Quantum annealing and the random field Ising model

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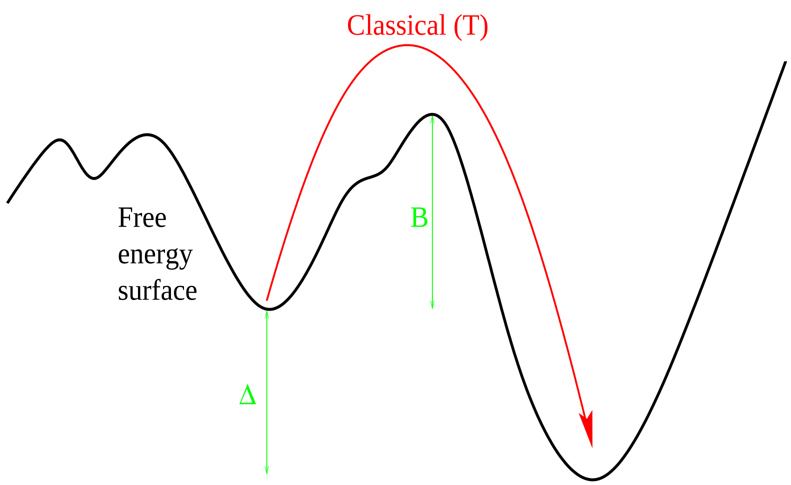
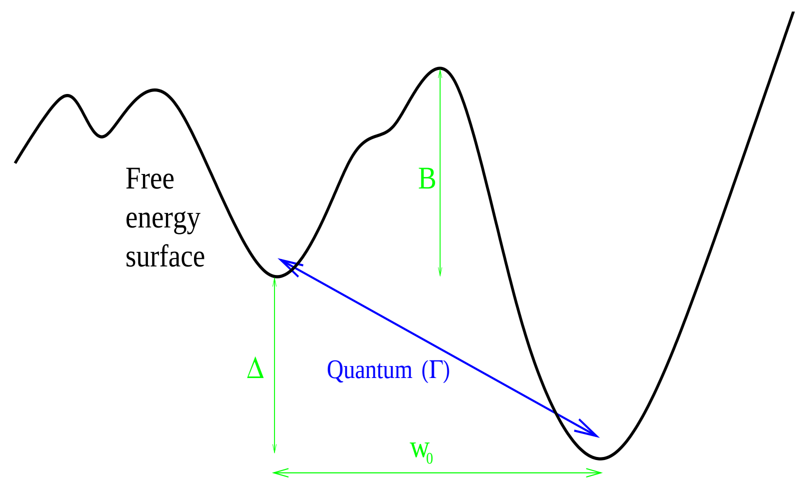
Quantum and classical annealing

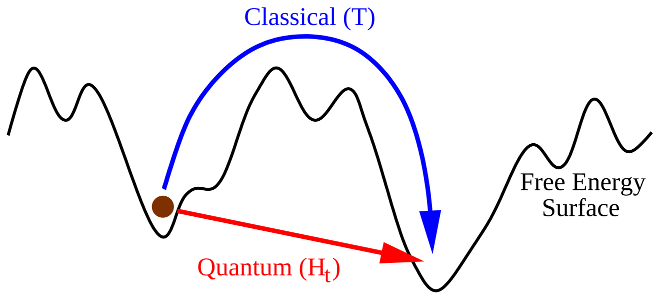


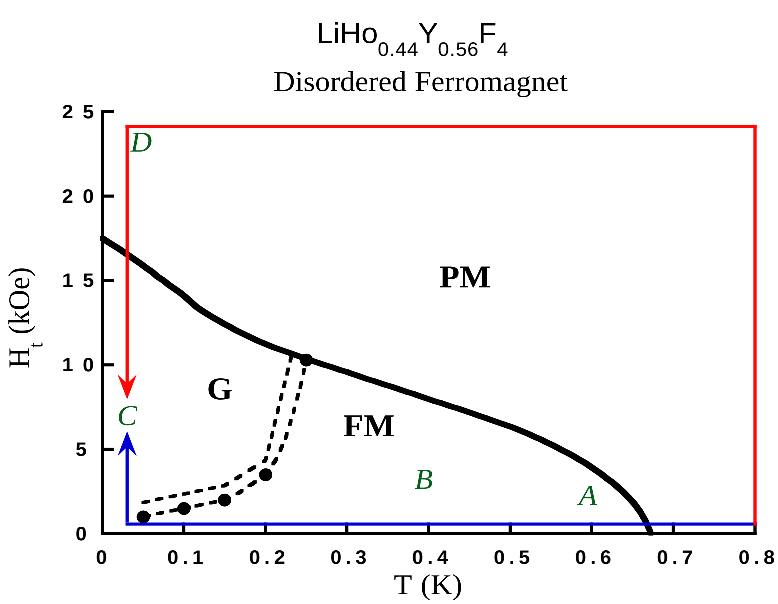
Classical energy landscape: use e.g. Simulated Annealing (lower the temperature).



Quantum version: modify a quantum parameter.

On Quantum Annealing



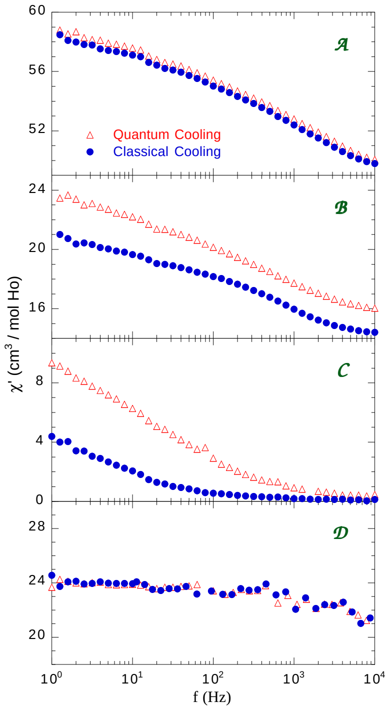


- Quantum annealing: system moves in the energy landscape by quantum tunneling.
- Small temperature, a finite quantum parameter Γ. Take Γ or ϵ slowly towards zero $(H = H_0 + \epsilon H_1)$.
- QM picture: induced (Landau-Zener) level crossings, follow lowest energy state.

Basic questions:

1. A quantum system $\to$ a classical one ($\Gamma = 0$).
2. "quantum computing" - see Brooke et al. Science/Nature.
3. how does this work vs. classical dynamics? In particular, if one wants to "optimize".
4. this is what we study here!

Experimental case-study



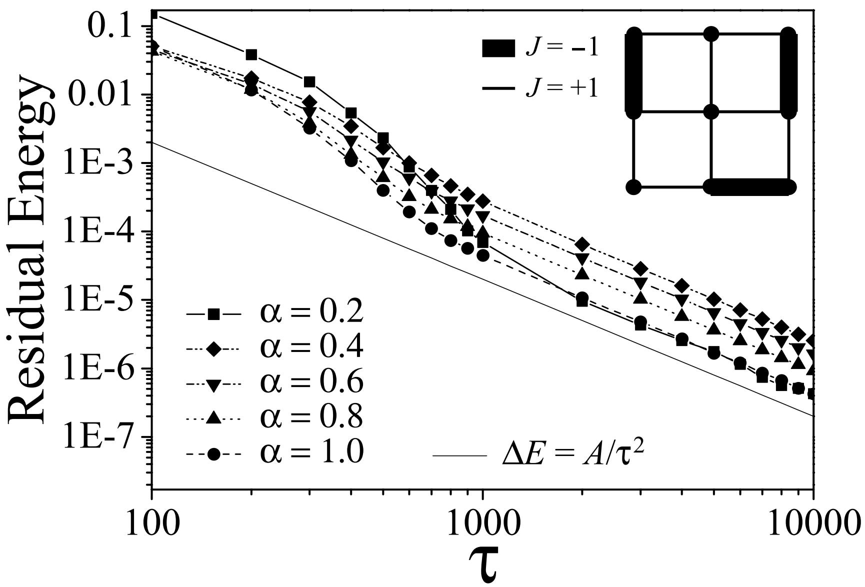
$H = \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z - \sum_i \Gamma \sigma_i^x$

σ Pauli spin matrices.

$LiHoF_4$:

tetragonal insulating FM.

Simple disordered Ising



(From cond-mat/0502....)

Residual energy:

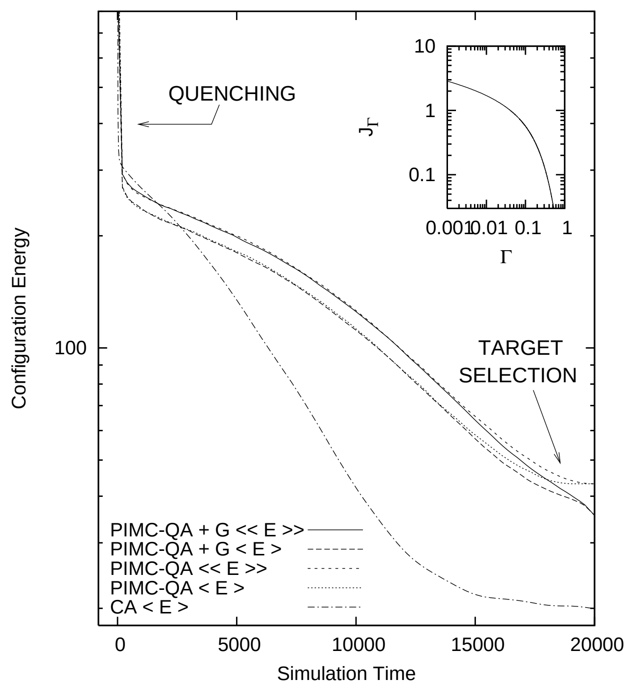
$\epsilon_{res} = (E_s - E_{gs})/N$,

actual vs. groundstate energy.

We want to optimize!

Here, ϵ_{res} polynomial?

Satisfiability problem(s)

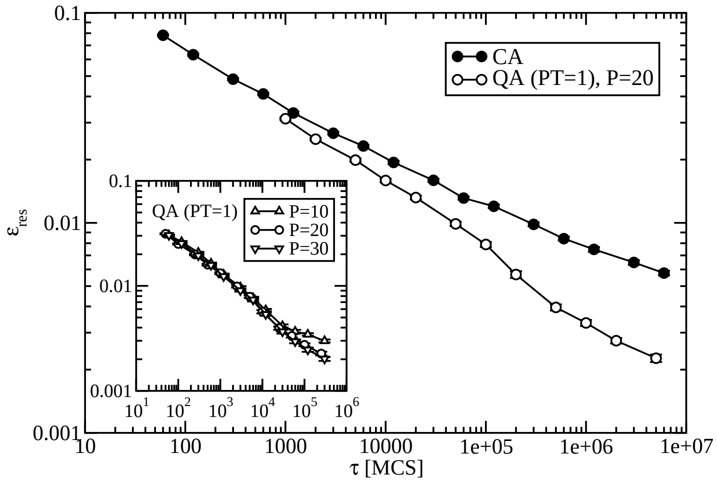


Santoro et al., cond-mat....

Apply quantum annealing to a NP-complete, K-SAT(isfiability) problem.

But: Classical Annealing better.

QA and a spin glass



Santoro et al. (Science 2002).
QA faster than CA
(simulated annealing).
Scaling theory: $\epsilon_{res} \sim \log N_{MC}{}^{-\zeta}$.
For QA: $2 \leq \zeta \leq 6$,
based on a Landau-Zener picture.

Random field Ising model

The Hamiltonian : $H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z$

h_i are the random fields at site i, a (Gaussian) probability distribution $P(h_i)$ w/ standard deviation δ_h.

The GS computed via the mapping to the max-flow/min-cut problem.

Why the RFIM?

- It is exactly solvable (and easily).
- It has a non-trivial energy landscape.
- RFIM: paramagnetic in 1D, 2D, phase transition in 3D. Overlap of GS with thermal state non-zero (q),
- Poorly known when quantum annealing is superior to classical. No real theory.

RFIM & QM

- QM effects can be tuned by varying a transverse field Γ.

  The Hamiltonian reads:

$$\mathcal{H} = J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z - \Gamma \sum_i \sigma_i^x ,$$

  $\Gamma \sum_i \sigma_i^x$ stands for the quantum effects.

- QA: decrease Γ from an initial value towards zero. $\Gamma(t)$? Usually, we use a logarithmical cooling schedule.
- $T > 0$: a d-dimensional quantum lattice Hamiltonian maps to a $d+1$-dimensional classical (Suzuki-Trotter).

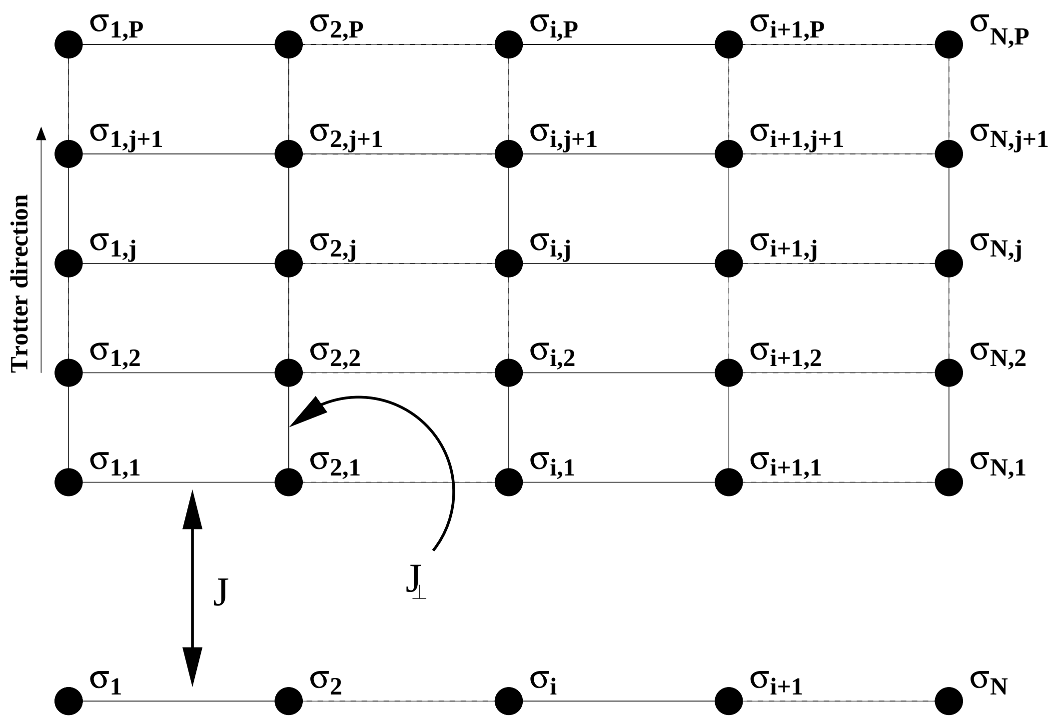
Suzuki-Trotter

$$\mathcal{H}_{eff} = -\sum_{k=1}^{P}(J\sum_{\langle ij\rangle}\sigma_i^k\sigma_j^k - \sum_i h_i\sigma_i^k + J_\perp\sum_i \sigma_i^k\sigma_i^{k+1})$$

P is the number of Trotter slices, $J_\perp = -\frac{T_{eff}}{2}\ln(\tanh\frac{\Gamma}{T_{eff}})$ is the coupling between the spins in Trotter direction.

H_{eff} simulated at the effective temperature $T_{eff} = PT$.

Suzuki-Trotter II

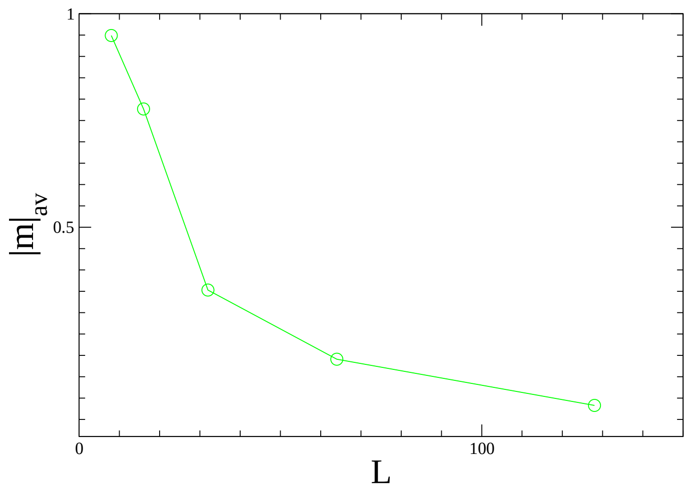


An example of the mapping.

Typical run (1D)

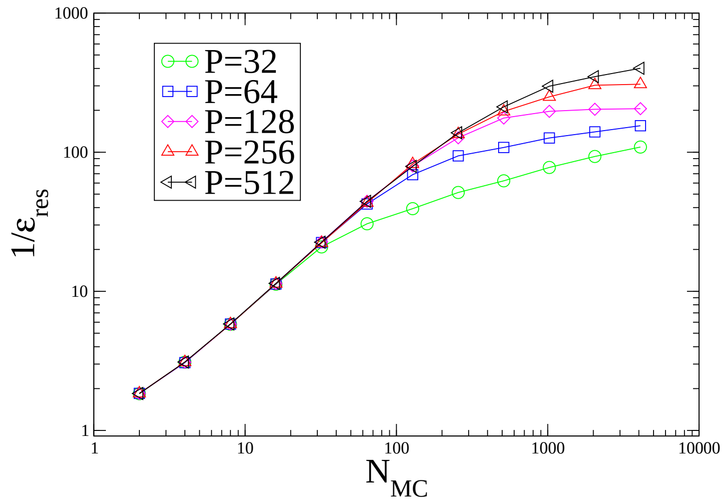
Spin orientations vs. GS at 3 times

More details (1D)



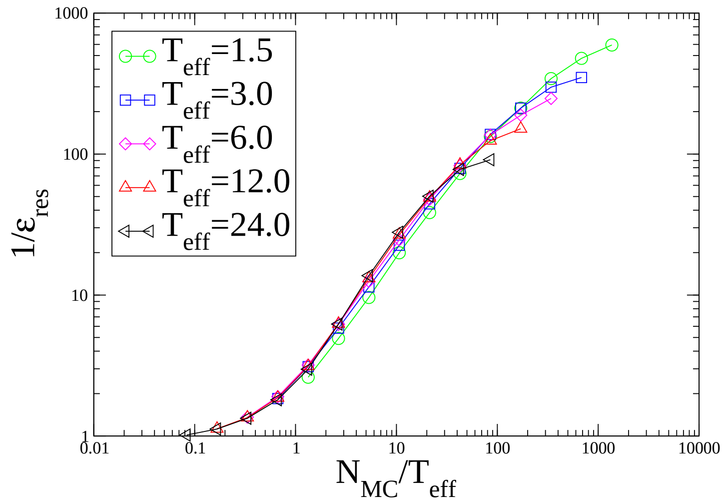
An example of the decay of the magnetization. Recall: 1D paramagnetic.

Adding slices



More slices: better optimization. Recall that the size of the system becomes P-fold.

Effective temperature



Again, a smaller T_{eff} results in a better ϵ_{res} but more slowly.

Paramagnetic phase

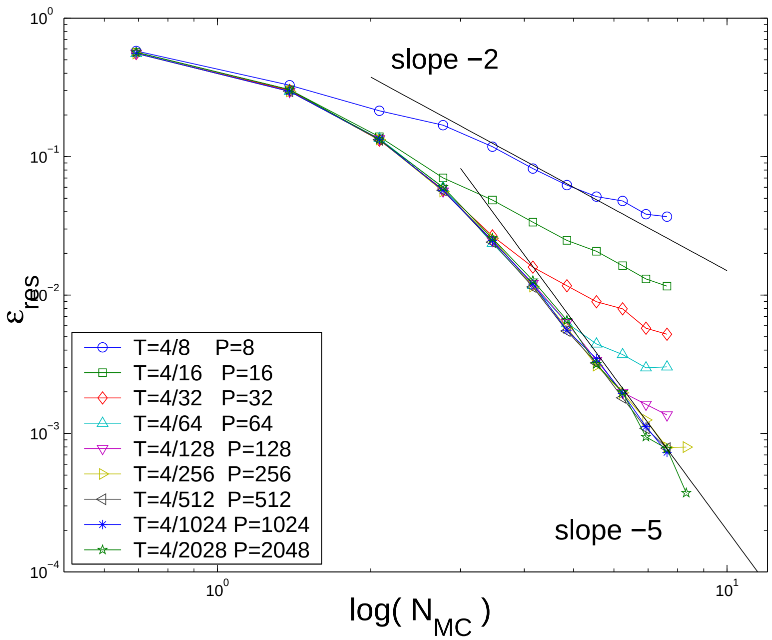
- Huse and Fisher [Phys. Rev. Lett. **57**, 2205 (1986)]: arbitrary two-level systems in the case of simulated annealing, with an energy barrier between the states. Their result:

  $\epsilon_{res} \sim \log(N_{MC})^{-\zeta}$

  where $\zeta = 2$.

- Santoro *et al.* [Science **295**, 2427 (2002)] again: 2D SG ($T = 0$-phase transition). $\zeta > 2$?

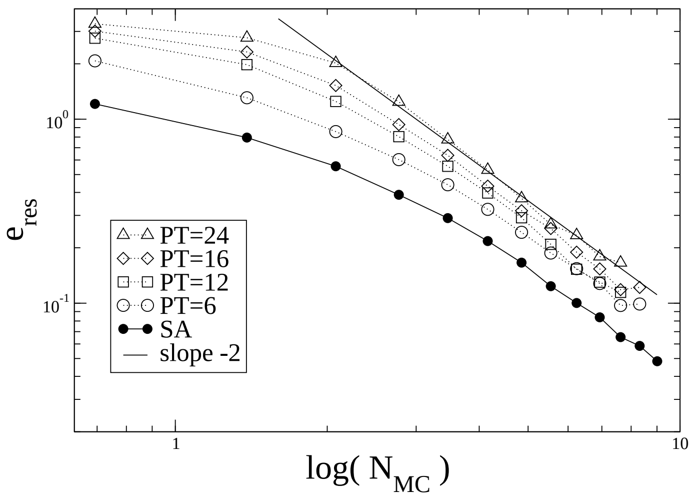
General picture (1D)



What a surprise: more slices (P) and a smaller temperature work!

- The essential behavior in the paramagnetic phase.
- Idea: use quantum fluctuations ($P \sim \sqrt{N_{MC}}$).
- "things go wrong" - SA result recovered $\zeta = 2$.
- Things go well: $\zeta \to 6$?.

2D case



In 2D, the RFIM "pseudo-FM" for L small.

Thus, a cross-over expected.

3D considerations

- Now, a phase transition ($T(\delta h)$) - PM/FM.
- Huse and Fisher: in FM phase

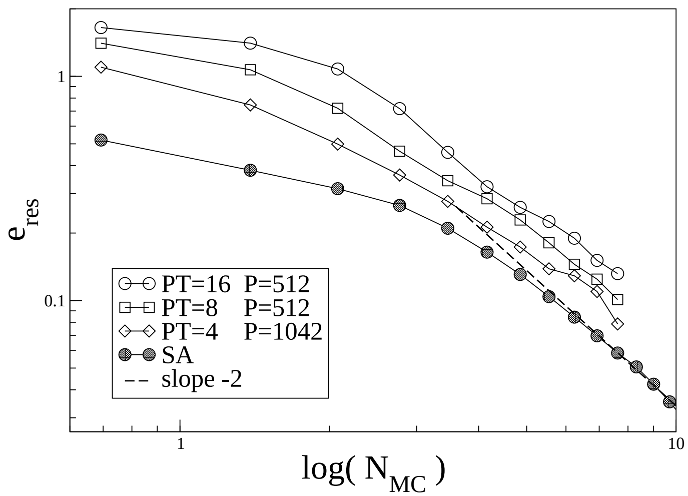
  $\epsilon_{res} \sim \log(N_{MC})^{-\zeta}$

  with $\zeta = 1$.
- Nash *et al.* [Phys. Rev. B **43**, 1272 (1991)]: activated scaling behavior observed, with $\zeta = 1.05 \pm 0.16$.

FM phase & Quantum annealing

- Also, a phase transition in 3D at some quantum parameter strength Γ_c.
- Using the energy barrier argument: the residual energy decreases as in classical case, but possibly with different exponent ζ_q [Phys. Rev. B **57**, 8375 (1998)].

3D data



Note the parameters - and the SA result.

Final thoughts

- Cooling schedule (in)dependence in PM phases.
- PM: QA better. FM: not so.
- Where is the theory, ζ?
- That is, no general knowledge how QA works for difficult problems (K-SAT, TSP etc.). Santoro...
- Use some kind of cluster algorithms?
- Bob Laughlin: “quantum computing is ...”