Surprises with Rotating Black Holes

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Łódź, 22.11.2006

Outline

1. Introduction
2. Non-Abelian Black Holes
   - Einstein-Yang-Mills Black Holes
   - Einstein-Yang-Mills-Dilaton Black Holes
3. Abelian Black Holes
   - $4D$ Einstein-Maxwell-Dilaton Black Holes
   - $5D$ Einstein-Maxwell-Chern-Simons Black Holes
   - Odd-D Einstein-Maxwell-Chern-Simons Black Holes
4. Conclusions and Outlook

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Summary: Einstein–Maxwell Black Holes

	static	rotating
spherically symmetric	Schwarzschild (M) Reissner-Nordström (M, Q, P)	–
axially symmetric	–	Kerr (M, J) Kerr–Newman (M, Q, P, J)

- Uniqueness theorem

  black holes are uniquely determined by their mass M, angular momentum J, charges Q and P
- Israel's theorem

  static black holes are spherically symmetric
- Staticity theorem

  stationary black holes with non-rotating horizon are static

Werner Israel

*1931

Outline

Einstein-Yang-Mills Theory

Einstein-Yang-Mills action

$$\mathcal{S} = \int \Big\{ \underbrace{\frac{R}{16\pi G}}_{\text{gravity}} - \underbrace{\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang−Mills (YM)}} \Big\} \sqrt{-g}\, d^4x$$

- YM gauge potential $A_\mu = A_\mu^a \frac{\tau^a}{2}$
- YM field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]$

Einstein equations

Einstein tensor $\longrightarrow$ $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ $\longleftarrow$ stress–energy tensor

Yang-Mills field equations

$$\frac{1}{\sqrt{-g}} D_\mu(\sqrt{-g} F^{\mu\nu}) = 0$$

Static Spherically Symmetric EYM Solutions

globally regular solutions: Bartnik, McKinnon 1988

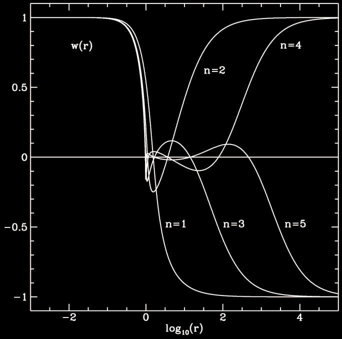
metric:
$$ds^2 = -A^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2$$

gauge potential:
$$A_\mu dx^\mu = \frac{1 - w_k(r)}{2}\left[\tau_\varphi d\theta - \sin\theta \tau_\theta d\varphi\right]$$

- regular at $r = 0$
- asymptotically flat
- node number k

  $k = 1, ..., \infty$
- dimensionless mass M_k

  $M_1 = 0.83, ..., M_\infty = 1$
- no charge



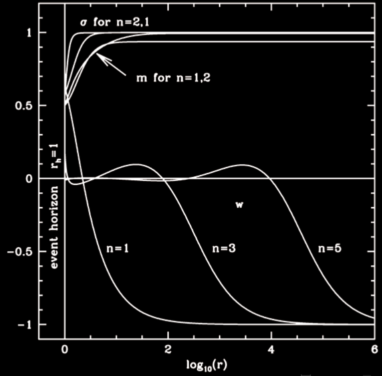
Static Spherically Symmetric EYM Solutions

black hole solutions: Volkov, Gal'tsov 1989, et al.

metric: $$ds^2 = -A^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2$$

gauge potential: $$A_\mu dx^\mu = \frac{1 - w_k(r)}{2}\left[\tau_\varphi d\theta - \sin\theta \tau_\theta d\varphi\right]$$

- regular at $r = r_{\rm H}$
- asymptotically flat
- node number k
  $k = 1, ..., \infty$
- limiting solution
  $k \to \infty$: RN
- no charge
- no uniqueness



Perturbative Rotating EYM Black Holes

perturbative rotating solutions: Brodbeck, Heusler, Straumann, Volkov 1997

metric: $$ds^2 = -A^2 N dt^2 + \frac{1}{N} dr^2 + r^2 d\Omega^2 - 2A^2 N\beta \sin^2\theta dt d\varphi$$

gauge field: $$A_\mu dx^\mu = \frac{1}{2}(1-w)\left[\tau_\varphi d\theta - \sin\theta \tau_\theta d\varphi\right] + \delta A_0 dt$$

$$\left.\begin{array}{lcl} A^2 N\beta & \rightarrow & 2J/r + O(1/r^2) \\ \delta A_0 & \rightarrow & (A_\infty - Q/r)\tau_z \end{array}\right\}$$

J angular momentum
Q electric YM charge

- black hole solutions

type 1:	$A_\infty = 0,\ J \neq 0,\ Q \neq 0$	rotating, charged
type 2:	$A_\infty \neq 0,\ J \neq 0,\ Q = 0$	uncharged, $E \neq 0$
type 3:	$A_\infty \neq 0,\ J = 0,\ Q \neq 0$	non-rotating, non-static

- regular solutions

$$A_\infty \neq 0, \quad J \neq 0, \quad Q \neq 0$$

Ansätze for Static Axially Symmetric EYM Solutions

Killing vectors: $\xi = \partial_t \ , \ \eta = \partial_\varphi$

metric: Lewis-Papapetrou form in isotropic coordinates

$$ds^2 = -f dt^2 + \frac{m}{f}\left[dr^2 + r^2 d\theta^2\right] + \sin^2\theta r^2 \frac{l}{f} d\varphi^2$$

gauge potential:

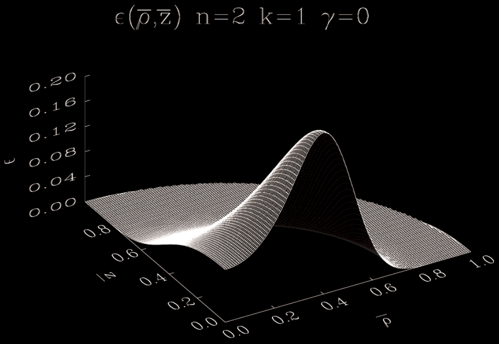
$$\begin{aligned} A_\mu dx^\mu &= A_\varphi d\varphi + \left(\frac{H_1}{r} dr + (1 - H_2) d\theta\right) \frac{\tau_\varphi^n}{2e} \\ A_\varphi &= -\sin\theta \left[H_3 \frac{\tau_r^n}{2e} + (1 - H_4) \frac{\tau_\theta^n}{2e}\right] \end{aligned}$$

$$\begin{aligned} \tau_r^n = \tau \cdot e_r^n &= \tau \cdot (\sin\theta\cos n\varphi, \sin\theta\sin n\varphi, \cos\theta) \\ \tau_\theta^n = \tau \cdot e_\theta^n &= \tau \cdot (\cos\theta\cos n\varphi, \cos\theta\sin n\varphi, -\sin\theta) \\ \tau_\varphi^n = \tau \cdot e_\varphi^n &= \tau \cdot (-\sin n\varphi, \cos n\varphi, 0) \end{aligned}$$

Static Axially Symmetric EYM Solutions

globally regular solutions: Kleihaus, Kunz 1997

- regular
- asymptotically flat
- node number k
- winding number n
- no charge

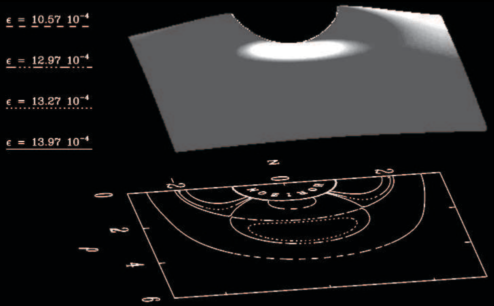


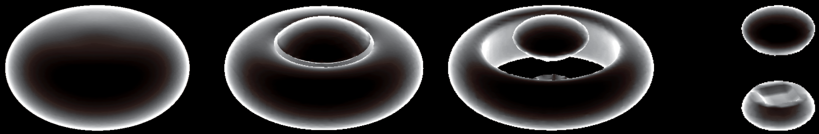
$$\epsilon = -T_0^0$$

Static Axially Symmetric Black Holes

black hole solutions: Kleihaus, Kunz 1997

- regular horizon
  $f(r_{\rm H}) = 0$
- asymptotically flat
- node number k
- winding number n
- no charge
- no uniqueness





$\epsilon = 10.57 \cdot 10^{-4}$ $\epsilon = 12.97 \cdot 10^{-4}$ $\epsilon = 13.27 \cdot 10^{-4}$ $\epsilon = 13.97 \cdot 10^{-4}$

Static Axially Symmetric EYM Black Holes

black hole solutions: Kleihaus, Kunz 1997

circumferences of horizon:

$$L_e = \int_0^{2\pi} \sqrt{\frac{l}{f}}\, x \, \sin\theta \, d\varphi \, , \quad L_p = 2 \int_0^{\pi} \sqrt{\frac{m}{f}}\, x \, d\theta$$

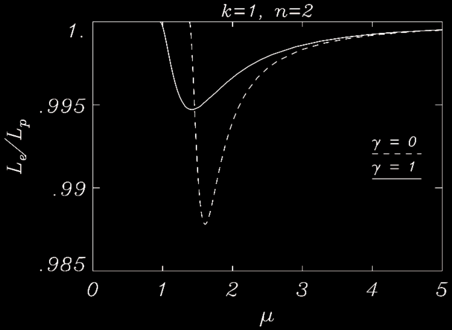
spherical symmetry:

$L_e = L_p$

prolate black holes:

$L_e < L_p$

Israel's theorem does not hold



Ansätze for Stationary EYM Solutions

Killing vectors: $\xi = \partial_t \ , \ \eta = \partial_\varphi$

metric: Lewis-Papapetrou form in isotropic coordinates

$$ds^2 = -f dt^2 + \frac{m}{f}\left[dr^2 + r^2 d\theta^2\right] + \sin^2\theta r^2 \frac{l}{f}\left[d\varphi - \frac{\omega}{r}dt\right]^2$$

gauge potential:

$$A_\mu dx^\mu = \psi dt + A_\varphi \left(d\varphi - \frac{\omega}{r}dt\right) + \left(\frac{H_1}{r}dr + (1-H_2)d\theta\right)\frac{\tau_\varphi^n}{2e}$$

$$A_\varphi = -\sin\theta\left[H_3\frac{\tau_r^n}{2e} + (1-H_4)\frac{\tau_\theta^n}{2e}\right] , \qquad \psi = B_1\frac{\tau_r^n}{2e} + B_2\frac{\tau_\theta^n}{2e}$$

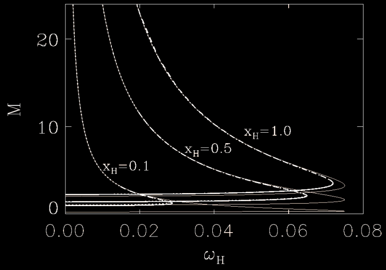
$$\tau_r^n = \tau \cdot e_r^n = \tau \cdot (\sin\theta\cos n\varphi, \sin\theta\sin n\varphi, \cos\theta)$$
$$\tau_\theta^n = \tau \cdot e_\theta^n = \tau \cdot (\cos\theta\cos n\varphi, \cos\theta\sin n\varphi, -\sin\theta)$$
$$\tau_\varphi^n = \tau \cdot e_\varphi^n = \tau \cdot (-\sin n\varphi, \cos n\varphi, 0)$$

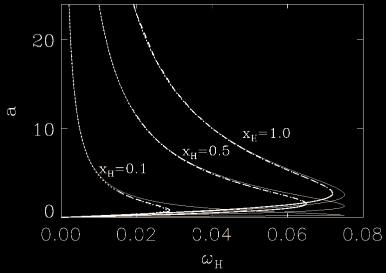
Global Properties of Rotating EYM Black Holes

nonperturbative rotating black holes: Kleihaus, Kunz 2001
Kleihaus, Kunz, Navarro-Lérida 2002

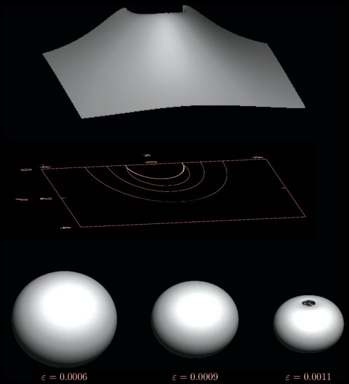
metric:
asymptotically flat

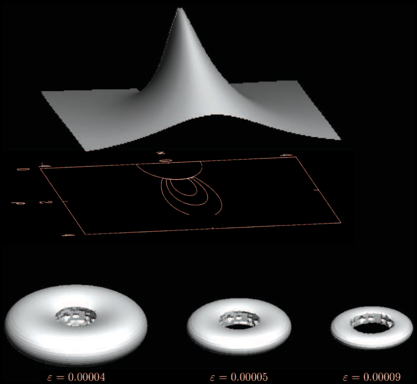
$$f \to 1 - \frac{2M}{x} \,, \qquad m \to 1 \,, \qquad l \to 1 \,, \qquad \omega \to \frac{2J}{x^2}$$





Energy Density of Rotating EYM Black Holes





$$M = 2.4$$
$$J = 1.9$$

slow

$$M = 10.3$$
$$J = 103$$

fast

Non-Abelian Charges of Rotating EYM Black Holes

gauge potential:

- magnetic charge $P = 0$

$$\mathcal{P}^{\mathrm{YM}} = \frac{1}{4\pi}\oint\sqrt{\sum_i\left(F^i_{\theta\varphi}\right)^2}d\theta d\varphi = \frac{|P|}{e}$$

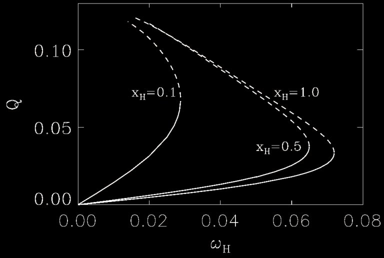
$$\begin{array}{ll} H_1 \to 0 & H_2 \to (-1)^k \\ H_3 \to 0 & H_4 \to (-1)^k \end{array}$$

- electric charge $Q \neq 0$

$$\mathcal{Q}^{\mathrm{YM}} = \frac{1}{4\pi}\oint\sqrt{\sum_i\left({}^*F^i_{\theta\varphi}\right)^2}d\theta d\varphi = \frac{|Q|}{e}$$

$$B_1 \to \frac{Q\cos\theta}{x}$$

$$B_2 \to -(-1)^k\frac{Q\sin\theta}{x}$$

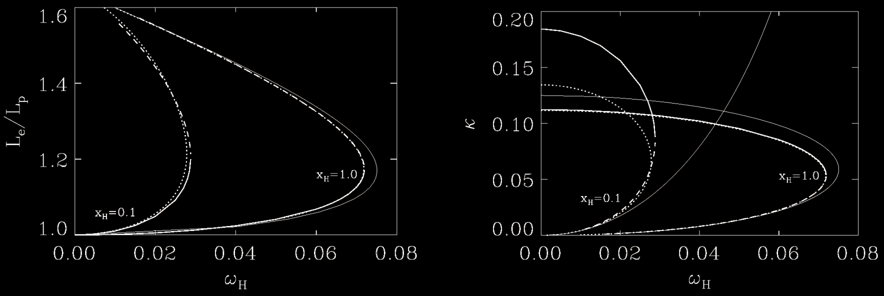


asymptotic expansion with non-integer powers

$$\alpha = \sqrt{9-4Q^2}$$
$$\beta = \sqrt{25-4Q^2}$$

Horizon Properties of Rotating EYM Black Holes

horizon deformation $L_\mathrm{e}/L_\mathrm{p}$ surface gravity κ_sg



- only black holes of type 1: $A_\infty = 0$, $J \neq 0$, $Q \neq 0$
- no regular solutions

van der Bij, Radu 2002

Outline

Einstein-Yang-Mills-Dilaton Theory

Einstein-Yang-Mills-dilaton action

$$\mathcal{S} = \int \Big\{ \underbrace{\frac{R}{16\pi G}}_{\text{gravity}} - \underbrace{\frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi}_{\text{dilaton}} - \underbrace{\frac{1}{2} e^{2\kappa\Phi} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu})}_{\text{Yang-Mills (YM)}} \Big\} \sqrt{-g} d^4x$$

- dilaton field Φ
- dilaton coupling constant κ

Einstein equations

Einstein tensor $\longrightarrow$ $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ $\longleftarrow$ stress–energy tensor

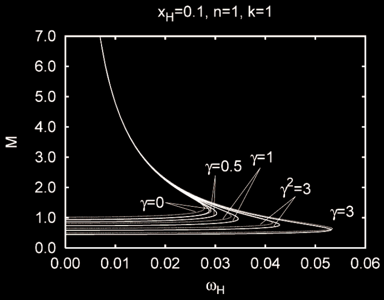
matter field equations

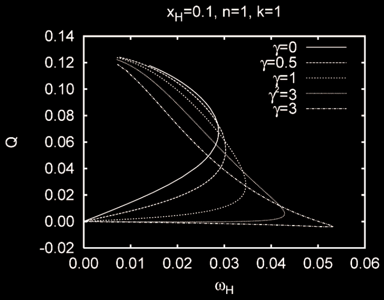
$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \partial^\mu \Phi \right) = \kappa e^{2\kappa\Phi} \mathrm{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right)$$

$$\frac{1}{\sqrt{-g}} D_\mu (\sqrt{-g} e^{2\kappa\Phi} F^{\mu\nu}) = 0$$

Global Properties of Rotating EYMD Black Holes

Kleihaus, Kunz, Navarro-Lérida 2003





dimensionless dilaton coupling constant $\gamma = \kappa/\sqrt{4\pi G}$

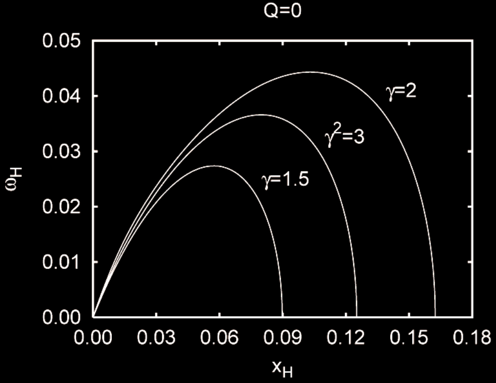
$\gamma = 0$: Einstein-Yang-Mills theory

$\gamma = 1$: string theory

$\gamma = \sqrt{3}$: Kaluza-Klein theory

New Type of Rotating Black Holes

- dilaton: black holes of type 2: $A_\infty \neq 0,\ J \neq 0,\ Q = 0$



$n = k = 1$

- dilaton charge $\Phi \to -\dfrac{D}{x}$
- mass formula?
- uniqueness?

Mass of Rotating Abelian Black Holes

Smarr formula holds also for EMD black holes

$$M = 2\,T\,S + 2\,\Omega\,J + \psi_{\rm el,H}\,Q + \psi_{\rm mag,H}\,P$$

- magnetic potential $\psi_{\rm mag}$: $\partial_\mu \psi_{\rm mag} = e^{2\gamma\phi}\ \chi^\nu\ {}^*\mathcal{F}_{\nu\mu}$
- electric charge Q: $\tilde{Q} = -\frac{1}{4\pi}\int e^{2\gamma\phi}({}^*\mathcal{F}_{\theta\varphi})d\theta d\varphi = Q$

EMD black holes satisfy another mass formula

$$M = 2\,T\,S + 2\,\Omega\,J + 2\,\psi_{\rm el,H}\,Q + \frac{D}{\gamma}$$

$$\frac{D}{\gamma} = \psi_{\rm mag,H}\,P - \psi_{\rm el,H}\,Q$$

Mass of Rotating Non-Abelian Black Holes

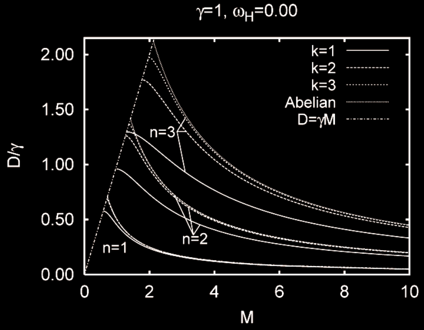
New Mass Formula

EYMD black holes satisfy the mass formula

$$M = 2\,T\,S + 2\,\Omega\,J + 2\,\psi_{\mathrm{el,H}}\,Q + \frac{D}{\gamma}$$

Uniqueness of Rotating Non-Abelian Black Holes?

uniqueness of static black holes?



curves with the same n do not intersect

topological number $N = n$

Ashtekar

pull-back of F to horizon H:

$$F_{\rm H} = F_{\theta\phi}|_{\rm H} d\theta \wedge d\phi$$

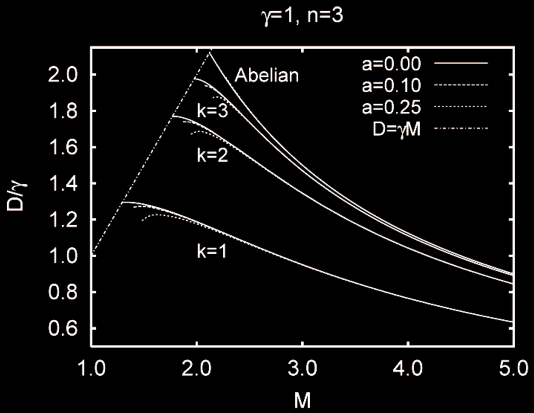
map $S^2 \rightarrow S^2$:

$$\sigma = \frac{{}^*F_{\rm H}}{|{}^*F_{\rm H}|} = \cos\Theta \frac{\tau_z}{2} + \sin\Theta \left(\cos n\varphi \frac{\tau_x}{2} + \sin n\varphi \frac{\tau_y}{2} \right)$$

$$N = \frac{1}{4\pi} \int_{\rm H} \frac{1}{2} \varepsilon_{ijk} \sigma^i d\sigma^j \wedge d\sigma^k = n$$

Uniqueness of Rotating Non-Abelian Black Holes?

non-Abelian uniqueness conjecture



black holes are uniquely determined by their mass M, angular momentum J, electric charge Q, dilaton charge D, topological charge N

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Einstein-Maxwell-Dilaton Theory

Einstein-Maxwell-dilaton action

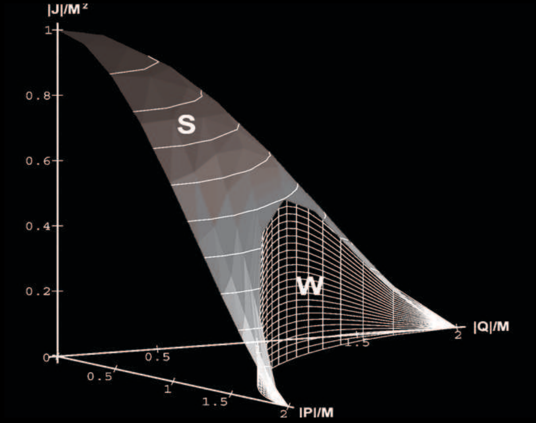
$$S = \int \Big\{ \underbrace{\frac{R}{4}}_{\text{gravity}} \underbrace{- \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi}_{\text{dilaton}} \underbrace{- \frac{1}{4} e^{2\gamma\Phi} (\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu})}_{\text{Maxwell}} \Big\} \sqrt{-g} d^4x$$

dimensionless dilaton coupling constant γ

- $\gamma = 0$: Einstein-Maxwell theory
- $\gamma = 1$: string theory
- $\gamma = \sqrt{3}$: Kaluza-Klein theory
- $\gamma > \sqrt{3}$

Kaluza-Klein Black Holes

Surfaces of extremal solutions in Kaluza-Klein theory: Rasheed 1995



vertical wall W:

stationary $\Omega = 0$ solutions

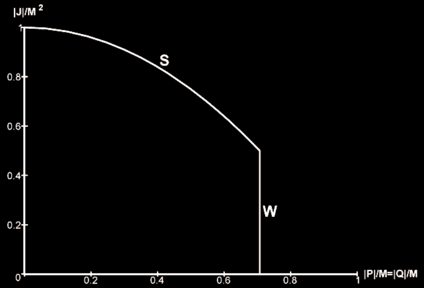
$$\left(\frac{P}{M}\right)^{\frac{2}{3}} + \left(\frac{Q}{M}\right)^{\frac{2}{3}} = 2^{\frac{2}{3}}$$

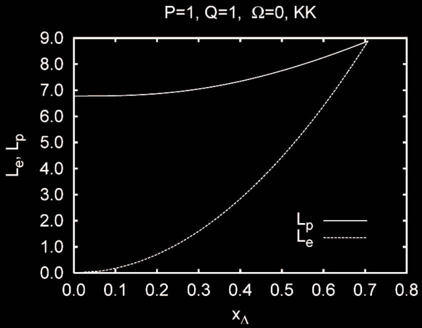
$$J \le PQ$$

J increases, $M = \text{const}$

Kaluza-Klein Black Holes

extremal $|P| = |Q|$ solutions





vertical wall W:

stationary $\Omega = 0$ solutions

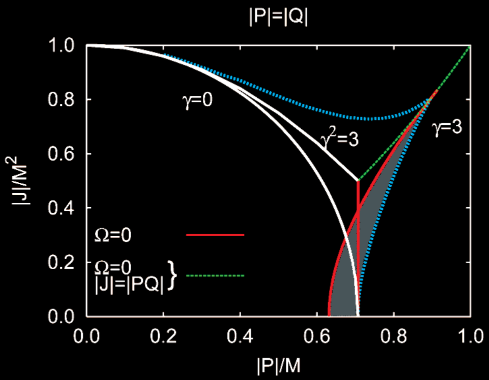
horizon circumferences:

L_e and L_p

prolate deformation

Rotating EMD Black Holes

Kleihaus, Kunz, Navarro-Lérida 2004



extremal: $|P| = |Q|$

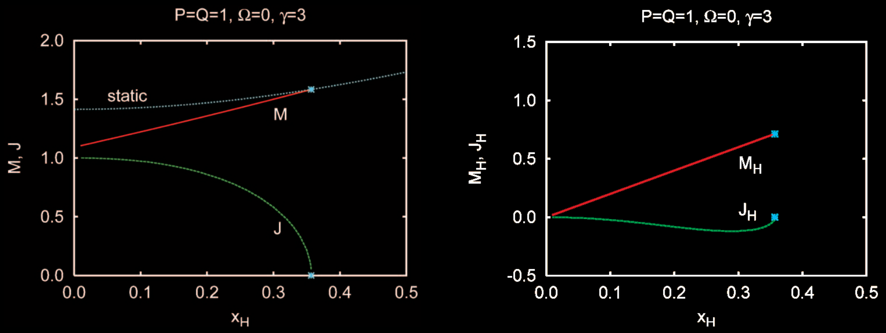
stationary: $\Omega = 0$

stationary: $\Omega = 0$, $J = PQ$

what is in the shaded region?

Non-Rotating Stationary EMD Black Holes

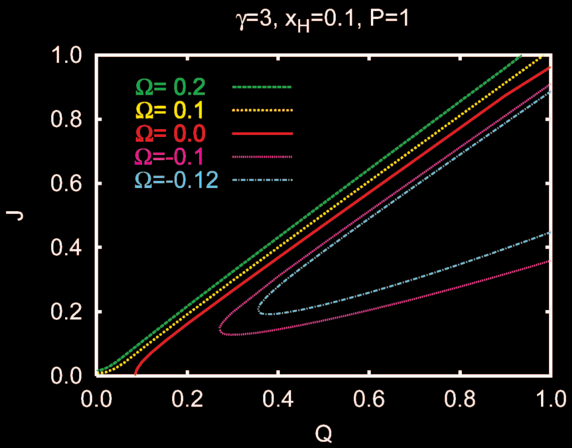
non-extremal stationary $\Omega = 0$ black holes



mass and angular momentum | horizon mass and angular momentum

- as J increases, M decreases
- a negative fraction of J resides behind the horizon: $J_\mathrm{H} < 0$
- effect of the rotation: prolate deformation of the horizon

Counter-Rotating EMD Black Holes



angular momentum versus charge

- co-rotation

  $J > 0$: $\Omega > 0$

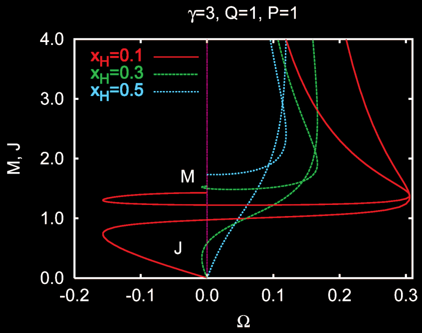
- non-rotating horizon

  $J > 0$: $\Omega = 0$

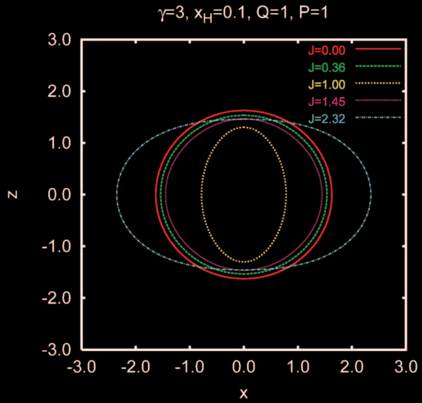
- counter-rotation

  $J > 0$: $\Omega < 0$

Shape of Counter-Rotating EMD Black Holes



angular momentum J versus Ω



embedding of the horizon shape

Outline

$D = 5$ Einstein-Maxwell-Chern-Simons Theory

In odd dimensions $D = 2n + 1$ the Einstein-Maxwell action may be supplemented by a 'AF^n' Chern-Simons term.

$D = 5$ Einstein-Maxwell-Chern-Simons action

$$S = \int \frac{1}{16\pi G_5} \Big\{ \sqrt{-g}\,(R - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) - \underbrace{\frac{2\lambda}{3\sqrt{3}} \varepsilon^{mnpqr} A_m F_{np} F_{qr}}_{\rm Chern-Simons} \Big\} d^5x$$

Chern-Simons coupling constant λ

$\lambda = 0$: Einstein-Maxwell theory

$\lambda = 1$: bosonic section of minimal $D = 5$ supergravity

$\lambda > 1$

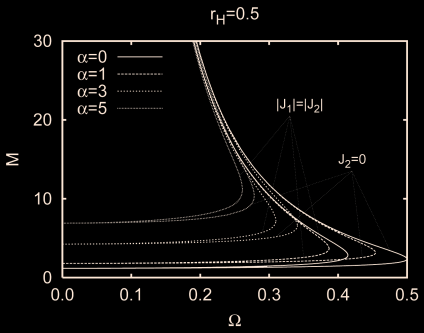
$\lambda = 0$: $D = 5$ Einstein-Maxwell Black Holes

- rotating vacuum black holes
  Myers, Perry 1986
  two angular momenta J_1, J_2
  rotation in two orthogonal planes

- rotating EM black holes
  surprise: no analytic solutions
  Kunz, Navarro-Lérida, Petersen 2005
  $g \neq 3$

$J_1 \neq 0$, $J_2 = 0$ black holes

$J_1 = J_2$ black holes



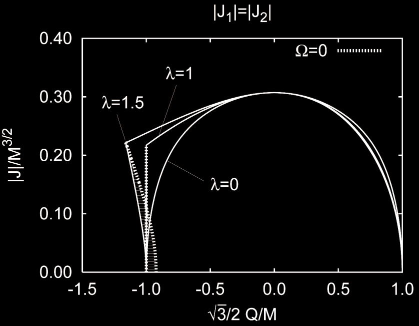
$\lambda = 1$: Supersymmetric Black Holes

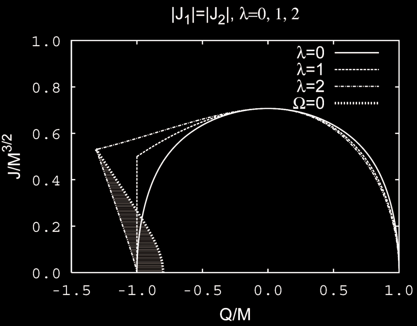
extremal $\lambda = 1$ EMCS black holes: Breckenridge, Myers, Peet, Vafa 1996

- mass saturates the bound: $M \geq \frac{\sqrt{3}}{2}|Q|$
- finite angular momenta: $|J| = |J_1| = |J_2|$
- angular momenta satisfy the bound: $|J| \leq \frac{1}{2}\left(\frac{\sqrt{3}}{2}|Q|\right)^{3/2}$
- horizon angular velocites vanish: $\Omega_i = 0,\ |J| \neq 0$
- angular momentum is stored in the Maxwell field
- negative fraction of the angular momentum is stored behind the horizon
- the effect of rotation is to deform the horizon into a squashed 3-sphere

$\lambda > 1$: Rotating $D = 5$ Black Holes

Kunz, Navarro-Lérida 2006





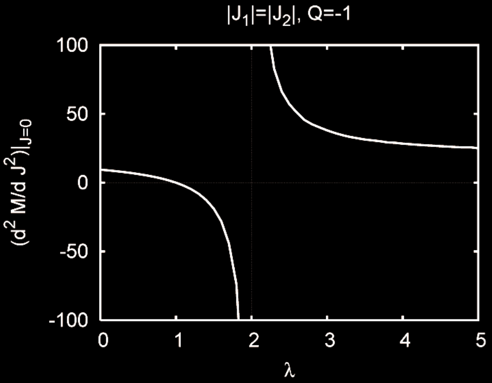
- black holes with $\Omega = 0,\, J \neq 0$
- black holes with $\Omega < 0,\, J > 0$

non-extremal

counter-rotating

Instability of $5D$ EMCS Black Holes

$\left.\frac{d^2 M}{dJ^2}\right|_{J=0}$ for extremal black holes

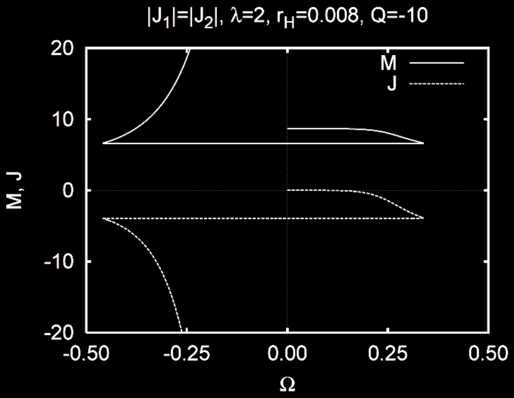


- instability beyond $\lambda = 1$

  supersymmetry marks the borderline between stability and instability
- $\lambda = 2$ is special

Non-Uniqueness of $5D$ EMCS Black Holes?

$\lambda = 2$ EMCS black holes

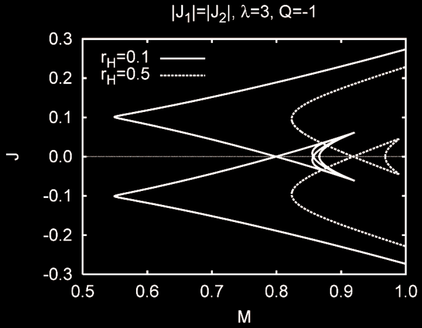


angular momentum and mass versus Ω

- $\lambda = 2$: set of extremal rotating $J = 0$ solutions appears to be present
- $\lambda = 2$: infinite set of extremal black holes with the same charges

Non-Uniqueness of $5D$ EMCS Black Holes

$\lambda > 2$ EMCS black holes

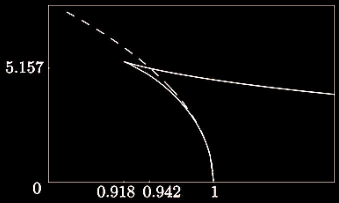


angular momentum versus mass

- black holes are not uniquely determined by M, J_i, Q

- non-uniqueness of $5D$ black holes with horizon topology of a sphere S^3
- non-uniqueness of $5D$ black holes and black rings ($S^1 \times S^2$)

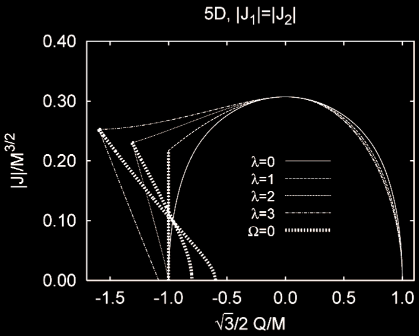
Emparan, Reall 2002



$$\frac{\mathcal{A}}{(GM)^{3/2}} \quad \text{versus} \quad \sqrt{\frac{27\pi}{32G}}\frac{J}{M^{3/2}}$$

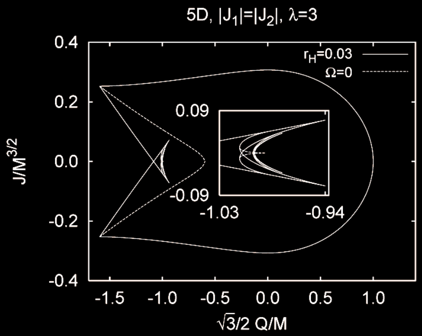
Domain of Existence of $5D$ EMCS Black Holes

$\lambda > 2$ EMCS black holes



domain of existence

- static extremal black hole is no longer on boundary

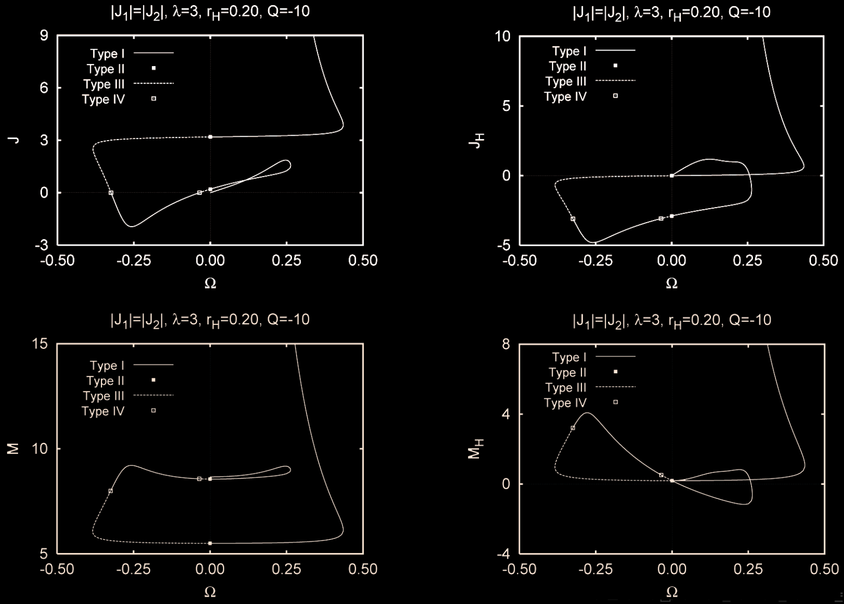


(almost) extremal black holes

- $J = 0, \Omega \neq 0$ (type 3)

  continuous set of black holes

Negative Horizon Mass of $5D$ EMCS Black Holes



Outline

Odd-D Einstein-Maxwell-Chern-Simons Theory

odd-D Einstein-Maxwell-Chern-Simons Lagrangian

$$L = \frac{1}{16\pi G_D}\Big\{R - \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \underbrace{\frac{8\tilde{\lambda}}{D+1}\epsilon^{\mu_1\mu_2\dots\mu_{D-2}\mu_{D-1}\mu_D} F_{\mu_1\mu_2}\dots F_{\mu_{D-2}\mu_{D-1}} A_{\mu_D}}_{\mathrm{Chern-Simons}}\Big\}$$

Chern-Simons coupling constant $\tilde{\lambda}$

$\tilde{\lambda} = 0$: Einstein-Maxwell theory

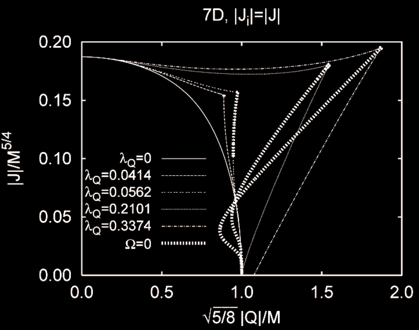
$\tilde{\lambda} \neq 0$: $\tilde{\lambda}$ dimensionful except for $D = 5$

scaling transformation: $D = 2N + 1$

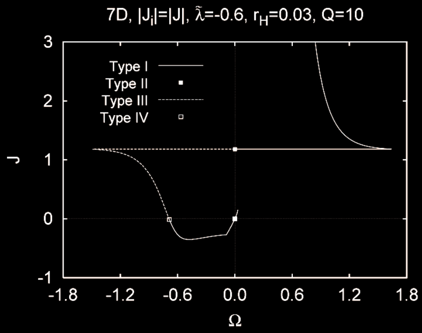
$$r_{\rm H} \to \gamma r_{\rm H}\ , \quad \Omega \to \Omega/\gamma\ , \quad \tilde{\lambda} \to \gamma^{N-2}\tilde{\lambda}\ , \quad Q \to \gamma^{D-3} Q\ , \dots$$

Rotating $D = 7$ EMCS Black Holes

Kunz, Navarro-Lérida 2006

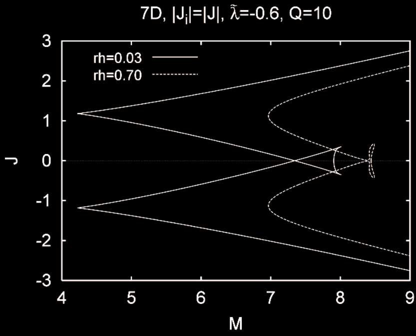


domain of existence

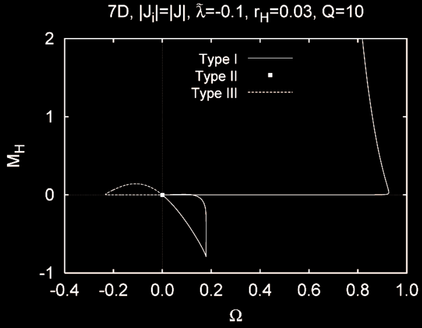


types of black holes

$\lambda > 1$: Rotating $D = 7$ Black Holes



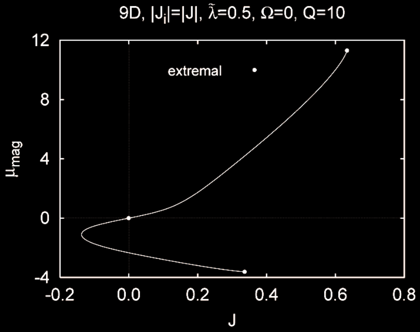
non-uniqueness



negative horizon mass

$\lambda > 1$: Rotating $D = 9$ EMCS Black Holes

Kunz, Navarro-Lérida 2006



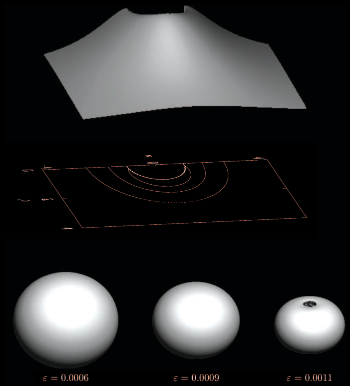
magnetic moment versus angular momentum

- non-static black holes with $J = 0$, $\Omega = 0$

Conclusions: Surprises with Rotating Black Holes

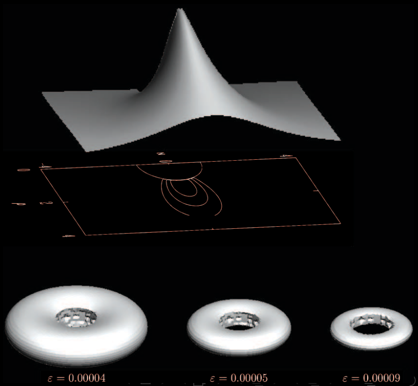
Einstein-Yang-Mills Black Holes

- rotating black holes carry hair
  no uniqueness theorem
- rotation induces electric charge
  no regular rotating solutions



EYM-dilaton Black Holes

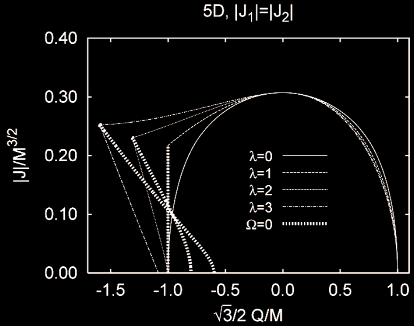
- rotating black holes with $Q = 0$
  dilaton charge D
- mass formula
  a uniqueness conjecture



Conclusions: Surprises with Rotating Black Holes

Einstein-Maxwell-Dilaton Black Holes

- $\Omega = 0$, $J > 0$ black holes
  stationary with static horizon
- $\Omega < 0$, $J > 0$ black holes
  counter-rotating black holes
- prolate horizon



$D = 5$ EM-Chern-Simons Black Holes

in addition: $\lambda \geq 2$

- $\Omega \neq 0$, $J = 0$ black holes
  rotating horizon, but vanishing J
- non-uniqueness of black holes
  with horizon topology S^3
- negative horizon mass

$D = 9$ EM-Chern-Simons Black Holes

in addition:

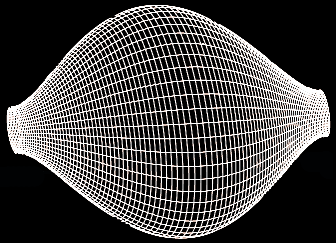
- $\Omega = 0$, $J = 0$ black holes
  stationary and non-static
- further surprises?

Outlook: Further Surprises?

higher dimensions:

- black holes

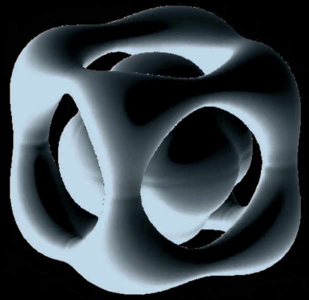
  different horizon topology?
- black strings



rotating non-uniform black strings

4 dimensions:

- platonic black holes?



- further surprises?