described for designing far-field broadband beamformers with an arbitrary spatial directivity pattern. For both techniques the resulting filter is the generalised eigenvector corresponding to the minimum generalised eigenvalue of two real, symmetric and positive definite matrices. In the conventional eigenfilter technique a reference point is needed, while in the eigenfilter technique based on a TLS (Total Least Squares) error criterion, this reference point is not needed. It is shown that linear constraints are easily incorporated into the design procedure. Both eigenfilter techniques are compared to the weighted least-squares filter design technique.

1. INTRODUCTION

In many speech communication applications, the recorded microphone signals are corrupted by acoustic background noise and reverberation. Both fixed and adaptive beamforming techniques are used for acoustic noise reduction and dereverberation [1]. Fixed beamforming is mainly used in highly reverberating acoustic environments and for creating a speech reference signal in an adaptive 'Generalised Sidelobe Canceller' (GSC) beamformer.
Several methods exist for the design of fixed broadband beamformers and are *e.g.* based on weighted least-squares (LS) filter design [2], non-linear optimisation [3], a broadband maximum energy array [4] or frequency-invariant beamforming [5].
This paper describes two eigenfilter techniques for the design of a fixed broadband beamformer having a desired spatial and frequency pattern $D(\omega,\theta)$. Eigenfilters have been introduced for designing 1-D linear phase FIR filters [6]. Their main advantage is the fact that no matrix inversion is required (as in LS filter design) and that time-domain and frequency-domain constraints are easily incorporated. Eigenfilters have also been used for designing 2-D and spatial filters [7][8]. Recently a new eigenfilter, based on a TLS (Total Least Squares) error criterion, has been proposed [9].
In section 2 the general broadband beamforming setup is discussed and some notational conventions are given. This section also defines the stopband and passband errors and discusses different linear constraints which can be imposed on the filters.
Section 3 describes the design of broadband beamformers using the weighted least-squares filter design technique and it is shown how linear constraints can be incorporated into the design.
Section 4 describes the design of broadband beamformers using two eigenfilter techniques: the conventional eigenfilter, which requires a reference frequency-angle point, and the TLS eigenfilter, not requiring a reference point. It is also shown that linear constraints can be easily incorporated into these design procedures.
In section 5 the simulation results are discussed, in which the eigenfilter-based techniques are compared with the conventional weighted least-squares technique.

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2. BROADBAND BEAMFORMING

2.1. Configuration and notation

Consider the linear microphone array depicted in figure 1, with N microphones and d_n the distance between the nth and the 0th microphone. The *spatial directivity pattern* $H(\omega,\theta)$ for a source $S(\omega)$ at an angle θ from the microphone array, is defined as

$$H(\omega,\theta)=\frac{Z(\omega,\theta)}{S(\omega)}=\frac{\sum_{n=0}^{N-1}W_n(\omega)Y_n(\omega,\theta)}{S(\omega)}\ , \qquad (1)$$

with $W_n(\omega)=\mathbf{w}_n^T\mathbf{e}(\omega)$ the frequency response of the L-dimensional filter $w_n(k)$, with

$$\mathbf{w}_n=\begin{bmatrix} w_n(0)\\ w_n(1)\\ \vdots\\ w_n(L-1)\end{bmatrix}\quad \mathbf{e}(\omega)=\begin{bmatrix}1\\ e^{-j\omega}\\ \vdots\\ e^{-j(L-1)\omega}\end{bmatrix}. \qquad (2)$$

In the far-field assumption the microphone signals $Y_n(\omega,\theta)$, $n=0\ldots N-1$, are delayed versions of the signal $S(\omega)$,

$$Y_n(\omega,\theta)=S(\omega)e^{-j\omega\tau_n(\theta)},\ -\pi\leq\omega\leq\pi,\ -\pi\leq\theta\leq\pi\ , \quad (3)$$

with the delay

$$\tau_n(\theta)=\frac{d_n\cos\theta}{c}f_s\ , \qquad (4)$$

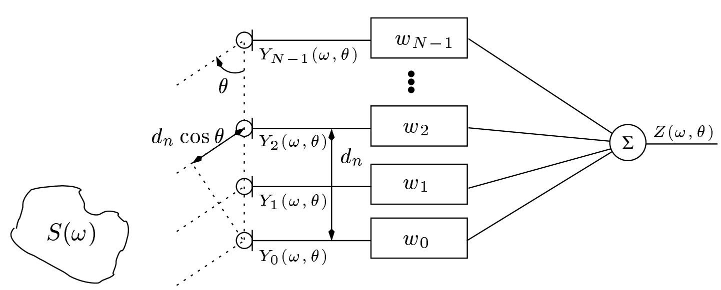

Figure 1: Microphone array configuration

with c the speed of sound ($c \simeq 340\frac{m}{s}$) and f_s the sampling frequency. For a uniform linear array the distances between two adjacent microphones are equal, such that $d_n = n \cdot d$, $n = 0 \ldots N-1$. Using (3) the spatial directivity pattern $H(\omega, \theta)$ can be written as

$$H(\omega,\theta) = \sum_{n=0}^{N-1} \mathbf{w}_n^T \mathbf{e}(\omega) e^{-j\omega\tau_n(\theta)} = \mathbf{w}^T \mathbf{g}(\omega,\theta) \;, \tag{5}$$

with the M-dimensional ($M = LN$) vectors $\mathbf{w}$ and $\mathbf{g}(\omega, \theta)$

$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_{N-1} \end{bmatrix} \quad \mathbf{g}(\omega,\theta) = \begin{bmatrix} \mathbf{e}(\omega) \\ \mathbf{e}(\omega)e^{-j\omega\tau_1(\theta)} \\ \vdots \\ \mathbf{e}(\omega)e^{-j\omega\tau_{N-1}(\theta)} \end{bmatrix} . \tag{6}$$

The design of a broadband beamformer consists of the calculation of the filter $\mathbf{w}$, such that $H(\omega, \theta)$ fits the *desired* spatial directivity pattern $D(\omega, \theta)$ as well as possible. Several design techniques exist, depending on the specific cost function being optimised.

For all techniques we will discuss the general case of an arbitrary $D(\omega, \theta)$ and then focus on the specific design case of a broadband beamformer having $D(\omega, \theta) = 0$ in the stopband region (Ω_s, Θ_s) and $D(\omega, \theta) = 1$ in the passband region (Ω_p, Θ_p).

2.2. Error criteria

In this section two error criteria are discussed, which will be used in the design procedures of sections 3 and 4. The first criterion is the *stopband error*, defined as

$$J_e^s = \int_{\Theta_s} \int_{\Omega_s} |H(\omega,\theta)|^2 d\omega d\theta, \quad 0 \le \Theta_s, \Omega_s \le \pi, \tag{7}$$

which specifies the energy in the stopband frequency-angle region (Ω_s, Θ_s). The integrand $|H(\omega, \theta)|^2$ can be written as

$$|H(\omega,\theta)|^2 = H(\omega,\theta)H^*(\omega,\theta) = \mathbf{w}^T \mathbf{G}(\omega,\theta)\mathbf{w} \;, \tag{8}$$

with $\mathbf{G}(\omega, \theta) = \mathbf{g}(\omega, \theta)\mathbf{g}^H(\omega, \theta)$. The (i, j)-element of $\mathbf{G}(\omega, \theta)$ is

$$\mathbf{G}^{ij}(\omega,\theta) = e^{-j\omega\left((k-l)+\frac{(d_n-d_m)\cos\theta}{c}f_s\right)}, \quad i,j = 1\ldots M \;, \tag{9}$$

with $k = \mod(i-1, L)$, $l = \mod(j-1, L)$, $n = \lfloor \frac{i-1}{L} \rfloor$ and $m = \lfloor \frac{j-1}{L} \rfloor$. The stopband error can now be written as

$$J_e^s = \mathbf{w}^T \Big(\int_{\Theta_s} \int_{\Omega_s} \mathbf{G}_R(\omega,\theta) d\omega d\theta \Big) \mathbf{w}^T = \mathbf{w}^T \mathbf{Q}_e^s \mathbf{w} \;, \tag{10}$$

with $\mathbf{G}_R(\omega, \theta)$ the real part of $\mathbf{G}(\omega, \theta)$, being a symmetric positive definite matrix. In a similar way the energy in the passband region (Ω_p, Θ_p) can be represented by $J_e^p = \mathbf{w}^T \mathbf{Q}_e^p \mathbf{w}$.

The second criterion, the *passband error*, is defined as

$$J_p = \int_{\Theta_p} \int_{\Omega_p} |H(\omega,\theta) - H(\omega_c,\theta_c)|^2 d\omega d\theta \;, \tag{11}$$

which specifies the error in the passband region (Ω_p, Θ_p) between the actual response $H(\omega, \theta)$ and the response at a reference point $H(\omega_c, \theta_c)$. Note that we do not define the value of $H(\omega_c, \theta_c)$. The integrand $|H(\omega, \theta) - H(\omega_c, \theta_c)|^2$ is equal to

$$\mathbf{w}^T \underbrace{\Big[\mathbf{g}(\omega_c,\theta_c)\mathbf{g}^H(\omega_c,\theta_c) - \mathbf{g}(\omega,\theta)\mathbf{g}^H(\omega_c,\theta_c) - \mathbf{g}(\omega_c,\theta_c)\mathbf{g}^H(\omega,\theta) + \mathbf{g}(\omega,\theta)\mathbf{g}^H(\omega,\theta)\Big]}_{\hat{\mathbf{G}}(\omega_c,\theta_c,\omega,\theta)} \mathbf{w} \;. \tag{12}$$

The passband error can now be written as

$$J_p = \mathbf{w}^T \Big(\int_{\Theta_p} \int_{\Omega_p} \hat{\mathbf{G}}_R(\omega_c,\theta_c,\omega,\theta) d\omega d\theta \Big) \mathbf{w}^T = \mathbf{w}^T \mathbf{Q}_p \mathbf{w} \;, \tag{13}$$

with $\hat{\mathbf{G}}_R(\omega_c, \theta_c, \omega, \theta)$ the real part of $\hat{\mathbf{G}}(\omega_c, \theta_c, \omega, \theta)$, being a symmetric positive definite matrix.

2.3. Linear constraints

Linear constraints (*e.g.* point, derivative, line constraints) can be imposed on the filter $\mathbf{w}$. These constraints can be written as

$$\mathbf{C}\mathbf{w} = \mathbf{b} \;, \tag{14}$$

with $\mathbf{C}$ a $K \times M$-dimensional matrix ($K \le M$) and $\mathbf{b}$ a K-dimensional vector.

A *point constraint*, constraining the response at a point $H(\omega_c, \theta_c)$ to be equal to a complex scalar $b = b_R + jb_I$, corresponds to 2 constraints (a real one and an imaginary one),

$$\underbrace{\begin{bmatrix} \mathbf{g}_R^T(\omega_c,\theta_c) \\ \mathbf{g}_I^T(\omega_c,\theta_c) \end{bmatrix}}_{\mathbf{C}} \mathbf{w} = \underbrace{\begin{bmatrix} b_R \\ b_I \end{bmatrix}}_{\mathbf{b}} . \tag{15}$$

A *line constraint* constrains $H(\omega, \theta)$ to be equal to a predefined frequency response $F(\omega) = \sum_{k=0}^{L-1} f(k)e^{-jk\omega}$ at the angle θ_c. If we assume that the filter $\mathbf{f}$ is real, we actually have $2L$ constraints (L real and L imaginary),

$$\underbrace{\begin{bmatrix} \cos(\omega\tau_0)I_L & \cos(\omega\tau_1)I_L & \ldots & \cos(\omega\tau_{N-1})I_L \\ \sin(\omega\tau_0)I_L & \sin(\omega\tau_1)I_L & \ldots & \sin(\omega\tau_{N-1})I_L \end{bmatrix}}_{\mathbf{C}} \mathbf{w} = \underbrace{\begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}}_{\mathbf{b}}, \tag{16}$$

This equation has to hold **for all** ω. However, because $K \le M$, in general these constraints can only be satisfied for $N/2$ frequencies. An exception is the angle $\theta_c = \pi/2$ (broadside direction), since in this case $\tau_n(\theta_c) = 0$, $n = 0 \ldots N-1$, such that (16) reduces to

$$\begin{bmatrix} I_L & I_L & \ldots & I_L \end{bmatrix} \mathbf{w} = \mathbf{f} \;. \tag{17}$$

3. WEIGHTED LEAST-SQUARES FILTER DESIGN

In this technique the least-squares (LS) error

$$e(\omega,\theta) = |D(\omega,\theta) - H(\omega,\theta)| \tag{18}$$

is minimised. This corresponds to minimising the cost function

$$\begin{aligned} J_{LS} &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)\big[D(\omega,\theta) - H(\omega,\theta)\big]^2 d\omega d\theta \qquad (19) \\ &= \mathbf{w}^T \mathbf{Q}_{LS}\mathbf{w} - 2\mathbf{w}^T\mathbf{a} + d \qquad (20) \end{aligned}$$

with $W(\omega, \theta)$ a positive weighting function [2]. For the specific design case we will use a weighting constant α for the passband and β for the stopband. The matrix $\mathbf{Q}_{LS}$, vector $\mathbf{a}$ and scalar d are equal to

$$\begin{aligned} \mathbf{Q}_{LS} &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)\mathbf{G}_R(\omega,\theta)d\omega d\theta = \alpha\mathbf{Q}_e^p + \beta\mathbf{Q}_e^s \\ \mathbf{a} &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)D(\omega,\theta)\mathbf{g}_R(\omega,\theta)d\omega d\theta \\ &= \alpha\int_{\Theta_p}\int_{\Omega_p} \mathbf{g}_R(\omega,\theta)d\omega d\theta \\ d &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)D^2(\omega,\theta)d\omega d\theta = \int_{\Theta_p}\int_{\Omega_p} \alpha d\omega d\theta \;. \end{aligned}$$

The filter $\mathbf{w}_{LS}$ minimising J_{LS} is given by

$$\mathbf{w}_{LS} = \mathbf{Q}_{LS}^{-1}\mathbf{a} \; . \tag{21}$$

If we incorporate K linear constraints $\mathbf{C}\mathbf{w} = \mathbf{b}$, it can be easily proven that the constrained LS solution is given by

$$\mathbf{w}_{LS}^{c} = \mathbf{Q}_{LS}^{-1}\mathbf{C}^{T}\left(\mathbf{C}\mathbf{Q}_{LS}^{-1}\mathbf{C}^{T}\right)^{-1}\mathbf{b} + \mathbf{C}_{a}^{T}\left(\mathbf{C}_{a}\mathbf{Q}_{LS}\mathbf{C}_{a}^{T}\right)^{-1}\mathbf{C}_{a}\mathbf{a} \; ,$$

with $\mathbf{C}_a$ the $(M-K) \times M$-dimensional null space of $\mathbf{C}$. As can be seen, the (numerically unstable) inverse of $\mathbf{Q}_{LS}$ is required.

4. EIGENFILTER TECHNIQUES

4.1. Conventional eigenfilter technique

In the conventional eigenfilter technique a reference point (ω_c, θ_c) is chosen and the filter $\mathbf{w}$ is calculated such that the relative response $H(\omega,\theta)/H(\omega_c,\theta_c)$ fits $D(\omega,\theta)/D(\omega_c,\theta_c)$ as well as possible. This can be achieved by minimising the cost function

$$\begin{aligned} J_{eig} &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)\Big[\frac{D(\omega,\theta)}{D(\omega_c,\theta_c)}H(\omega_c,\theta_c) - H(\omega,\theta)\Big]^2 d\omega d\theta \\ &= \mathbf{w}^T\mathbf{Q}_{eig}\mathbf{w} \end{aligned} \tag{22}$$

For the general and the specific design case, the matrix $\mathbf{Q}_{eig}$ is

$$\begin{aligned} \mathbf{Q}_{eig} &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)\Big[\frac{D(\omega,\theta)}{D(\omega_c,\theta_c)}\mathbf{g}(\omega_c,\theta_c) - \mathbf{g}(\omega,\theta)\Big] \cdot \\ &\quad \Big[\frac{D(\omega,\theta)}{D(\omega_c,\theta_c)}\mathbf{g}(\omega_c,\theta_c) - \mathbf{g}(\omega,\theta)\Big]^H d\omega d\theta \qquad (23) \\ &= \alpha\mathbf{Q}_p + \beta\mathbf{Q}_e^s \; , \qquad (24) \end{aligned}$$

because $D(\omega,\theta) = D(\omega_c,\theta_c)$ in the passband (assuming (ω_c,θ_c) lies in the passband) and $D(\omega,\theta) = 0$ in the stopband.
In order to avoid the trivial solution $\mathbf{w} = \mathbf{0}$, a constraint is added. Usually the unit-norm constraint $\mathbf{w}^T\mathbf{w} = 1$ is added [6][7], such that the solution is the eigenvector of $\mathbf{Q}_{eig}$, corresponding to the smallest eigenvalue (hence the name eigenfilters). In the 1-D FIR filter case, this constraint corresponds to the total energy under the amplitude response being 1, while for the broadband beamformer, this constraint apparently does not have a physical meaning. We will therefore use the constraint $J_e^{tot} = \mathbf{w}^T\mathbf{Q}_e^{tot}\mathbf{w} = 1$, constraining the energy under the amplitude response in the total frequency-angle plane. The filter minimising J_{eig} under this constraint is the generalised eigenvector of $\mathbf{Q}_{eig}$ and $\mathbf{Q}_e^{tot}$, corresponding to the smallest generalised eigenvalue. This eigenvector still has to be scaled such that $\mathbf{w}^T\mathbf{g}(\omega_c,\theta_c) = D(\omega_c,\theta_c)$.

4.2. Eigenfilter based on TLS error criterion

Recently an eigenfilter, based on a TLS error criterion, has been described [9]. The advantage of this eigenfilter is that no reference point is required. Instead of minimising the weighted LS error (see section 3), the weighted TLS error is used and the cost function

$$\int_{\Theta}\int_{\Omega} W(\omega,\theta)\frac{\big[D(\omega,\theta) - H(\omega,\theta)\big]^2}{\mathbf{w}^T\mathbf{w}+1} d\omega d\theta \tag{25}$$

has to be minimised. As in the conventional eigenfilter technique we replace $\mathbf{w}^T\mathbf{w}$ with $\mathbf{w}^T\mathbf{Q}_e^{tot}\mathbf{w}$ and minimise

$$\begin{aligned} J_{TLS} &= \int_{\Theta}\int_{\Omega} W(\omega,\theta)\frac{\big[D(\omega,\theta) - H(\omega,\theta)\big]^2}{\mathbf{w}^T\mathbf{Q}_e^{tot}\mathbf{w}+1} d\omega d\theta \\ &= \frac{\hat{\mathbf{w}}^T\hat{\mathbf{Q}}_{TLS}\hat{\mathbf{w}}}{\hat{\mathbf{w}}^T\hat{\mathbf{Q}}_e^{tot}\hat{\mathbf{w}}} \; , \end{aligned} \tag{26}$$

with

$$\hat{\mathbf{w}} = \begin{bmatrix} \mathbf{w} \\ -1 \end{bmatrix}, \; \hat{\mathbf{Q}}_{TLS} = \begin{bmatrix} \mathbf{Q}_{LS} & \mathbf{a} \\ \mathbf{a}^T & d \end{bmatrix}, \; \hat{\mathbf{Q}}_e^{tot} = \begin{bmatrix} \mathbf{Q}_e^{tot} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} .$$

The filter $\hat{\mathbf{w}}_{TLS}$ minimising J_{TLS} is the generalised eigenvalue of $\hat{\mathbf{Q}}_{TLS}$ and $\hat{\mathbf{Q}}_e^{tot}$, corresponding to the smallest generalised eigenvalue. After scaling the last element of $\hat{\mathbf{w}}_{TLS}$ to -1, the actual solution $\mathbf{w}_{TLS}$ is obtained as the first M elements of $\hat{\mathbf{w}}_{TLS}$. In [9] it is shown that linear constraints $\mathbf{C}\mathbf{w} = \mathbf{b}$ can be rewritten as

$$\hat{\mathbf{C}}\hat{\mathbf{w}} = \mathbf{0}, \qquad \hat{\mathbf{C}} = \begin{bmatrix} \mathbf{C} & \mathbf{b} \end{bmatrix}, \tag{27}$$

such that the constrained problem can be transformed to the unconstrained problem

$$\min_{\mathbf{h}} \frac{\mathbf{h}^T\mathbf{B}^T\hat{\mathbf{Q}}_{TLS}\mathbf{B}\mathbf{h}}{\mathbf{h}^T\mathbf{B}^T\hat{\mathbf{Q}}_e^{tot}\mathbf{B}\mathbf{h}} \; , \tag{28}$$

with $\hat{\mathbf{w}} = \mathbf{B}\mathbf{h}$ and $\mathbf{B}$ the null space of $\hat{\mathbf{C}}$. The solution $\mathbf{h}_{TLS}$ of the unconstrained minimisation problem is the generalised eigenvector of $\mathbf{B}^T\hat{\mathbf{Q}}_{TLS}\mathbf{B}$ and $\mathbf{B}^T\hat{\mathbf{Q}}_e^{tot}\mathbf{B}$, corresponding to the minimum generalised eigenvalue, such that the solution $\hat{\mathbf{w}}_{TLS}^c$ of the constrained minimisation problem is equal to $\hat{\mathbf{w}}_{TLS}^c = \mathbf{B}\mathbf{h}_{TLS}$.

5. SIMULATION RESULTS

In our simulations we have used a linear uniform microphone array with $N = 5$ microphones and microphone distance $d = 4$ cm. The filter length $L = 20$ and the sampling frequency $f_s = 8$ kHz. The passband area $(\Omega_p, \Theta_p) = (300\text{–}4000\,\text{Hz}, 70°\text{–}110°)$ and stopband area $(\Omega_s, \Theta_s) = (300\text{–}4000\,Hz, 0°\text{–}60° + 120°\text{–}180°)$.
We have designed a broadband beamformer using the LS, eigenfilter and TLS design procedure, with and without linear constraints. The reference point (ω_c, θ_c) used for the eigenfilter technique is $(1500\,\text{Hz}, 90°)$. The linear constraint is a line constraint at $90°$ with unit frequency response, such that $\mathbf{C} = \begin{bmatrix} I_L & I_L & \ldots & I_L \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}^T$. The weighting constants for the passband and the stopband are $\alpha = 1$, $\beta = 2$ and $\beta = 0.01$.
Table 1 shows the stopband and passband errors for the different design techniques. The differences for the stopband and the passband errors between the LS, eigenfilter and TLS techniques are small and depend on the value of β. For $\beta = 2$ the stopband errors are smaller than for $\beta = 0.01$. Constrained design gives rise to larger stopband errors and smaller passband errors than unconstrained design.
Figures 2-4 show the spatial directivity pattern of the LS, eigenfilter and TLS design techniques without linear constraints and $\beta = 2$. The LS and TLS design give rise to similar results, while in the eigenfilter design the high frequencies are amplified. Figure 5 shows the result of the TLS technique with a line constraint for $\beta = 0.01$, giving rise to a better passband behaviour.

Algo	J_e^s $(\beta=2)$	J_p $(\beta=2)$	J_e^s $(\beta=0.01)$	J_p $(\beta=0.01)$
LS	0.1320	0.0662	0.9990	0.0033
LS + constr	0.3904	0.0674	1.0921	0.0018
EIG	0.1491	0.1576	1.1160	0.0010
TLS	0.1886	0.0998	1.0862	0.0020
TLS + constr	0.3989	0.0584	1.1897	0.0010

Table 1: Stopband and passband error for different techniques

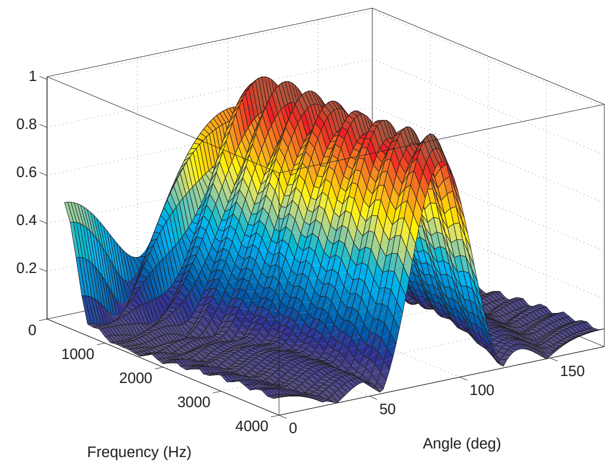

Figure 2: LS design without constraints ($\beta = 2$)

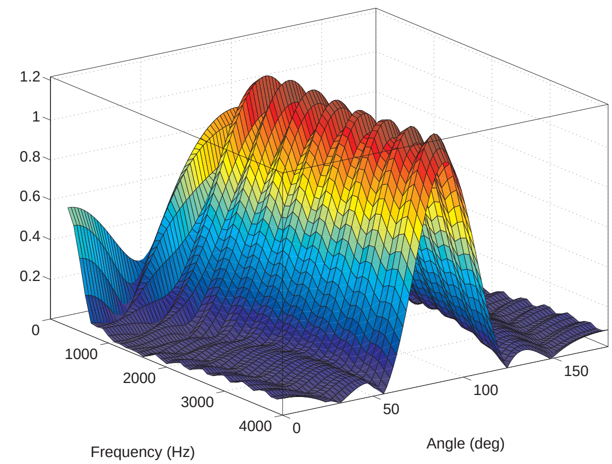

Figure 4: TLS design without constraints ($\beta = 2$)

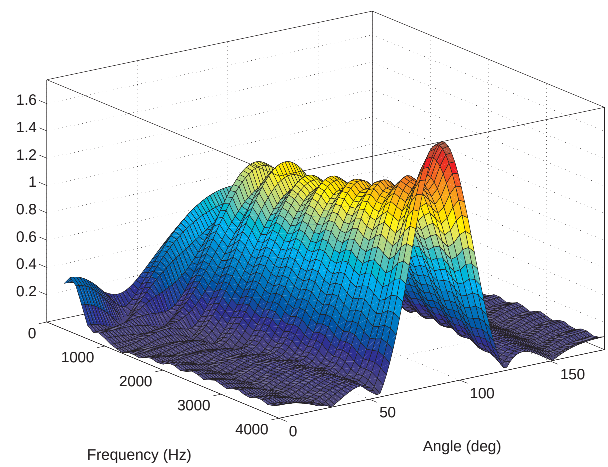

Figure 3: Eigenfilter design without constraints ($\beta = 2$)

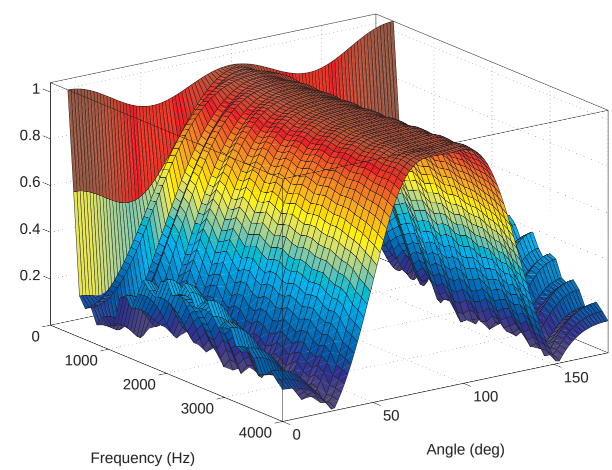

Figure 5: TLS design with line constraint ($\beta = 0.01$)

6. CONCLUSION

In this paper different techniques have been discussed for the design of fixed broadband beamformers. Two eigenfilter techniques have been introduced and compared with the conventional weighted least-squares design technique.

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