Black holes and ultra-compact objects in Einstein-scalar-Gauss-Bonnet theories

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Gravity Seminar, University of Vienna (2022)

Outline

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1. Introduction
2. Scalarized BHs
   - EdGB BHs
   - EsGB BHs
   - EsGB+R BHs
3. UCOs
   - Wormholes
   - Particle-like
4. Conclusions

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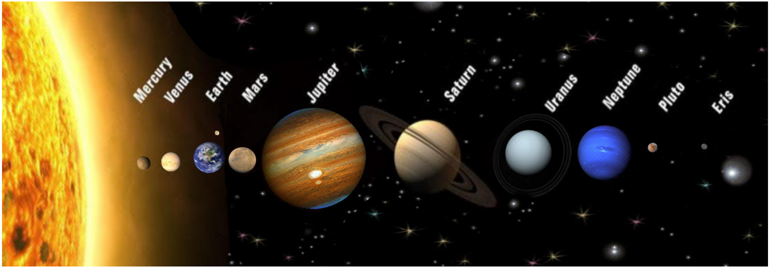
Introduction

Generalized Theories of Gravity



- Compatible with all solar system tests!
- Strong gravity?
  - Black holes
  - Neutron stars
  - Exotic compact objects
- Cosmology?

Einstein-scalar-Gauss-Bonnet Theories

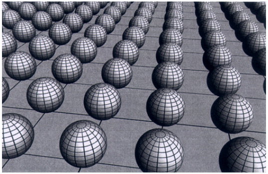
EsGB action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial_\mu \varphi)^2 + f(\varphi) R^2_{\rm GB} \right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

coupling function $f(\varphi)$



In 4 spacetime dimensions the coupling to the scalar is needed.

The resulting set of equations of motion are of second order (Horndeski).

Einstein-scalar-Gauss-Bonnet Theories

Gregory Horndeski, ‘Horndeski Scalar Theory, Past, Present and Future’

Einstein-scalar-Gauss-Bonnet Theories

generalized Einstein equations

$$\begin{aligned} G_{\mu\nu} &= -\frac{1}{4} g_{\mu\nu} \partial_\rho \varphi \partial^\rho \varphi + \frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi \\ &- \frac{1}{2} \left(g_{\rho\mu} g_{\lambda\nu} + g_{\lambda\mu} g_{\rho\nu} \right) \eta^{\kappa\lambda\alpha\beta} \tilde{R}^{\rho\gamma}{}_{\alpha\beta} \nabla_\gamma \partial_\kappa f(\varphi) \end{aligned}$$

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

crucial: choice of coupling function $f(\varphi)$

- GR black hole solutions do not remain solutions
  $\Longrightarrow$ only hairy black holes result
- GR black hole solutions do remain solutions
  $\Longrightarrow$ in addition spontaneously scalarized black holes emerge

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EdGB black holes

Kanti et al. hep-th/9511071, Torii et al. gr-qc/9606034

coupling function

$$f(\phi) = \frac{\alpha}{4} e^{-\gamma\phi}$$

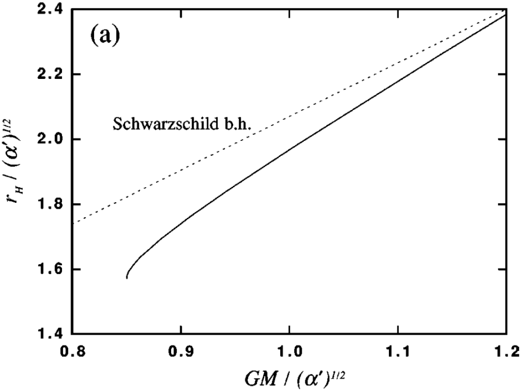
static black holes

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4} e^{2\phi_h}}$$

lower bound
on the horizon size
for fixed α'



lower bound on the mass

EdGB black holes

Kanti et al. hep-th/9511071, Antoniou et al. 1711.03390

coupling function

$$f(\phi) = \frac{\alpha}{4} e^{-\gamma\phi}$$

static black holes

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4} e^{2\phi_h}}$$

lower bound
on the horizon size
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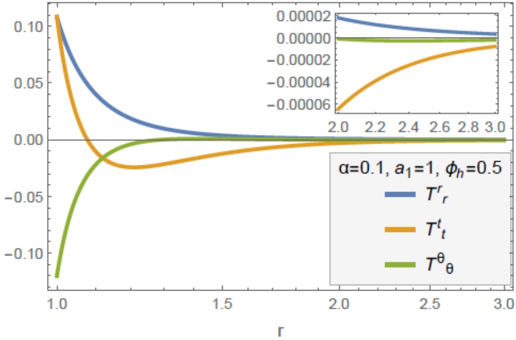
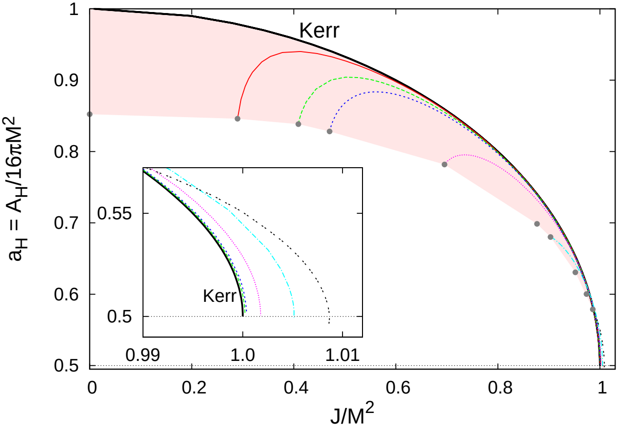


negative effective energy density

EdGB black holes

Kleihaus et al. 1101.2868

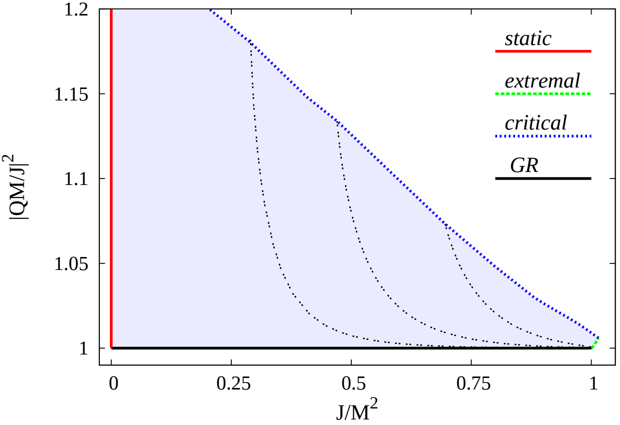
horizon area versus angular momentum



EdGB black holes

Kleihaus et al. arXiv:1101.2868

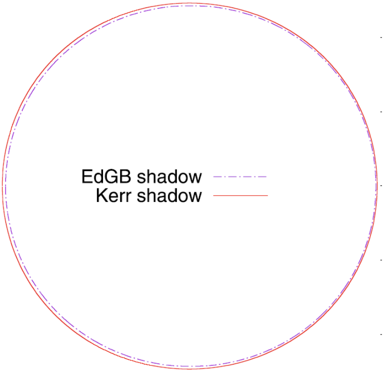
quadrupole moment versus angular momentum



EdGB black holes

Cunha et al. arXiv:1701.00079

shadow



$$\alpha/M^2 = 0.172, \ J/M^2 = 0.41$$

EdGB black holes

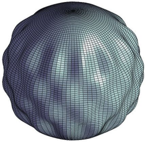
perturbation theory: damped oscillations

metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \varphi)$$

scalar

$$\phi = \phi_0(r) + \epsilon\delta\phi(t, r, \theta, \varphi)$$



polar modes: even-parity perturbations

axial modes: odd-parity perturbations (pure space-time modes)

master equation: Schrödinger-like equation

eigenvalue ω

$$\omega = \omega_R + i\omega_I$$

frequency: ω_R

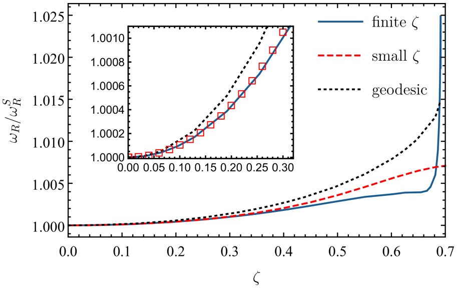
decay time: $\tau = 1/\omega_I$

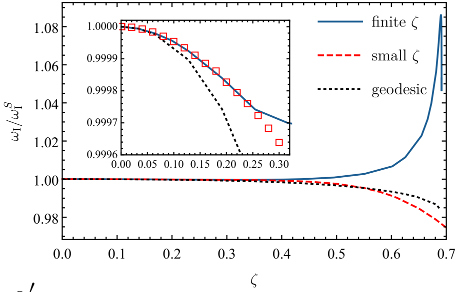
EdGB black holes

Blazquez-Salcedo et al. 1609.01286

quasi-normal mode (axial $l = 2$) versus coupling constant

normalized to the Schwarzschild values





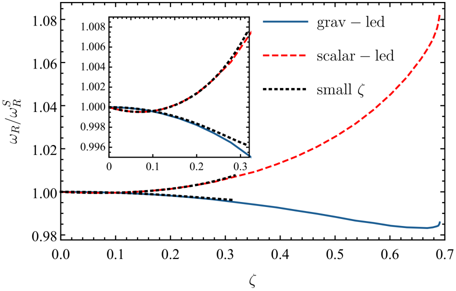
real part $\qquad \zeta = \dfrac{\alpha'}{M^2} \qquad$ imaginary part

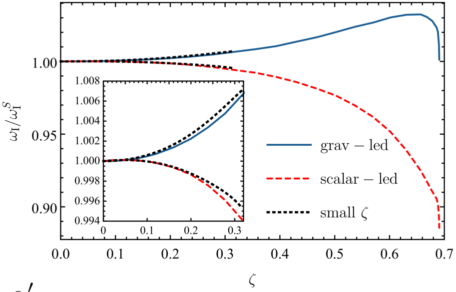
EdGB black holes

Blazquez-Salcedo et al. 1609.01286

quasi-normal mode (polar $l = 2$) versus coupling constant

normalized to the Schwarzschild values





real part $\qquad \zeta = \dfrac{\alpha'}{M^2} \qquad$ imaginary part

Outline

Scalar-Tensor Theories: Spontaneous Scalarization

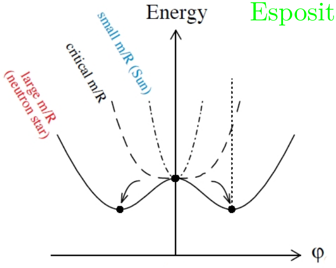
VOLUME 70, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1993

Nonperturbative Strong-Field Effects in Tensor-Scalar Theories of Gravitation

Thibault Damour
Institut des Hautes Etudes Scientifiques, 91440 Bures sur Yvette, France
and Département d'Astrophysique Relativiste et de Cosmologie, Observatoire de Paris,
Centre National de la Recherche Scientifique, 92195 Meudon, France

Gilles Esposito-Farèse
Centre de Physique Théorique, Centre National de la Recherche Scientifique, Luminy, Case 907,

Esposito-Farese (lecture)



matter induced "spontaneous scalarization"

Static curvature induced scalarized black holes

Doneva et al. 1711.01187, Silva et al. 1711.02080, Antoniou et al. 1711.03390

curvature induced scalarized black holes

Einstein equations

$$G_{\mu\nu} = T_{\mu\nu}$$

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

GR solutions remain solutions: $\varphi = 0, \ \frac{df(\varphi)}{d\varphi} = 0$

Gauss-Bonnet: Schwarzschild

$$R^2_{\rm GB} = \frac{48M^2}{r^6} > 0$$

tachyonic instability

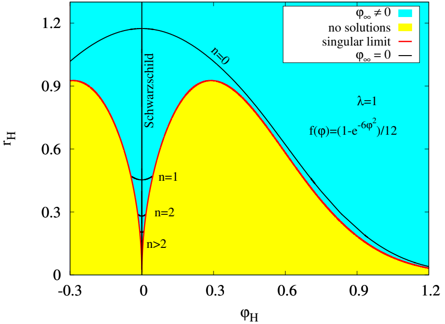
effective mass

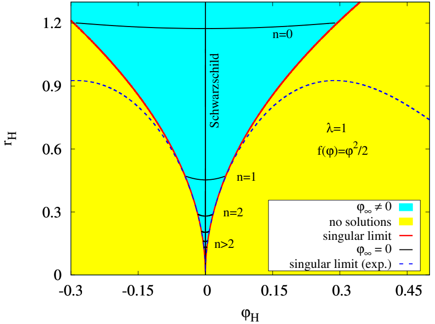
$$m^2_{\rm eff} = -\eta R^2_{\rm GB} < 0 \ , \quad \text{if } \eta > 0$$

Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755

domain of existence of spontaneously scalarized static black holes



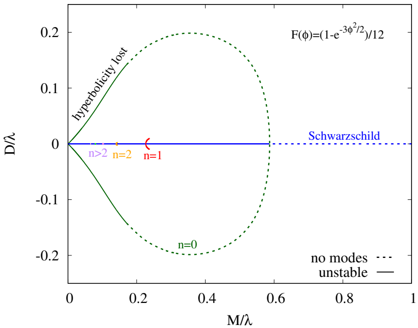


$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \qquad f(\varphi) = \frac{\lambda^2}{2}\varphi^2$$

spontaneously scalarized black holes, $\varphi_\infty \neq 0$, radicand negative

Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755



solutions

Schwarzschild blue

scalarized $n = 0$ dark green

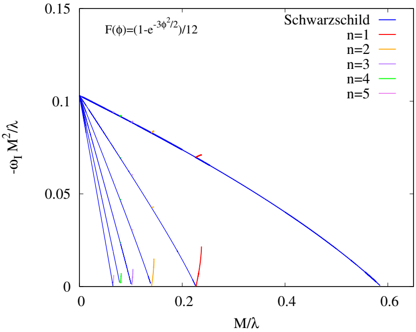
scalarized $n > 0$ ...

unstable radial modes

Schwarzschild blue

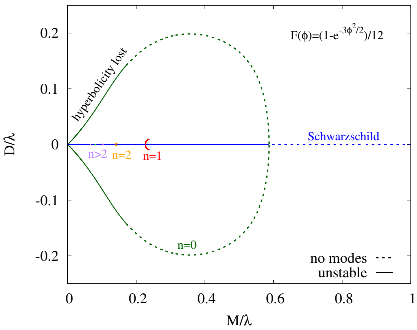
scalarized $n = 0$ –

scalarized $n > 0$ ...



Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755



solutions

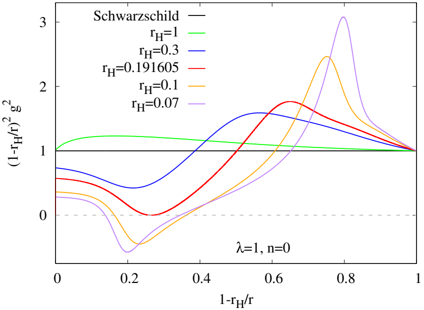
Schwarzschild blue

scalarized $n = 0$ dark green

scalarized $n > 0$ ...

scalar equation

$$g^2(r)\ddot{\varphi}_1 - \varphi_1'' + C_1(r)\varphi_1' + U(r)\varphi_1 = 0$$

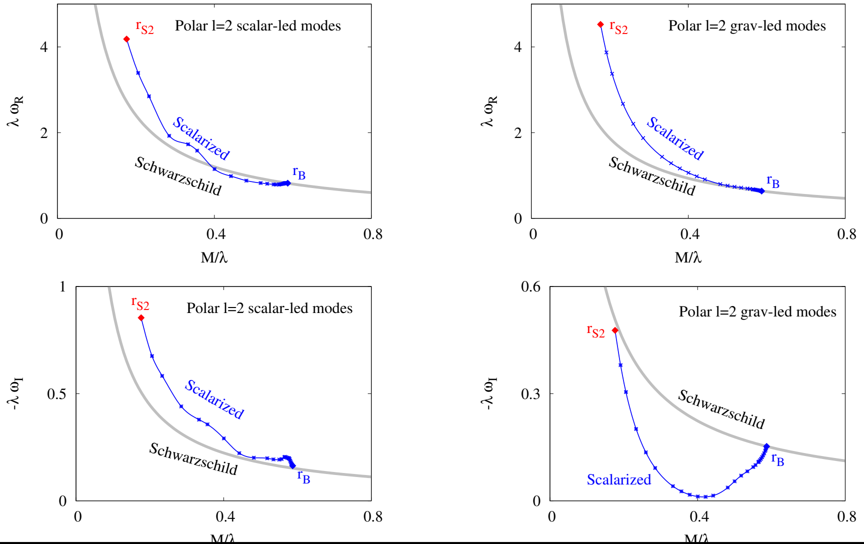


$(1 - \frac{r_H}{r})^2 g^2$ vs $1 - r_H/r$

lost hyperbolicity

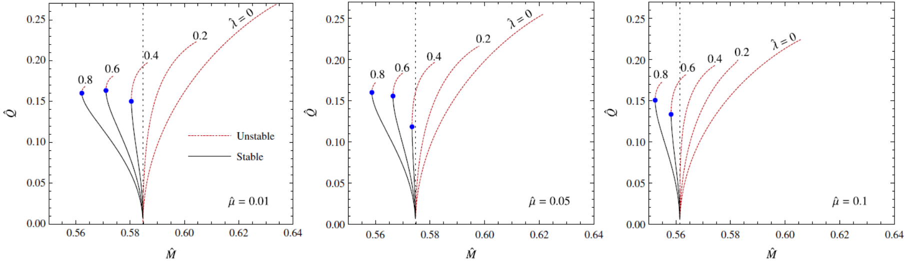
Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 2006.06006



Static curvature induced scalarized black holes

Macedo et al. arXiv:1903.06784



quadratic coupling function

scalar field potential

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{8}\lambda\varphi^4$$

radial stability: small mass, large self-interaction

Rotating curvature induced scalarized black holes

Cunha et al. 1904.09997, Collodel et al. 1912.05382, Dima et al. 2006.03095

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

Gauss-Bonnet: Kerr

$$R^2_{\rm GB} = \frac{48M^2}{(r^2+\chi^2)^6}\left(r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6\right) \ , \quad \chi = a\cos\theta$$

effective mass

$$m^2_{\rm eff}(r) = -\eta R^2_{\rm GB} < 0$$

- $\eta > 0$

  $\Longrightarrow$ spin suppresses scalarization

- $\eta < 0$

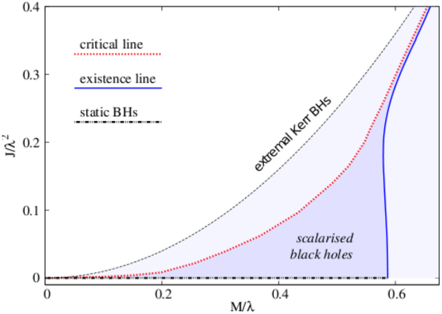
  $\Longrightarrow$ spin induces scalarization

Rotating curvature induced scalarized black holes

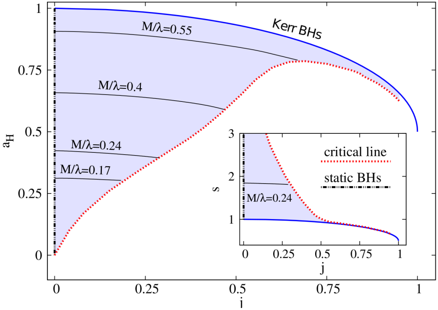
Cunha et al. arXiv:1904.09997

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) , \quad \eta > 0 , \quad V(\varphi) = 0$$



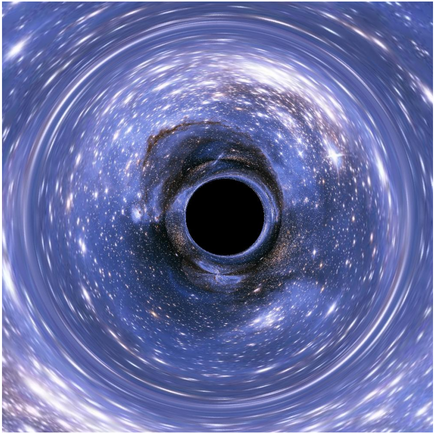
angular momentum vs mass



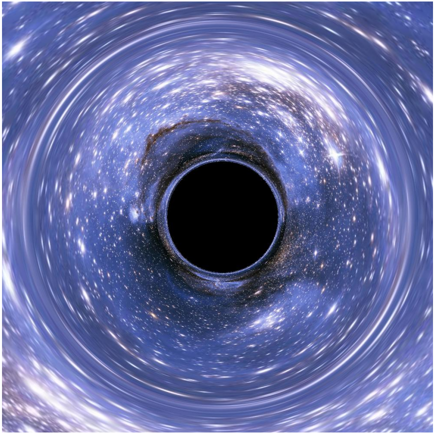
area/entropy vs angular momentum

Rotating curvature induced scalarized black holes

Cunha et al. arXiv:1904.09997



EsGB



Kerr

$$M/\lambda = 0.237 (j = 0.24)$$

Rotating spin induced scalarized black holes

Dima et al. arXiv:2006.03095

$|\phi| \sim \exp(t/\tau)$

coupling function

$$f(\varphi) = -\eta\,\varphi^2$$

$$V(\varphi) = 0$$

onset of scalarization

even scalar field

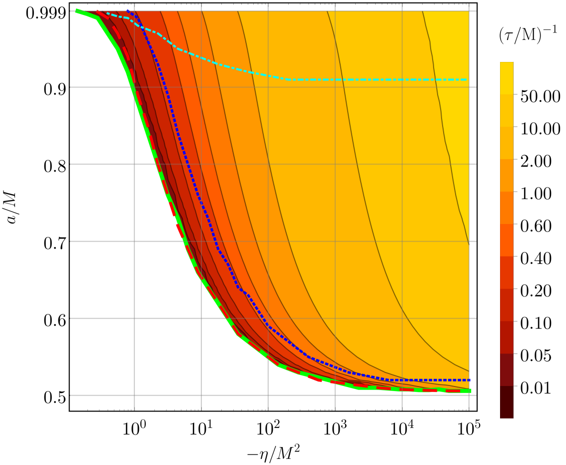
$$\varphi(\pi - \theta) = +\varphi(\theta)$$

odd scalar field

$$\varphi(\pi - \theta) = -\varphi(\theta)$$

range

$$0.5 \leq j \leq 1$$

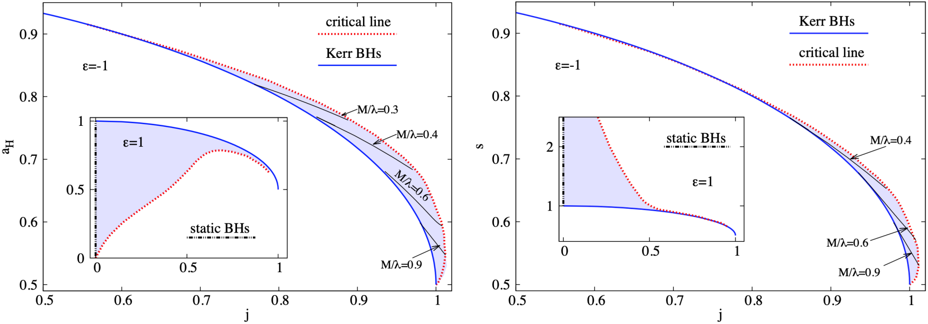


Rotating spin induced scalarized black holes

Herdeiro et al. arXiv:2009.03904, Berti et al. arXiv:2009.03905

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \ , \quad \eta < 0 \ , \quad V(\varphi) = 0$$



area vs angular momentum

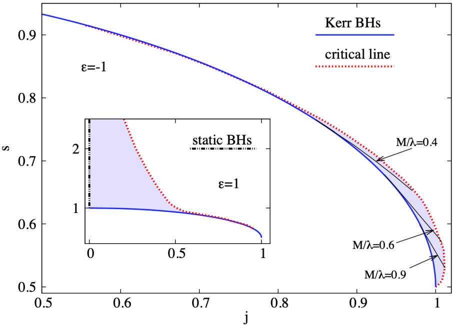
entropy vs angular momentum

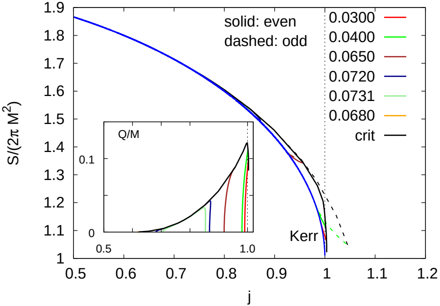
even scalar field

Rotating spin induced scalarized black holes

Herdeiro et al. arXiv:2009.03904

Berti et al. arXiv:2009.03905





entropy vs angular momentum

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$$

entropy vs angular momentum

$$f(\varphi) = \frac{\lambda^2}{8}\varphi^2$$

Outline

Einstein-scalar-Gauss-Bonnet with Ricci coupling

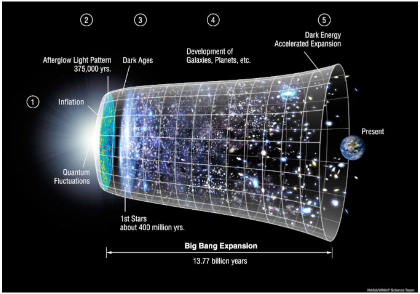
Antoniou et al. 2004.14985

Compact object scalarization with general relativity as a cosmic attractor

EsGB with Ricci action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{\varphi^2}{2}\left(\alpha R^2_{\mathrm{GB}} - \frac{\beta}{2}R\right)\right]$$

coupling function $f(\varphi) = \frac{\varphi^2}{2}$



Einstein-scalar-Gauss-Bonnet with Ricci coupling

Antoniou et al. 2004.14985

scalar equation in a cosmological background with Hubble function H

$$\ddot{\varphi} + 3H\dot{\varphi} + m_{\text{eff}}^2 \varphi = 0$$

effective mass

$$m_{\text{eff}}^2 = \frac{\beta}{2} R - \alpha R_{\text{GB}}^2$$

Ricci scalar and Gauss-Bonnet term

$$R = 6\left(2H^2 + \dot{H}\right)$$

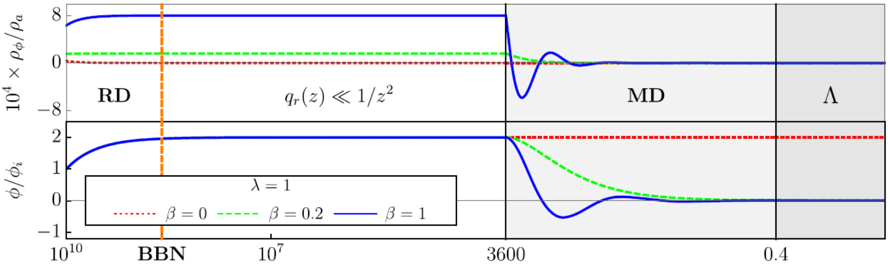
$$R_{\text{GB}}^2 = 24H^2\left(H^2 + \dot{H}\right)$$

for spontaneously scalarized black holes $\varphi(\infty) = 0$ needed

general relativistic black hole solutions should remain solutions

Einstein-scalar-Gauss-Bonnet with Ricci coupling

Antoniou et al. 2004.14985



top: energy density ratio of scalar ρ_φ and cosmic fluid ρ_a vs redshift z

bottom: evolution of scalar field φ in units of its initial value φ_i

RD: radiation dominated, MD: matter dominated, Λ: Λ dominated
BBN: big bang nucleosysthesis
Figure with $\bar{\beta}$: $2\bar{\beta} = \beta$

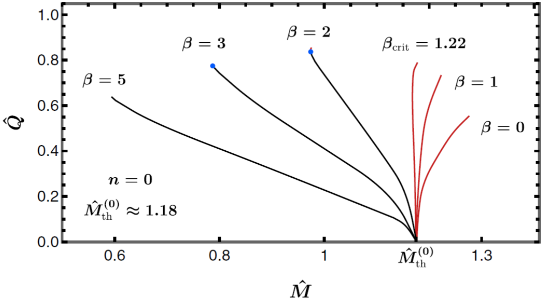
Static curvature induced scalarized black holes

Antoniou et al. 2105.04479

domain of existence of spontaneously scalarized static black holes

$$m_{\rm eff}^2 = \frac{\beta}{2} R - \alpha R_{\rm GB}^2 < 0$$

tachyonic instability: independent of β $(R = 0)$



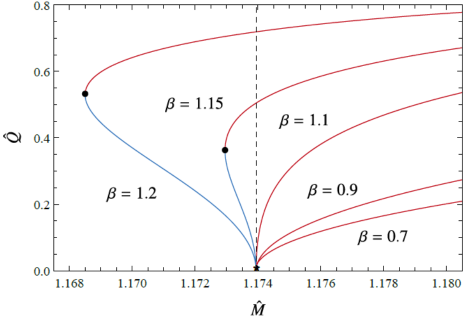
scaled scalar charge vs scaled mass for varying Ricci coupling β

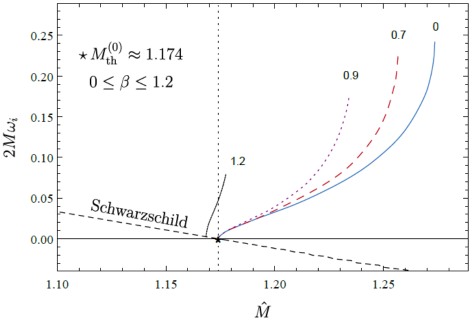
endpoint: onset of instability

Static curvature induced scalarized black holes

Antoniou et al. 2204.01684

stability of Schwarzschild and spontaneously scalarized static black holes





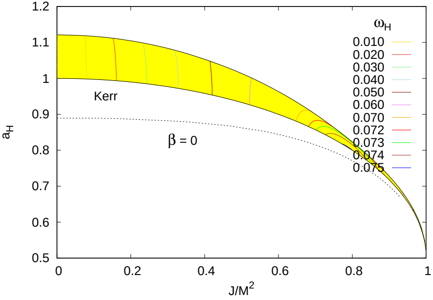
charge vs mass radial mode vs mass

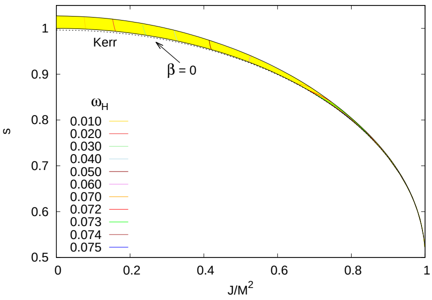
- Schwarzschild black holes unstable vor $\hat{M} < 1.174$
- scalarized black holes always unstable for $\beta = 0, 0.7, 0.9$
- scalarized black holes in part radially stable for $\beta = 1.2$

Rotating curvature induced scalarized black holes

preliminary results

Ricci coupling $\beta = 5$





angular momentum vs mass area/entropy vs angular momentum

scalarized black holes are entropically preferred

Outline

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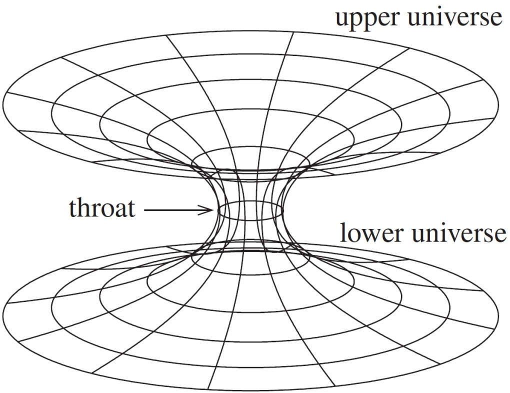
Wormholes



embedding diagram

- 2 asymptotically flat regions
- sphere of minimal surface/radius
- no horizon
- no singularity

violation of the energy conditions

Static EdGB Wormholes

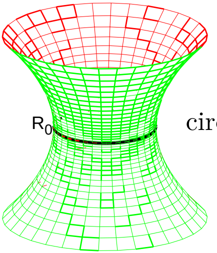
Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091

static spherically symmetric wormholes

$$ds^2 = -e^{F_0} dt^2 + e^{F_1} \left[d\eta^2 + h \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]$$

$$h = \eta^2 + \eta_0^2$$

$$-\infty < \eta < \infty$$



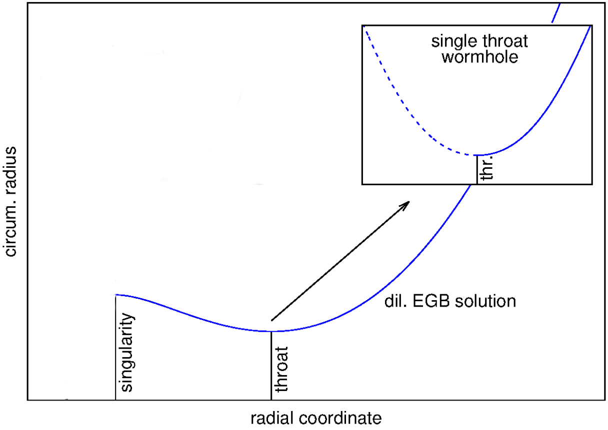
circumferential radius

$$R = \sqrt{e^{F_1} h}$$

embedding of the throat of the wormhole

Static EdGB Wormholes

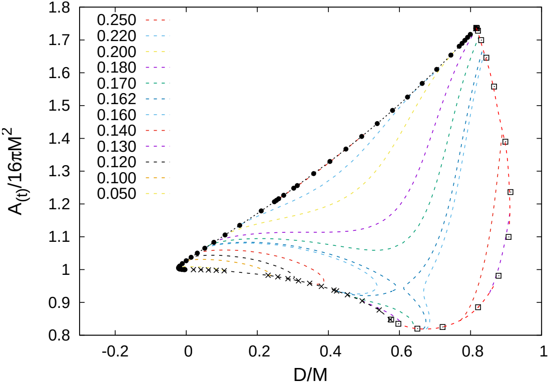
Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091



junction conditions: thin shell of ordinary matter needed

Static EdGB Wormholes

Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091



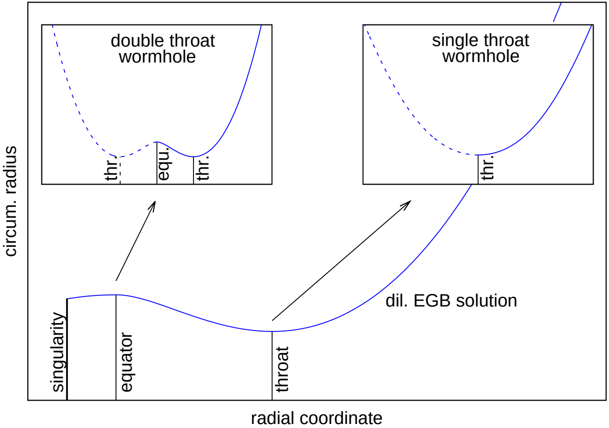
throat area vs dilaton charge

domain of existence

- dashed lines
  $\bar{\alpha} = \frac{\alpha}{\eta_0^2} = \text{const}$
- lower boundary:
  black hole
- right boundary:
  singularity
- left boundary:
  $R' = 0, \; R'' = 0$

Double Throat EdGB Wormholes

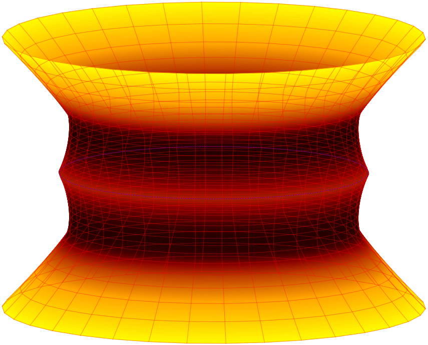
Antoniou et al. 1904.13091



junction conditions: thin shell of ordinary matter needed

Double Throat EdGB Wormholes

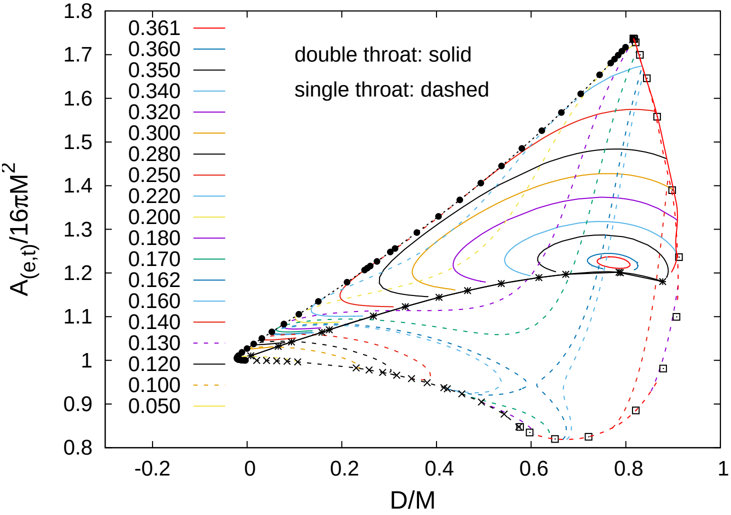
Antoniou et al. 1904.13091



embedding of double throat wormhole

Double Throat EdGB Wormholes

Antoniou et al. 1904.13091



- stars
  singularity
  equator
- squares
  cusp
  singularity
  elsewhere

Geodesics of EdGB Wormholes

Kanti et al. 1108.3003, 1111.4049

- geodesics from Lagrangian

$$\mathcal{L} = \frac{1}{2} e^{-2\beta\phi} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

$\beta = const.$ (= 1/2 for heterotic string theory)

- conjugate momenta $p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$

$$p_t = -e^{-2\beta\phi} e^{F_0} \dot{t} \ , \quad p_\varphi = e^{-2\beta\phi} e^{F_1} (\eta_0^2 + \eta^2) \dot{\varphi}$$

$$p_\eta = e^{-2\beta\phi} e^{F_1} \dot{\eta}$$

- first integrals

$$p_t = const. = -E \ , \quad p_\varphi = const. = L$$

- time-like geodesics

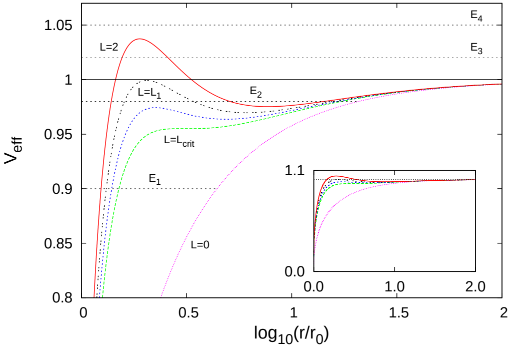
$$2\mathcal{L} = -e^{2\beta\phi} e^{-F_0} E^2 + e^{-2\beta\phi} e^{F_1} \dot{\eta}^2 + e^{2\beta\phi} e^{-F_1} \frac{L^2}{\eta_0^2 + \eta^2} = -1$$

Geodesics of EdGB Wormholes

Kanti et al. 1108.3003, 1111.4049

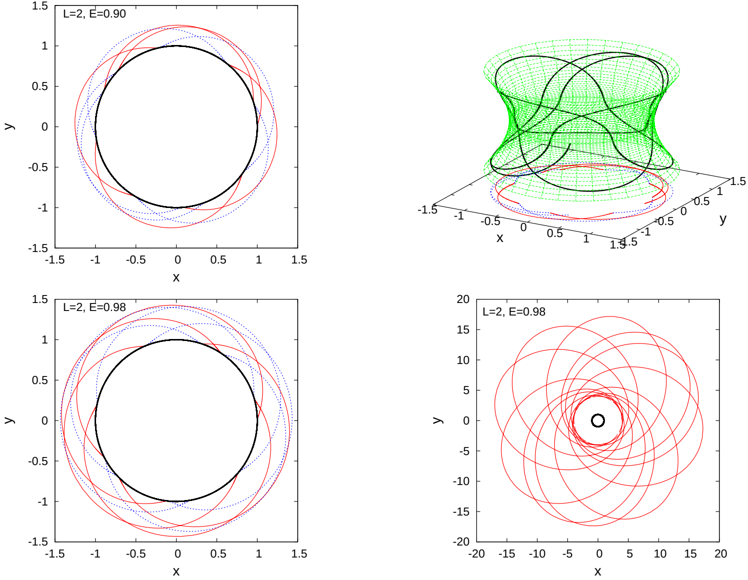
- radial equation: $\dot{\eta}^2 = e^{4\beta\phi} e^{-F_0 - F_1} \left[E^2 - V_{\rm eff}^2(\eta, L)\right]$

- effective potential: $V_{\rm eff}^2(\eta, L) = e^{F_0}\left(e^{-2\beta\phi} + e^{-F_1}\dfrac{L^2}{\eta_0^2 + \eta^2}\right)$



- $E^2 \geq V_{\rm eff}^2(\eta, L)$
- turning points η_i:
  $E^2 - V_{\rm eff}^2(\eta_i, L) = 0$
- no horizon
- bound orbits:
  motion around the throat
  motion across the throat

Geodesics of Dilatonic EGB Wormholes



Traversable EdGB Wormholes?

Kanti et al. 1108.3003, 1111.4049

acceleration of a traveler at the throat?

- string theory
  $\alpha \sim \ell_P^2 \Longrightarrow r_0 \sim 10\,\ell_P$
  acceleration $(10^{51} - 10^{52})\, g_\oplus$
  $g_\oplus$: acceleration of gravity at the surface of the earth

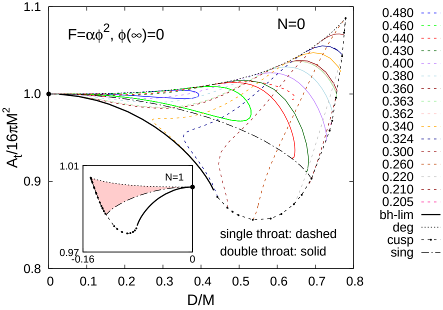
- acceleration on the order of $g_\oplus$:
  throat radius $(10 - 100)$ light-years

Cuyubamba et al. 1804.11170

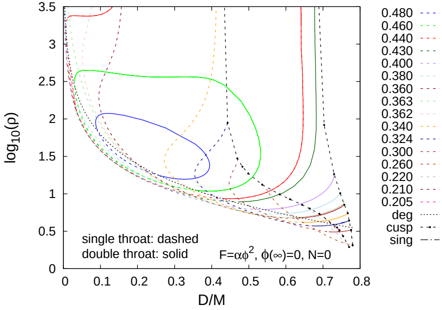
EsGB Wormholes

Antoniou et al. 1904.13091

no spontaneous scalarization: no GR wormholes



domain of existence



matter at throat

$$F(\varphi) = \alpha\varphi^2 \ , \quad \varphi(\infty) = 0$$

Outline

EsGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

metric

$$ds^2 = -e^{f_0} dt^2 + e^{f_1} \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]$$

coupling $F(\phi) = \alpha\phi^n$, $n \geq 2$

expansion at origin

$$\begin{aligned} f_0 &= f_{0c} + \frac{e^{f_{1c}}}{64\alpha} r^2 + \frac{e^{f_{1c}}\phi_c}{96\alpha c_0} r^3 + \mathcal{O}(r^4) \\ f_1 &= f_{1c} + \frac{e^{f_{1c}}}{64\alpha} r^2 + \frac{\nu_3}{6} r^3 + \mathcal{O}(r^4) \\ \phi &= -\frac{c_0}{r} + \phi_c - \frac{e^{f_{1c}} c_0}{256\alpha} r + \frac{32\alpha c_0 \nu_3 - e^{f_{1c}}\phi_c}{768\alpha} r^2 + \mathcal{O}(r^3) \end{aligned}$$

f_{0c}, f_{1c}, ν_3, ϕ_c, and c_0 are constants

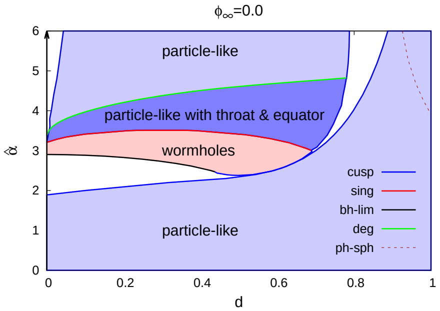
stress-energy tensor $(n = 2)$

$$T_t^t(0) = \frac{3}{32\alpha} \ , \quad T_r^r(0) = T_\theta^\theta(0) = T_\varphi^\varphi(0) = \frac{2}{32\alpha}$$

EsGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

globally regular metric, regular stress-energy tensor, diverging scalar (origin)

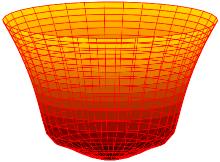


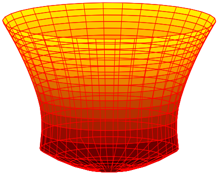
domain of existence: $\hat{\alpha} = \dfrac{\alpha}{M^2}$ vs $d = \dfrac{D}{M}$

$$F(\varphi) = \alpha\varphi^2 \ , \quad \varphi(\infty) = 0$$

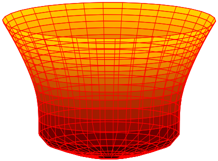
EsGB Particle-like Solutions

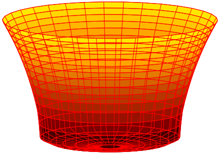
Kanti et al. 1910.02121, 2005.07650





$$F(\varphi) = \alpha\varphi^2$$



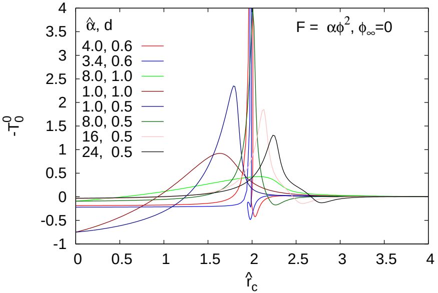


embeddings

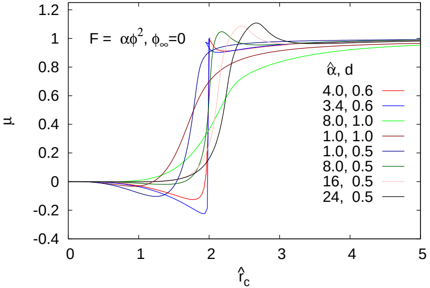
EsGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

ECOs and UCOs



energy density $\rho = -T_0^0$



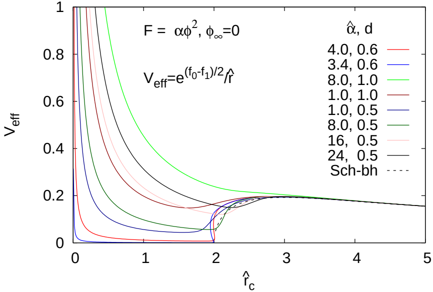
mass function $\mu(\hat{r}_c)$

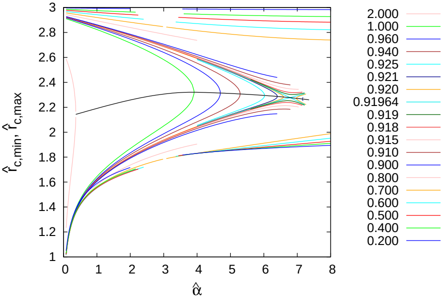
vs circumferential radius $\hat{r}_c = r_c/M$

EsGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

UCOs: pairs of light-rings





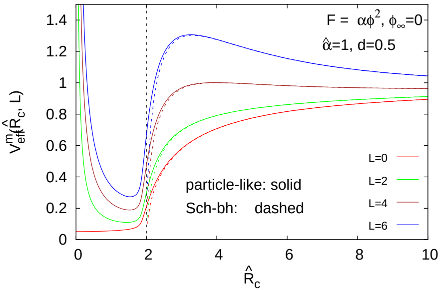
photon effective potential V_{eff} local extrema $\hat{r}_{\text{c,max}}$ and $\hat{r}_{\text{c,min}}$

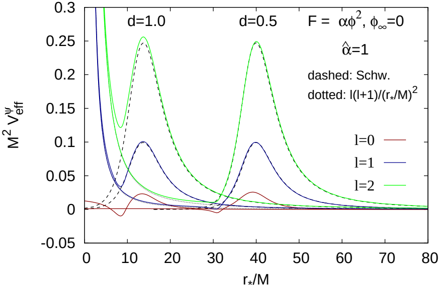
vs circumferential radius $\hat{r}_{\text{c}} = r_c/M$

Cardoso et al. 1406.5510 , Cunha et al. 1708.04211

EsGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650





effective potential $V_{\rm eff}^m$

(massive particles)

vs circumferential radius $\hat{R}_c = R_c/M$

effective potential $V_{\rm eff}^\psi$

(test scalar particle)

vs tortoise coordinate r^*/M

Cardoso et al. 1608.08637 echoes of ECOs

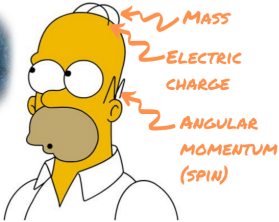
Outline

Conclusions

Beyond General Relativity: Scalar EGB Theories

black holes

- dilatonic
- spontaneously scalarized
  - curvature induced
  - spin induced



wormholes

- static
  - single throat
  - double throat
- geodesics
- rotating?

particle-like solutions

- regular metric, regular $T_{\mu\nu}$
- UCOs with pairs of light-rings
- echoes of ECOs
- rotating?

THANKS