

## Conservation of Momentum and Energy/ Law of Collisions

### Keywords:

Conservation of momentum, conservation of energy, elastic, inelastic, and totally inelastic collisions, laws of collision, lab frame, centre of momentum system, scattering angle, digital video analysis.

### Measuring program:

Measurement of velocities on the basis of momentum conservation, analysis of oblique elastic collisions on an air cushion table.

### References:

- /1/ ALONSO, M., FINN, E. J.: „Fundamental University Physics, Vol. 1: Mechanics“, Addison-Wesley Publ. Comp., Reading (Mass.) among others
- /2/ STÖCKER, H.: „Taschenbuch der Physik“, Harri Deutsch, Frankfurt
- /3/ GERTHSEN, C. et al.: „Physik“, Springer-Verlag, Berlin among others

## 1 Introduction

Two experiments are described in the following which will contribute to the understanding of the conservation of momentum and of energy. In particular, it will appear that momentum conservation (vector having magnitude proportional to velocity) and the conservation of kinetic energy (scalar, proportional to the velocity squared) are two quite different things.

## 2 Experiment I: Measurement of Velocities on the Basis of Momentum Conservation

Quite a few (and correspondingly expensive) instruments are required to measure high velocities of small masses directly in the laboratory. Simple light barriers e.g., as used in the laboratory course, react too slowly to be applied for such measurements. Instead, photo detectors would be required with fast response times, i.e. providing impulses with very steep edges which would have to be processed by appropriately „fast“ electronic counters or storage oscilloscopes. Since such instruments are not available in the laboratory course, we have to employ a trick: The rapid movement of a small mass is converted into the slow movement of a large mass. In the following experiment this technique is applied to determine the muzzle velocity of air gun bullets.<sup>1</sup>

### 2.1 Theory

A bullet of mass  $m$  hits a mass  $M$  (Fig. 1), on a resting block, at velocity  $v$ . The bullet follows a line connecting the centres of gravity of the bullet and of the block; both hit each other *centrally*. Following the impact, block and bullet will continue to move at the common velocity  $u$ . Therefore we are dealing with a *total inelastic* collision here.

#### Question 1:

- What are the properties of an *elastic*, of an *inelastic*, and a *total inelastic* collision?

The momentum conservation law may be written in scalar form in this case of a central collision:

$$(1) \quad mv = (M + m)u$$

Considering the energy  $D$  which is transformed into deformation and heat during the inelastic collision, the energy conservation law reads:

$$(2) \quad \frac{1}{2}mv^2 = \frac{1}{2}(M + m)u^2 + D$$

#### Question 2:

- How would the momentum and energy conservation laws read in the case of an elastic collision?

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<sup>1</sup> Since we do not consider the air gun to be a weapon but rather a fun fair article, we think its use in the laboratory course is justifiable, particularly since it is the cheapest device which produces sufficiently high and adequately reproducible velocities.

From Eq. (1) we can determine the velocity  $v$  if  $M$ ,  $m$  and  $u$  are known.  $M$  and  $m$  may be determined by simple weighing. In order to determine  $u$ , the least possible friction is required, which is achieved, for example, by the use of an air cushion table. We will, however, try a less complicated method: we suspend the block by means of a long thread of length  $l$  such that it performs a pendulum motion following the collision (Fig. 1). Neglecting friction effects, the maximum kinetic energy of the block is equal to its maximum potential energy, i.e.:

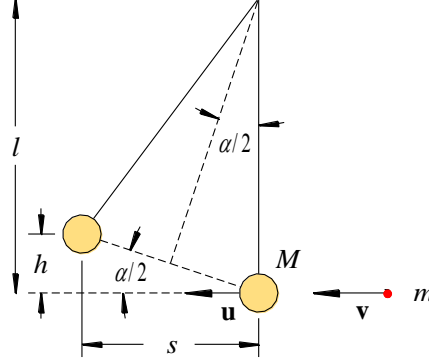


Fig. 1: Central collision between a ball (red) of mass  $m$  having the velocity  $\mathbf{v}$  and a pendulum body (beige) of mass  $M$  departing from its initial position after the collision with the velocity  $\mathbf{u}$ . For the remaining labels, refer to the text.

$$(3) \quad \frac{1}{2}(M+m)u^2 = (M+m)gh$$

In this case  $u$  is the velocity at which the block leaves its resting position,  $h$  is the maximum vertical deflection of the block from the zero position, and  $g$  is the gravitational acceleration.

For small pendulum deflections at angle  $\alpha$ , the approximation  $\tan\alpha \approx \alpha$  can be used and from Fig. 1 follows:

$$(4) \quad \tan\frac{\alpha}{2} = \frac{h}{s} \approx \frac{\alpha}{2}$$

$s$  being the maximum horizontal deflection of the body from the zero position. For  $l \gg s$ , we can approximate:

$$(5) \quad \alpha \approx \frac{s}{l}$$

Inserting Eqs. (4) and (5) into Eq. (3), we obtain:

$$(6) \quad u^2 \approx \frac{g}{l} s^2$$

For the period of oscillation we use the relationship valid for small deflection angles  $\alpha$ :

$$(7) \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Solving Eq. (7) for  $g/l$  and inserting the result into Eq. (6), we obtain:

$$(8) \quad u \approx \frac{2\pi}{T} s$$

Finally we insert Eq. (8) into Eq. (1) and obtain the required relationship for determining the velocity  $v$  from the measured values of  $m$ ,  $M$ ,  $s$  and  $T$ :

$$(9) \quad v \approx \frac{M + m}{m} \cdot \frac{2\pi}{T} s$$

## 2.2 Experimental Procedure

### Equipment:

Air gun in adjustable holder, protecting device, calibration bar, bullets, bifilarly suspended pendulum bob, U-track with scale and displaceable bar for measuring the horizontal pendulum deflection, stand, stopwatch, laboratory balance, foil, adhesive tape, double-sided tape.

➤ *The utmost caution is advised for this experiment. Always make sure that nobody comes into the line of fire! Do not touch the trigger while the gun is open! Prior to first firing inform the assistant!*

First the average bullet mass  $m$  of ten bullets and the mass  $M$  of the pendulum bob are determined.

Afterwards the gun is adjusted by means of inserting the calibration bar into the barrel such that the bullets hit the pendulum bob centrally. Only then a central collision is ensured. Otherwise, the pendulum bob would perform additional rotational and tilting motions and Eq. (9) would no longer be valid. The muzzle should have a distance of about 15 cm from the resting pendulum. On the opposite side a U-track is mounted centrally with adjustable bars to measure the maximum deflection  $s$ . The track is mounted such that deflections of up to 10 cm can be measured. Now we fire 15 times and measure the maximum deflection  $s$  incl. maximum error  $\Delta s$ , as well as the period of oscillation,  $T$ .  $T$  is determined with a stopwatch as mean of ten periods each, the maximum error  $\Delta T$  is assessed from the precision of the time measurement (pressing the stopwatch).

In the described procedure the mass  $M$  of the pendulum would increase with every shot, so we would have different experimental conditions for every shot. We shall avoid this by adding the bullets which have not yet been fired to the pendulum (fixed with double-sided adhesive tape symmetrically around the centre axis) thus keeping the mass  $M$  constant. Eq. (9) then becomes

$$(10) \quad v \approx \frac{M + 15m}{m} \cdot \frac{2\pi}{T} s$$

According to Eq. (7), it would be sufficient to measure  $T$  once because  $l$  does not change from shot to shot. However, in order to avoid errors, e.g. by wrong counting of the oscillation periods,  $T$  shall be determined for each shot.

For each shot (No.  $i$ ), the velocity  $v_i$  incl. maximum error  $\Delta v_i$  is calculated.  $v_i$  is plotted over  $i$  with error bars. The mean  $\bar{v}$  and its standard deviation are calculated and included in the diagram in the form of horizontal lines.

Finally, the values of the kinetic energy as well as of the momentum before and after the collision are calculated (Eq. (2), (16)-(18), (25)). For  $v$  the mean  $\bar{v}$  is used.  $u$  is calculated with Eq. (8) using the means of the single values  $s_i$  and  $T_i$  for  $s$  and  $T$ .

### Question 3:

- How can the result be interpreted by applying Eq. (2)? Which kind of collision is it?

## 3 Experiment II: Oblique Elastic Collisions on an Air Cushion Table

In the first experiment we dealt with central collisions, which mean that the colliding masses come into direct mechanical contact. Now we will observe oblique elastic collisions under the influence of magnetic interaction forces, which means that the masses do not contact each other. Non-contacting collisions between particles under the influence of interaction forces play a major part in atomic, nuclear and particle physics. We will simulate them on an air cushion table.

### 3.1 Theory

In accordance with Fig. 2 we observe two masses  $m_1$  and  $m_2$  from the centre of a resting coordinate system  $XY$  which move towards each other at velocities  $\mathbf{v}_1$  and  $\mathbf{v}_2$  at an arbitrary angle (*oblique collision*, characterized by the fact that the centres of gravity of the two masses do not move along a common line).

The momentum and energy conservation law is easy to describe in the coordinate system  $XY$ . When additional statements about *scattering angles* (see below) are to be made, however, the calculation becomes significantly easier with a coordinate system  $X_s Y_s$ , the origin of which lies in the common centre of gravity  $S$  of both masses. Such a coordinate system is called *centre-of-gravity system* (or *centre-of-mass system*). The index “ $s$ ” is used for all quantities in the centre-of-gravity system in the following.

Looked at from the coordinate system  $XY$ , the centre of gravity  $S$ , and thus the coordinate system  $X_s Y_s$ , moves at the velocity:

$$(11) \quad \mathbf{u} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2}$$

If a mass in the centre-of-gravity system  $X_s Y_s$  has the velocity  $\mathbf{v}_s$ , its velocity  $\mathbf{v}$  can be calculated in the coordinate system  $XY$  by means of simple vector addition:

$$(12) \quad \mathbf{v} = \mathbf{v}_s + \mathbf{u}$$

and hence:

$$(13) \quad \mathbf{v}_s = \mathbf{v} - \mathbf{u}$$

Looked at from  $X_s Y_s$ ,  $m_1$  and  $m_2$  have the following velocities prior to the collision:

$$(14) \quad \mathbf{v}_{s1} = \mathbf{v}_1 - \mathbf{u} = (\mathbf{v}_1 - \mathbf{v}_2) \frac{m_2}{m_1 + m_2}$$

$$(15) \quad \mathbf{v}_{s2} = \mathbf{v}_2 - \mathbf{u} = -(\mathbf{v}_1 - \mathbf{v}_2) \frac{m_1}{m_1 + m_2}$$

The velocities thus always point in opposite directions in the centre-of-gravity system.

By multiplying the velocities with the masses we obtain for the momenta prior to the collision in the centre-of-gravity system (Eqs. (14) and (15)):

$$(16) \quad \mathbf{p}_{s1} = \mu (\mathbf{v}_1 - \mathbf{v}_2)$$

$$(17) \quad \mathbf{p}_{s2} = -\mu (\mathbf{v}_1 - \mathbf{v}_2)$$

To simplify matters we have introduced the *reduced mass*  $\mu$ :

$$(18) \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Thus, the momenta in the centre-of-gravity system always point in opposite directions as well, and are furthermore equal in magnitude (Fig. 3). We obtain:

$$(19) \quad |\mathbf{p}_{s1}| = |\mathbf{p}_{s2}| \quad \text{or in another notation} \quad p_{s1} = p_{s2}$$

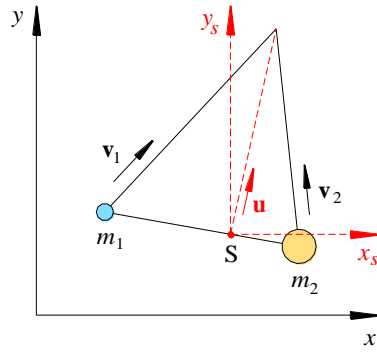


Fig. 2: Oblique collision of the masses  $m_1$  and  $m_2$  in the coordinate system  $XY$ . For all other labels refer to the text.

From Eqs. (16), (17) and (19) follows:

$$(20) \quad \mathbf{p}_{s1} + \mathbf{p}_{s2} = \mathbf{0}.$$

For reasons of momentum conservation Eq. (20) must hold after the collision as well (in the following the upper index "'' is used for all quantities *after* the collision). Hence it follows:

$$(21) \quad \mathbf{p}'_{s1} + \mathbf{p}'_{s2} = \mathbf{0}$$

and

$$(22) \quad |\mathbf{p}'_{s1}| = |\mathbf{p}'_{s2}| \quad \text{or in another notation} \quad p'_{s1} = p'_{s2}$$

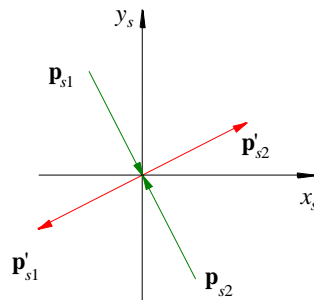


Fig. 3: Course of the momentum during the elastic collision of two masses in the centre-of-gravity system  $X_sY_s$ . The pairs of momenta before (green) and after (red) the collision each point in opposite directions and are equal in magnitude.

Let us now look at the *elastic collision*. It is characterized by the conservation of kinetic energy:

$$(23) \quad \frac{1}{2} m_1 v_{s1}^2 + \frac{1}{2} m_2 v_{s2}^2 = \frac{1}{2} m_1 v'_{s1}{}^2 + \frac{1}{2} m_2 v'_{s2}{}^2$$

Using the magnitudes of the momenta, Eq. (23) can also be written in the form:

$$(24) \quad \frac{p_{s1}^2}{m_1} + \frac{p_{s2}^2}{m_2} = \frac{p'_{s1}{}^2}{m_1} + \frac{p'_{s2}{}^2}{m_2}$$

Inserting Eqs. (19) and (22) into Eq. (24), we learn that in this case, the magnitudes of the momenta *prior* to and *after* collision also need to be equal in the centre-of-gravity system:

$$(25) \quad p_{s1} = p_{s2} = p'_{s1} = p'_{s2}$$

For the magnitudes of the velocities this in turn means:

$$(26) \quad v_{s1} = v'_{s1} \quad v_{s2} = v'_{s2}$$

The *scattering angle*  $\theta$  is per definition the angle between the momentum vector (or velocity vector) of a mass before and after the collision. In our case, we obtain in the centre-of-gravity system  $\theta_s$  according to Fig. 4:

$$(27) \quad \sin \frac{\theta_s}{2} = \frac{|\mathbf{v}'_{s1} - \mathbf{v}_{s1}|}{2|\mathbf{v}'_{s1}|} = \frac{|\mathbf{v}'_{s2} - \mathbf{v}_{s2}|}{2|\mathbf{v}'_{s2}|}$$

This equation must not be misinterpreted as a tool for „calculating“ the scattering angle  $\theta_s$  a priori. This would imply the knowledge of the direction of momenta and velocities, respectively, following collision. This direction, however, is dependent, e.g., on the exact form of the masses and their positions upon collision and is thus not exactly predictable in general. (For a number of idealized special cases, however, a calculation is feasible.)

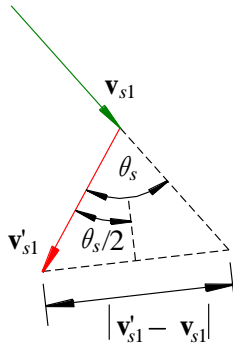


Fig. 4: Definition of the scattering angle  $\theta_s$  between the velocity vectors before (green) and after (red) the collision in the centre-of-gravity system.

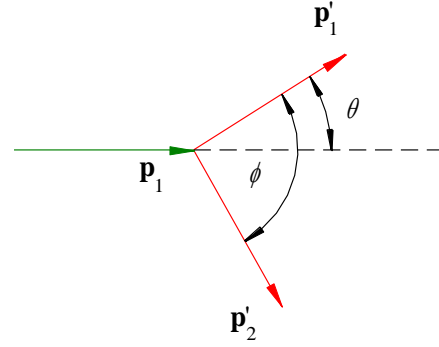


Fig. 5: Definition of the scattering angles  $\theta$  and  $\phi$  between the momentum vectors before (green) and after (red) the collision in the  $XY$  coordinate system (special case  $\mathbf{p}_2 = 0$ ).

The scattering angle  $\theta$  in the coordinate system  $XY$  is considerably more difficult to calculate. Let us confine ourselves to the simple special case of an oblique elastic collision, in which one of the masses is at rest prior to the collision ( $\mathbf{v}_2 = 0$ ; cf. Fig. 5). The momentum conservation law then yields:

$$(28) \quad \mathbf{p}_1 = \mathbf{p}'_1 + \mathbf{p}'_2 \quad \text{and} \quad \mathbf{p}'_2 = \mathbf{p}_1 - \mathbf{p}'_1$$

Squaring the right-hand term in Eq. (28) we obtain

$$(29) \quad p_2'^2 = (\mathbf{p}_1 - \mathbf{p}'_1)^2 = p_1^2 + p_1'^2 - 2p_1p_1' \cos \theta$$

and thus

$$(30) \quad \theta = \arccos \frac{p_1^2 + p_1'^2 - p_2'^2}{2p_1p_1'}$$

Finally, we consider the angle  $\phi$  between the velocity or momentum vectors of both masses after the collision in the coordinate system  $XY$  for the case  $v_2 = 0$  and, in addition,  $m_1 = m_2$ . For momentum conservation we again obtain Eq. (28):

$$(31) \quad \mathbf{p}_1 = \mathbf{p}'_1 + \mathbf{p}'_2$$

and hence

$$(32) \quad p_1^2 = (\mathbf{p}'_1 + \mathbf{p}'_2)^2 = p_1'^2 + p_2'^2 + 2\mathbf{p}'_1\mathbf{p}'_2$$

From the energy conservation law it follows in this case for the elastic collision:

$$(33) \quad p_1^2 = p_1'^2 + p_2'^2$$

Eqs. (32) and (33) together yield the condition

$$(34) \quad 2\mathbf{p}'_1\mathbf{p}'_2 = 2p_1'p_2' \cos \phi = 0$$

This equation is fulfilled for  $\phi = 90^\circ$ . The velocity and momentum vectors for both masses after the collision are perpendicular in this case ( $m_1 = m_2, v_2 = 0$ ).

### 3.2 Experimental Procedure

#### **Equipment:**

Air cushion table including accessory parts (fan, pucks), webcam Logitech C930, PC with image processing software *Viana.NET*, water balance, laboratory balance, glass cup, metal measuring tape.

#### **Attention:**

The pucks may only be moved on the table with the air supply switched on! Prior to the experiment the air cushion table is adjusted by the technical assistant such that the pucks are not accelerated within the measurement range.

Using an air cushion table on which special pucks can move nearly frictionless we will investigate the oblique elastic collision of two repulsive magnetic pucks, namely

- a) for the case  $m_1 \approx m_2, v_2 \approx 0$  and
- b) for the case  $m_1 \neq m_2, v_1 \neq 0, v_2 \neq 0$

In order to pursue and quantitatively evaluate the path of the pucks, movement is filmed with a webcam which is mounted on the ceiling above the air cushion table. The webcam was previously aligned by the technical assistants and must not be changed in its orientation.

Both the recording and the evaluation of the videos can be performed with the program *Viana.NET*. The program is already installed on the laptop and is also freely accessible for private use<sup>2</sup>.

As the overhead light causes disturbing reflections on the table, an indirect light source is used to obtain the necessary brightness in the room. In the video, the table should appear illuminated as evenly as possible. If necessary, the orientation of the indirect light source must be adjusted.

The masses of the pucks are weighed on the laboratory balance (put a large glass below them, otherwise the magnetic fields of the pucks influence the balance!).

The pucks are started manually; in case a) the second puck has to be kept in its position until the first puck starts, if necessary. The recording is started in the *Viana.NET* program ( $\rightarrow$  Start: "Record Video", see Fig. 6).<sup>3</sup> After the measurement, it is recommended to stop the recording until the next measurement is performed in order to reduce the amount of recorded data and to avoid confusion.

The storage directory of the videos is set in the menu "Record Video" (Fig. 7). The videos must be stored on a mobile data carrier or personal cloud for further evaluation. Cloud storage is available through Stud.IP for every student of the University of Oldenburg.<sup>4</sup>

<sup>2</sup> Available under: <http://viananet.de/downloads> (Version 5.5); the language can be switched to English under the *Options* menu.

<sup>3</sup> If the autofocus of the webcam is active, it can be switched off via the tab „Video  $\rightarrow$  Video Device“.

<sup>4</sup> In Stud.IP under the tab Webmail  $\rightarrow$  Cloud-Storage

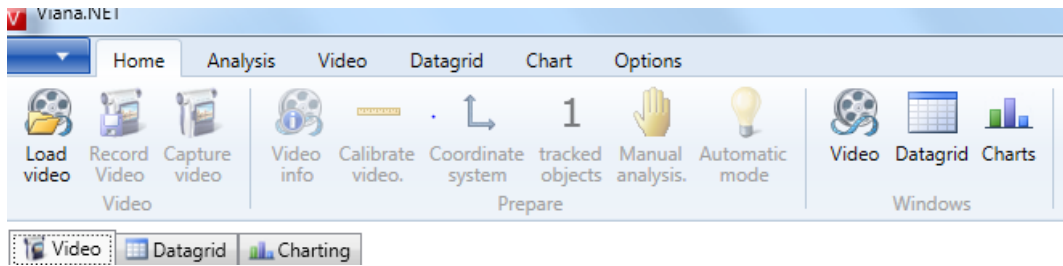


Fig. 6: Start menu of the program Viana.NET.

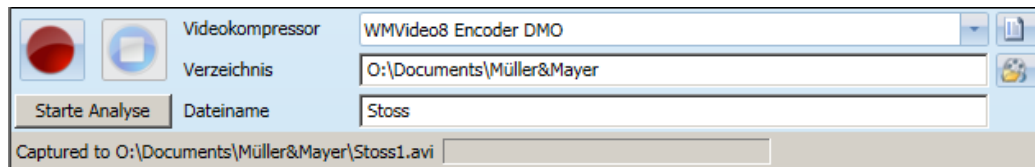


Fig. 7: Selection of the storage directory for the video.

### 3.3 Analysis

The following analysis can be done from home or from one of the PCs of basic laboratory.

For the cases a) and b) one suitable video can be quantitatively analysed. For this, the previously recorded video can be edited directly in Viana.NET. With the option "Load Video" one of the recorded videos can be loaded in the program. Then, the video must be calibrated ( $\rightarrow$ Home: "Calibrate Video") and the coordinate system must be added ( $\rightarrow$  Home: "Coordinate system").

To convert distances from pixel to meter, a known distance must be available as a reference distance, which can be seen in the video. This reference distance is used to convert a distance in pixels to a distance in mm.

For this, the origin of the coordinate system is placed in the lower left corner of the wire (i.e. at the bottom of the video frame, see Fig.8). The reference distance goes from the origin to the other end of the wire on the right side and has a length of 1085 mm. This length also corresponds to the X-axis of our reference coordinate system.

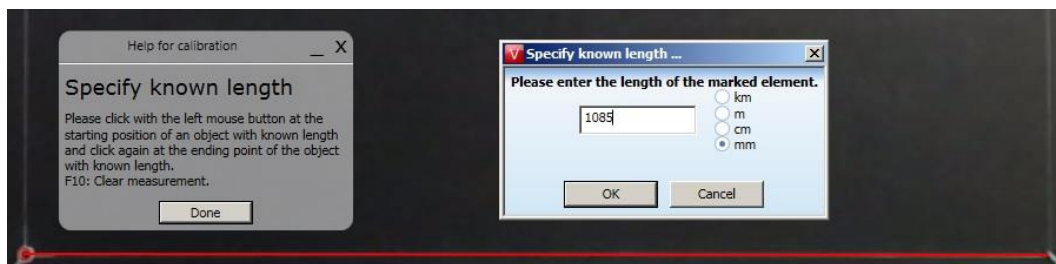


Fig. 8: Example for the distance conversion from pixel to meter. The red line represents the length of the reference distance (1085 mm).

The Y-axis of the coordinate system should also be chosen appropriately to be able to perform a meaningful analysis, and the number of tracked objects must be set to 2 ( $\rightarrow$  Start: "Track 2 objects").

The trajectories of the pucks are extracted with "Manual analysis". The center of each puck must be selected with the mouse alternately. If both centers of the pucks have been selected in a certain frame image, the program automatically jumps to the next frame. This procedure should be repeated until enough points are found for visualizing the movements of the pucks before and after the impact and to define the vectors of velocity (at least 5 points per vector). The extracted data are then displayed in two x-y diagrams, in which the trajectory for each object (a superposition of both objects in a common diagram is not possible) is displayed. Based on this diagram, a suitable choice of the coordinates for determining the respective momentum vectors of the pucks can be made. The diagram can be saved via the "Charts" tab and should be added to the report.



For the evaluation of the experiments, the momentum vectors of the pucks shall be given as column vectors before and after the collision, i.e. in the form:

$$\mathbf{p} = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

In addition, a vector diagram is to be created, in which the momenta before and after the collision are drawn, as well as their sums.

In order not to make the evaluation time-consuming, error theory is dispensable in this case. A plausible estimation of the maximum error for the different impulse components is sufficient.

The determined coordinates are stored under the tab "Data grid" after marking the trajectories of the pucks. They can be saved as a *txt* or *Excel* file. For the data export, it is sufficient to save the picture number, time and X & Y position. Based on this data, the momentum vectors of the respective pucks can be constructed both before and after the impact. The data table should also be listed in the appendix of your report.

Please carry out the evaluation of your results "manually" and document the details of your calculations in your report comprehensibly. In addition a MATLAB script "*law\_of\_collision*" is provided, which you can use to check your results and display the corresponding vector diagrams. The script is explained in the following to make it logically comprehensible.

Let us assume that the centre of puck 1 (mass  $m_1 = 0.2$  kg) before the collision at time  $t$  has the screen coordinates  $x_{11} = 210$  px,  $y_{11} = 320$  px (cf. Fig. 9).  $n$  frame images later at the time  $t + n/f$  ( $f$ : Frame rate in frames per second (fps), in our case  $f = 15$  Hz und  $n = 1$ ), the coordinates of the center of puck 1 is at  $x_{12} = 345$  px and  $y_{12} = 275$  px. The coordinates mentioned here are still given in pixels (px). After calibration of the video, the corresponding information is also available in meters.

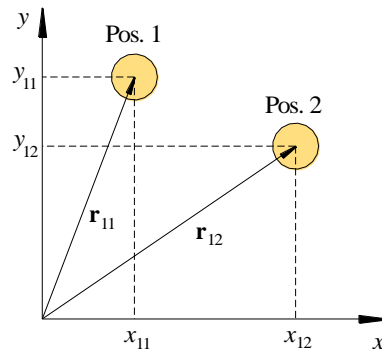


Fig. 9: Definition of values describing the position of puck 1 at time  $t$  (Pos. 1) and at time  $t + n/f$  (Pos. 2).  $\mathbf{r}_{11}$  and  $\mathbf{r}_{12}$  are the position vectors describing the puck's positions.

The *m*-file "*law\_of\_collision*" is opened in MATLAB. First, the parameters of the experiment (in SI-units) are entered into this file (reminder: the semicolon at the end of the line suppresses the output in the command window of MATLAB):

```
f = 15;
n = 1;
m1 = 0.2;
x11 = 210;
y11 = 320;
x12 = 345;
y12 = 275;
```

Now, we calculate the position vectors at the two positions of puck 1 prior to the collision.  $\mathbf{r}_{11}$  is the position vector at time  $t$ ,  $\mathbf{r}_{12}$  is the position vector at time  $t + n/f$ . The position vectors are given as column vectors (MATLAB notation [ $x$  component;  $y$  component]) and let them appear on the screen (therefore *no* semicolon at the end of the row):

```
r11 = [x11; y11]
r12 = [x12; y12]
```

From this we get the momentum vector  $\mathbf{p}_1$  for puck 1 before the collision as column vector:

$$p1 = m1 * (r12 - r11) / (n/f)$$

For the mentioned example we get in numbers:

$$\mathbf{p}_1 = \begin{pmatrix} 0.0771 \\ -0.0257 \end{pmatrix} \text{kg m s}^{-1}$$

In an analogous way we can calculate the momentum of puck 2 before the collision ( $\mathbf{p}_2$ ) and the momenta of the two pucks after the collision ( $\mathbf{p}_1'$  und  $\mathbf{p}_2'$ ) (in Matlab we use e.g. `p1s` for  $\mathbf{p}_1'$ , “s” denoting “dash”. From that we get the total momenta before and after the collision ( $\mathbf{p}$  und  $\mathbf{p}'$ ):

$$\begin{aligned} p &= p1 + p2 \\ ps &= p1s + p2s \end{aligned}$$

and we can check whether the momentum difference  $\Delta\mathbf{p}$  is zero:

$$\text{delta\_p} = p - ps$$

In order to check whether the momenta  $\mathbf{p}_1'$  and  $\mathbf{p}_2'$  are orthogonal to each other in experiment a), we calculate their dot product (here named `sk`) with the MATLAB command `dot`:

$$sk = \text{dot}(p1s, p2s)$$

For an angle of  $\phi = 90^\circ$  between the two vectors it follows that  $sk = 0$ . If the value of the dot product differs from zero, we can calculate  $\phi$  from the value for `sk` by using the well-known relationship for the dot product:

$$(35) \quad sk = \mathbf{p}_1' \cdot \mathbf{p}_2' = |\mathbf{p}_1'| |\mathbf{p}_2'| \cos \phi$$

and thus,

$$(36) \quad \phi = \arccos \left( \frac{sk}{|\mathbf{p}_1'| |\mathbf{p}_2'|} \right)$$

The magnitude of a vector is calculated in MATLAB using the `norm` function. Hence, Eq. (37) is expressed in MATLAB syntax as:

$$(37) \quad \text{phi} = \text{acos}(sk / (\text{norm}(p1s) * \text{norm}(p2s)))$$

#### **Question 4:**

- Is the momentum conservation law fulfilled in both cases? Is the kinetic energy conserved in both cases? How can possible deviations be explained?
- The scattering angle  $\phi$  is only calculated for case a). What is its value? Does the result correspond with the theoretical expectations according to Eq. (34)? If it does not - what could be the reasons why?

### **3.4 Vector diagrams**

2D vector diagrams of the momentum  $\mathbf{p}$  can easily be drawn with MATLAB using the `quiver` command. The command has the format `quiver(a, b, u, v, s)`. Here  $a$  and  $b$  are the coordinates of the starting point of the vector,  $u$  and  $v$  are its  $x$ - and  $y$ -components and  $s$  is a scaling factor. For more details have a look into the MATLAB script provided for the experiment.

Please add the vector diagrams to your experimental protocol.