Monte Carlo simulations of 2-d spin glasses

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Overview

- Model and motivation
- Simulation techniques
- Results for the ±J model
- Gaussian model, low energy excitations
- Conclusion and outlook

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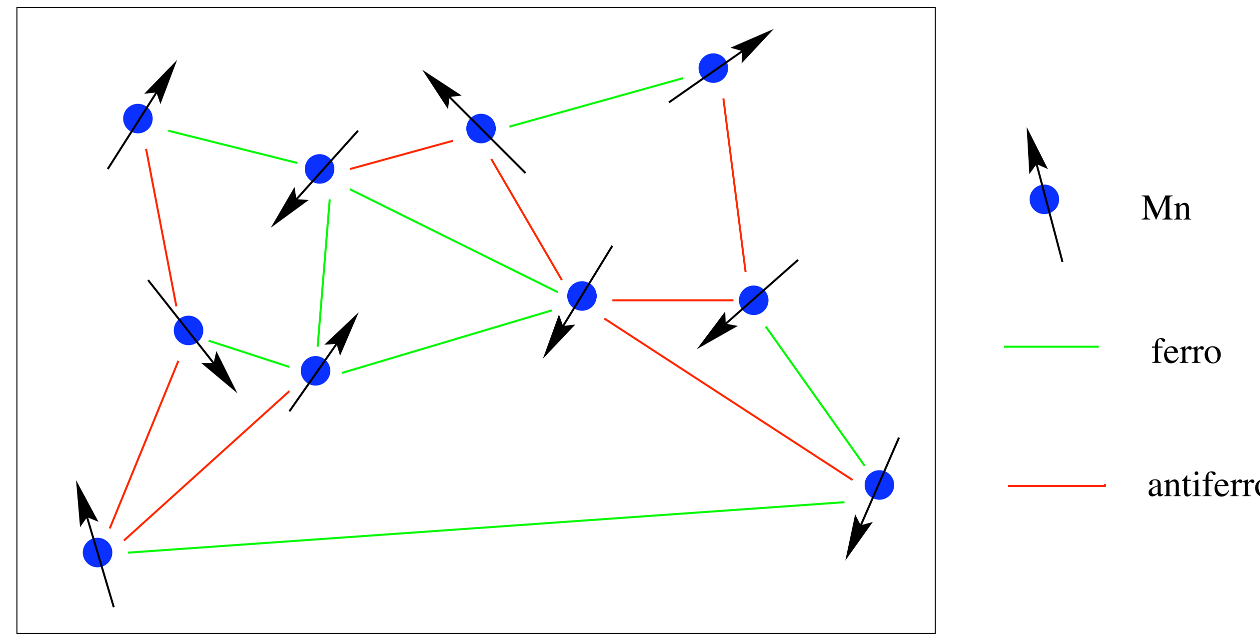
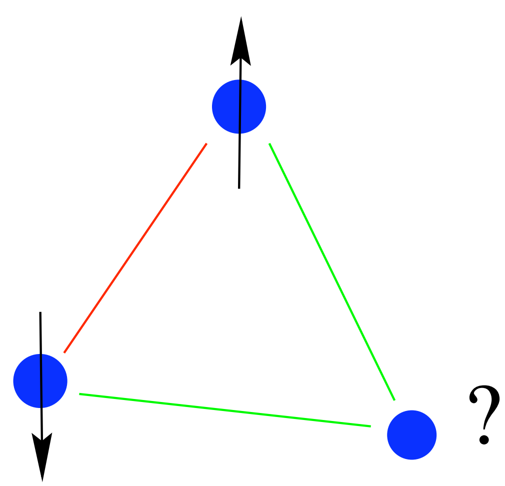
Spin Glasses

Disordered magnetic system
with random interactions:
ex. : $Cu_{1-x}Mn_x$ with x ~ 1%

Interactions RKKY: $J(r) \sim \dfrac{\cos kr}{r^3}$



Frustration:



- Disorder + frustration
  => spin glass phase (Tc~15°K)
- Spins are frozen without apparent order
- Complex structure (many pure states ?)
- Rich dynamics (aging, memory)

Edwards-Anderson Model

$$H_J(S) = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - B \sum_i S_i$$

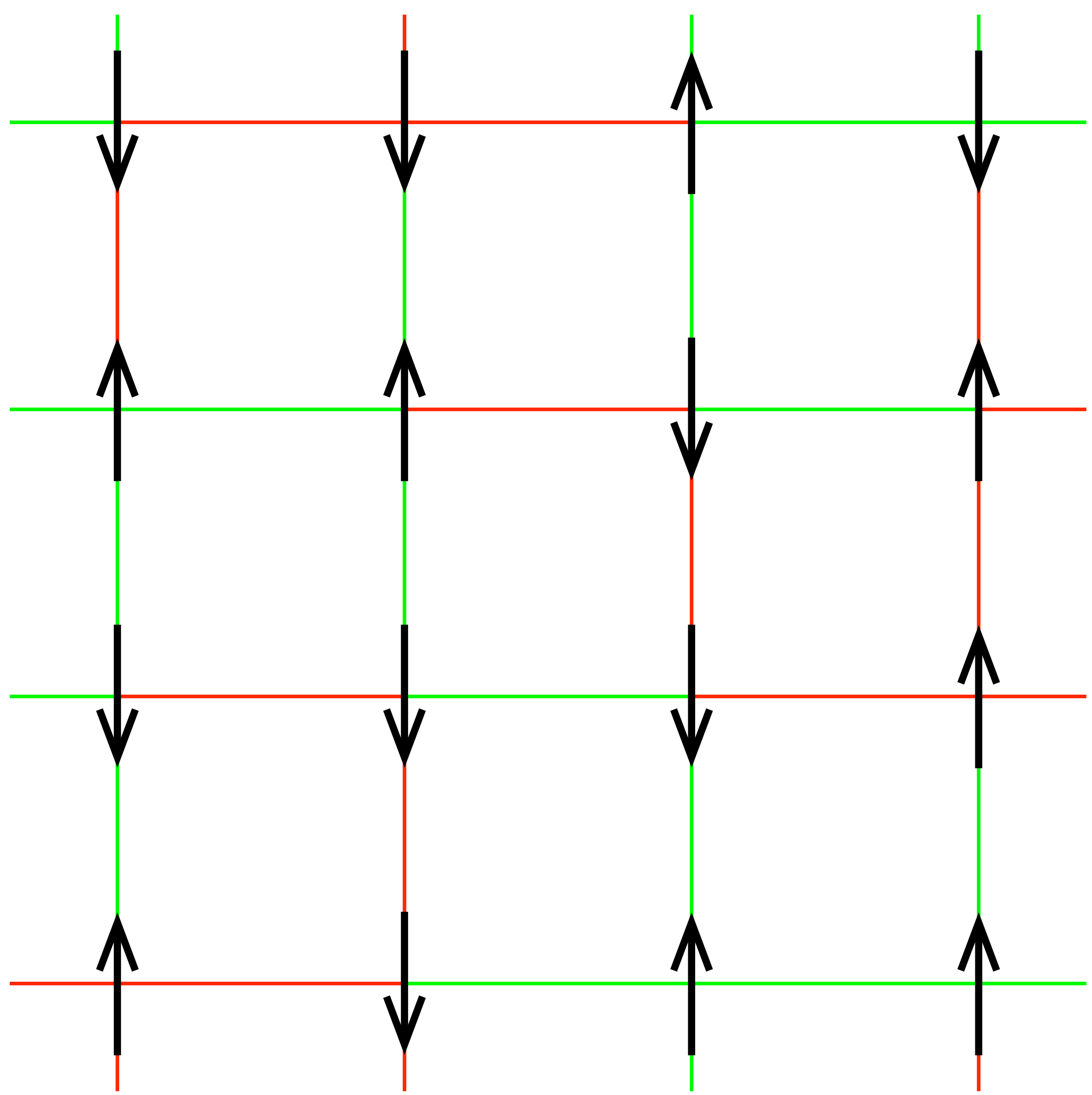
Spins: $S_i = \pm 1$

Random interactions:

$\langle J \rangle = 0$ and $\langle J^2 \rangle = 1$

typically $J = \pm 1$

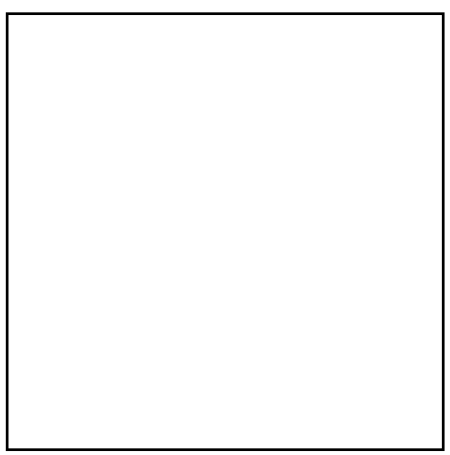
or $P(J) \sim \exp(-J^2/2)$



Overlaps

Overlap (=distance) between 2 configs α and β: $$q^{\alpha\beta} = \frac{1}{N}\sum_i S_i^\alpha S_i^\beta$$

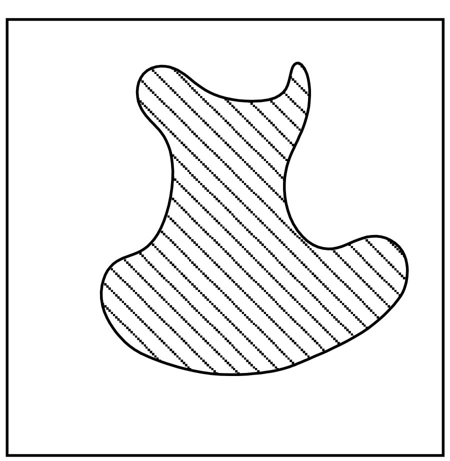
Link overlap: $$q_l^{\alpha\beta} = \frac{1}{dN}\sum_{\langle i,j\rangle} S_i^\alpha S_j^\alpha S_i^\beta S_j^\beta$$



Volume V, surface S

$$q = 1 - \frac{2V}{N}$$

different spins



$$q_l = 1 - \frac{2S}{dN}$$

different bonds

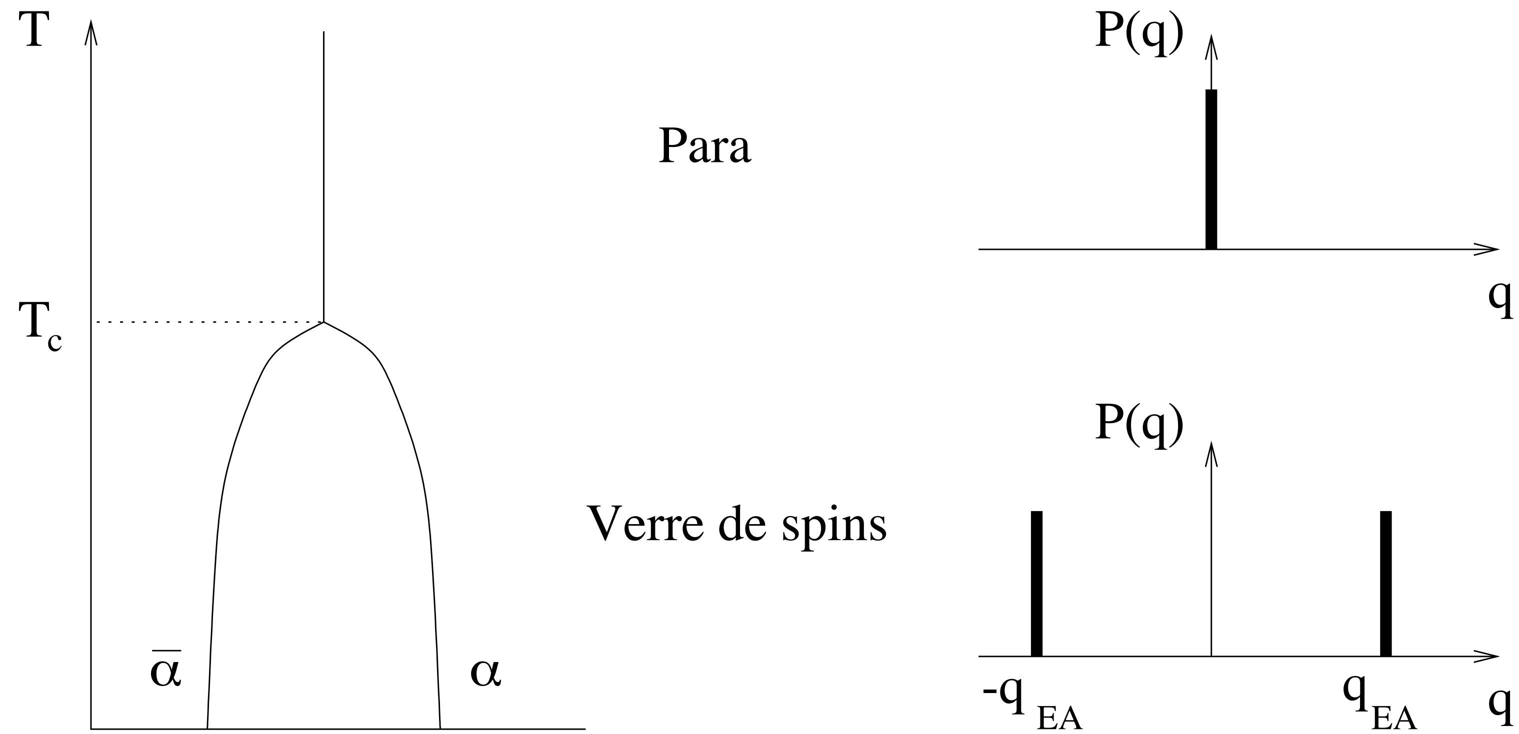
Droplet theory (d>2)

Only 2 pure states in the spin glass phase

Elementary excitations (droplets): compact with $E \sim L^{\theta}$

No spin glass phase in a magnetic field

P(q) is trivial



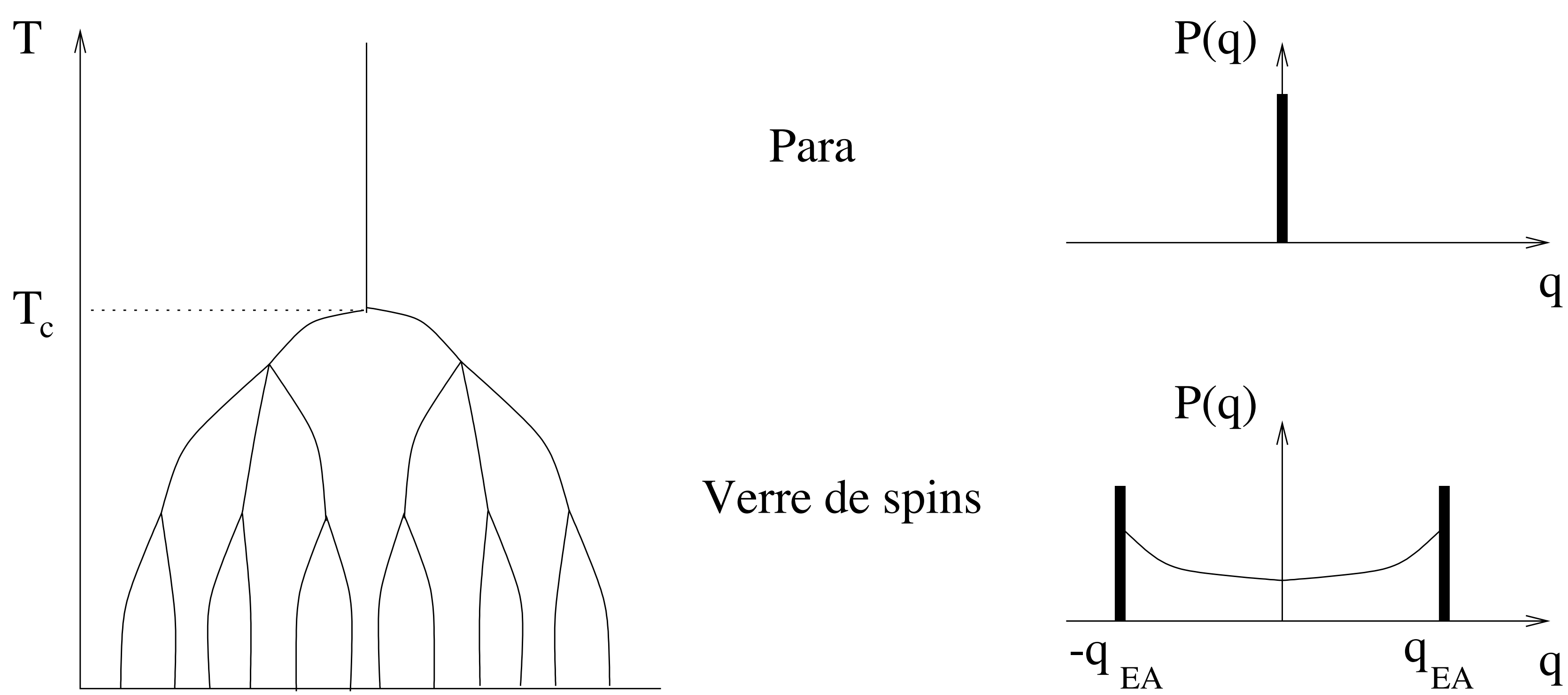
Mean field theory (d=∞)

Many pure states in the spin glass phase

Elementary excitations: system wide with E ~ 1

Spin glass phase under the A-T line in a magnetic field

P(q) is non - trivial



2-d Spin Glasses

- We can do it ! Ground states, Monte Carlo.
- Critical temperature Tc = 0 ? (Yes)
- Universality ?
- Is d = 2 the lower critical dimension ?
- Behavior of c, ξ, χ, P(q)... critical exponents ?
- Does the droplet theory apply in 2 d ?
- Nature of the low energy excitations (energy vs. size, fractal surface, scaling laws...).
- Does it say something for the 3-d case ?

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Standard Monte Carlo

Metropolis: choose a spin at random and flip it with probability

$$p = \min\left(1, e^{-\beta \Delta E}\right)$$

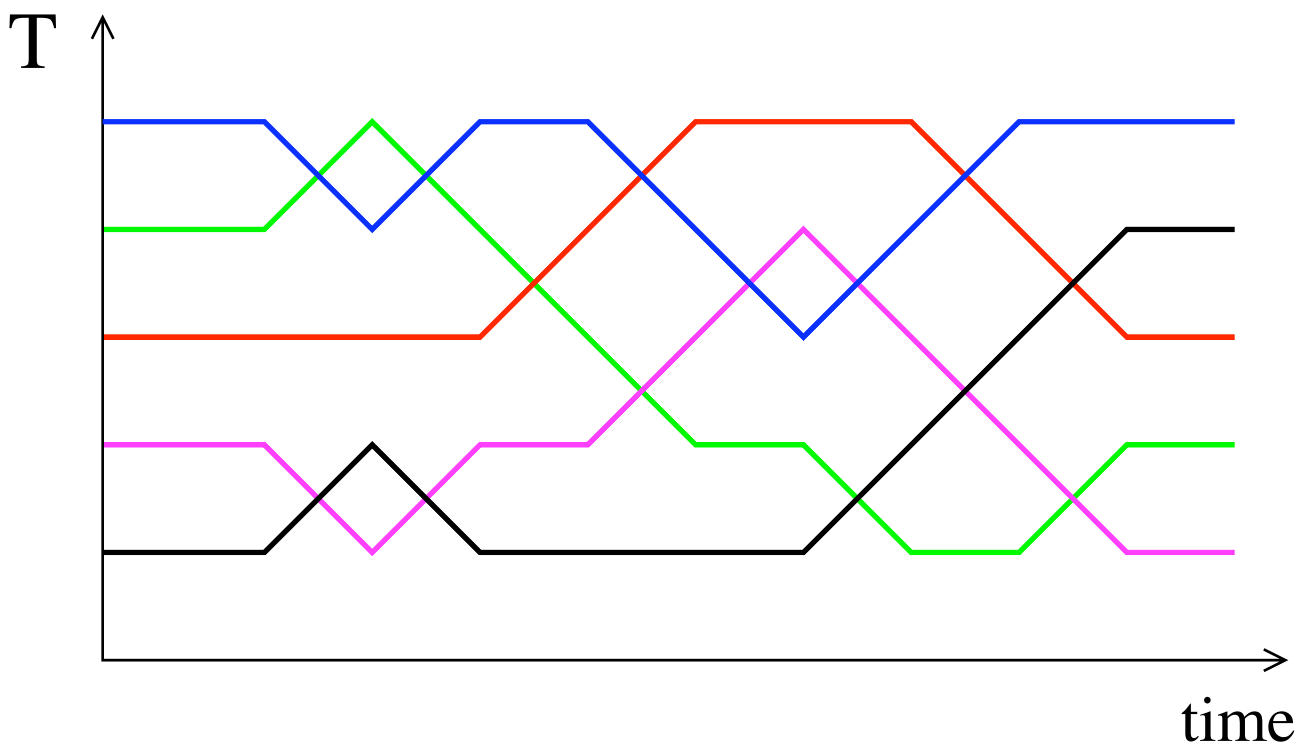
Problem : very slow dynamics

Parallel tempering

Simulate n temperatures in parallel with standard Monte Carlo

Exchange 2 configurations at different temperatures with probability

$$p = \min\left(1, e^{\Delta\beta\Delta E}\right)$$

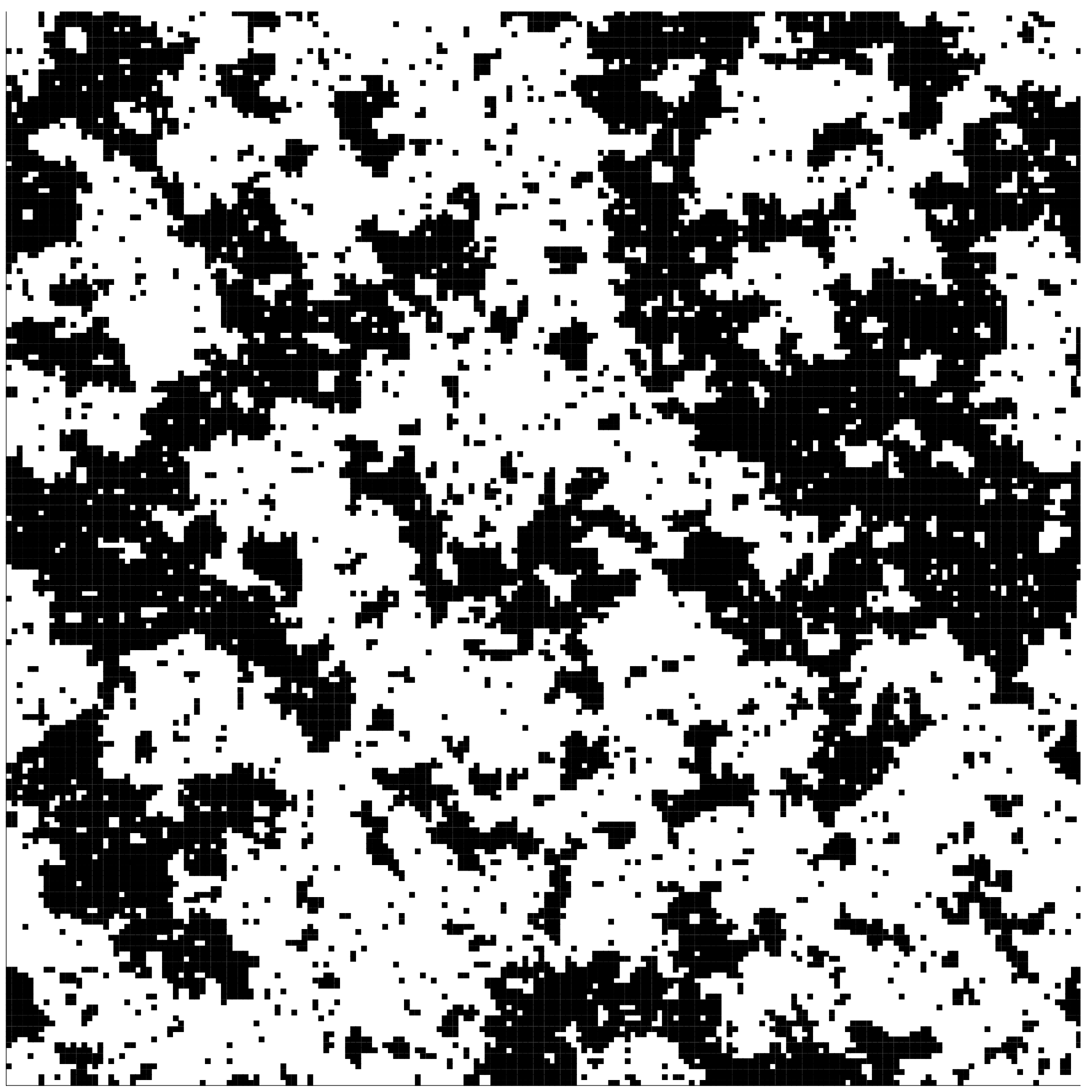


Allow system to pass energy barriers

Cluster moves

Local overlap between 2 configurations α and β: $q_i = S_i^{\alpha} S_i^{\beta}$

Cluster: connected domain with constant q_i



Simulate 2 configurations in parallel

Choose a spin i at random, find associated cluster and flip it in both configs (no rejection)

q_i, $q^{\alpha\beta}$ and $E = E^{\alpha} + E^{\beta}$ unchanged

Problems with cluster moves

When d > 2, qi defines only 2 clusters (percolation threshold<1/2)

Flipping one cluster = Exchanging the configs

Even at d = 2, it does not equilibrate q and E !

To equilibrate q: simulate more than 2 configs in parallel

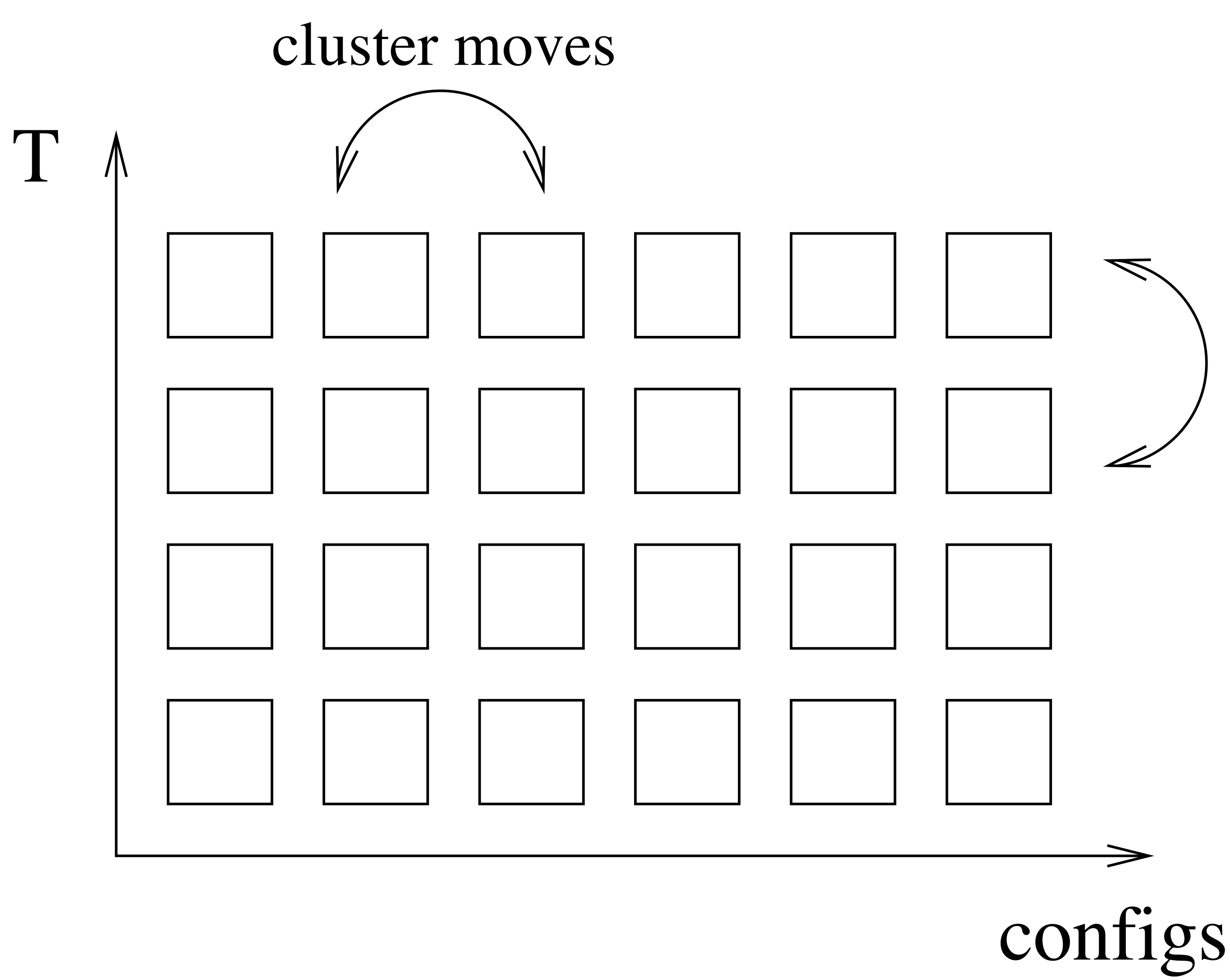
Flipping a cluster between α and β does not change $q^{\alpha\beta}$, but does change $q^{\alpha\gamma}$ and $q^{\beta\gamma}$

To equilibrate E: also use standard Monte Carlo and parallel tempering

Overview of the cluster algorithm

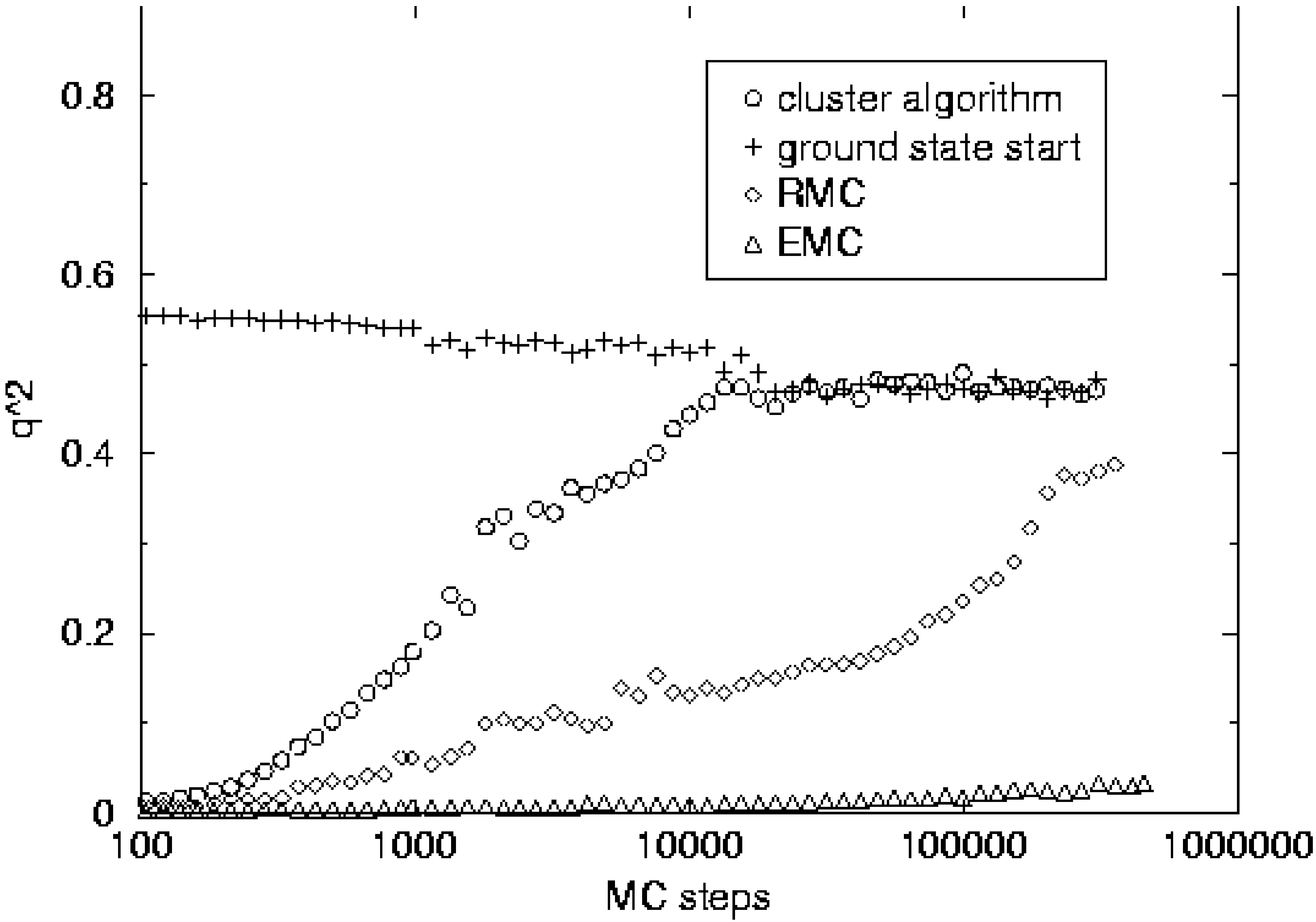
Simulate in parallel m configs for n temperatures with 3 moves:

- Standard Monte Carlo for all m x n configs
- Parallel tempering between the n temperatures
- Cluster moves between m configs for each temperature



Efficiency

Orders of magnitude faster (here β =10, L = 100)



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What we measure

Interactions ±J

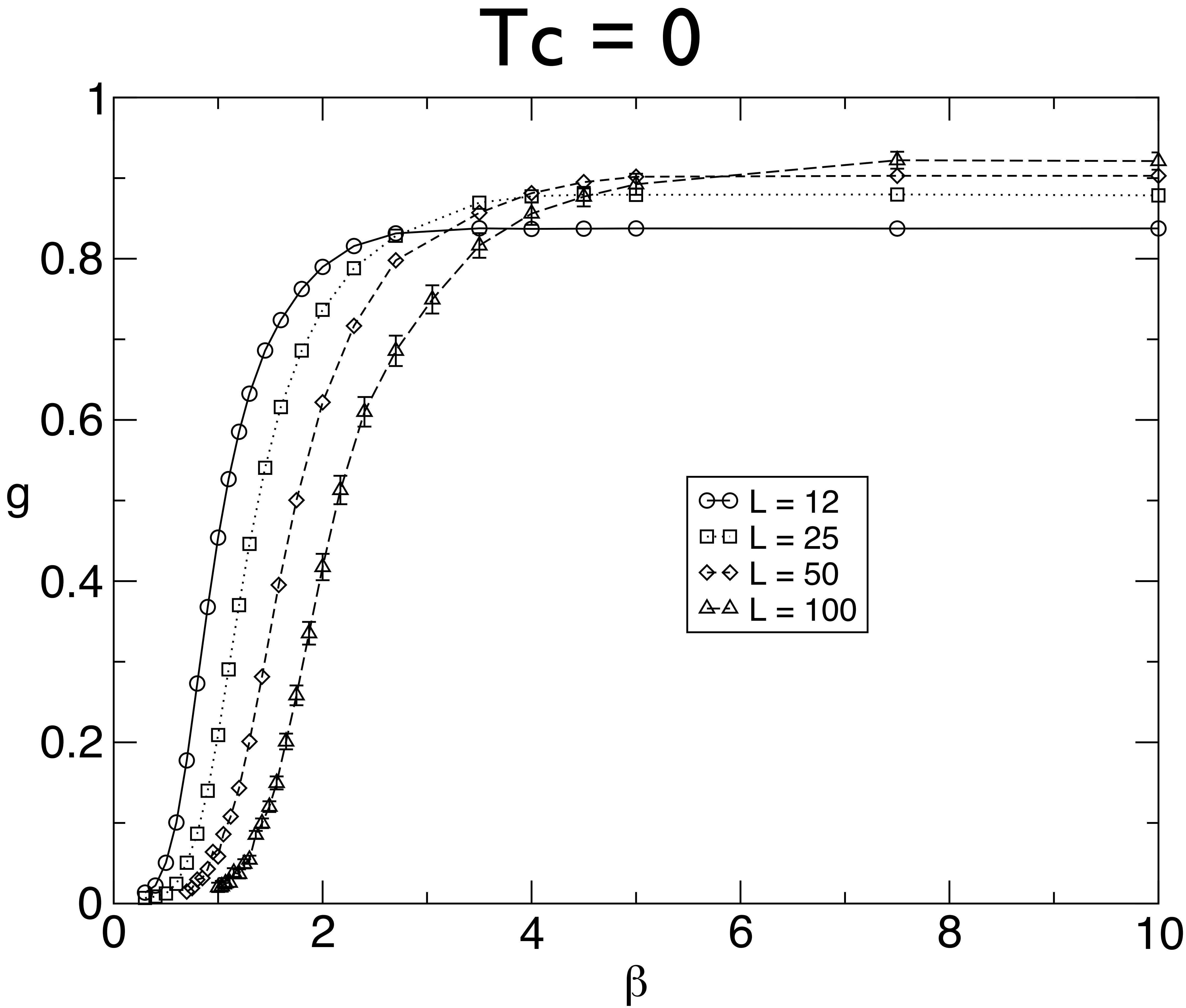
Questions: Tc = 0 ? Correlation length ξ behavior ? Critical exponents

We measure: spin glass susceptibility $\chi = N\overline{\langle q^2 \rangle}$

binder cumulant $g = \frac{1}{2}\overline{\left(3 - \frac{\langle q^4 \rangle}{\langle q^2 \rangle}\right)}$

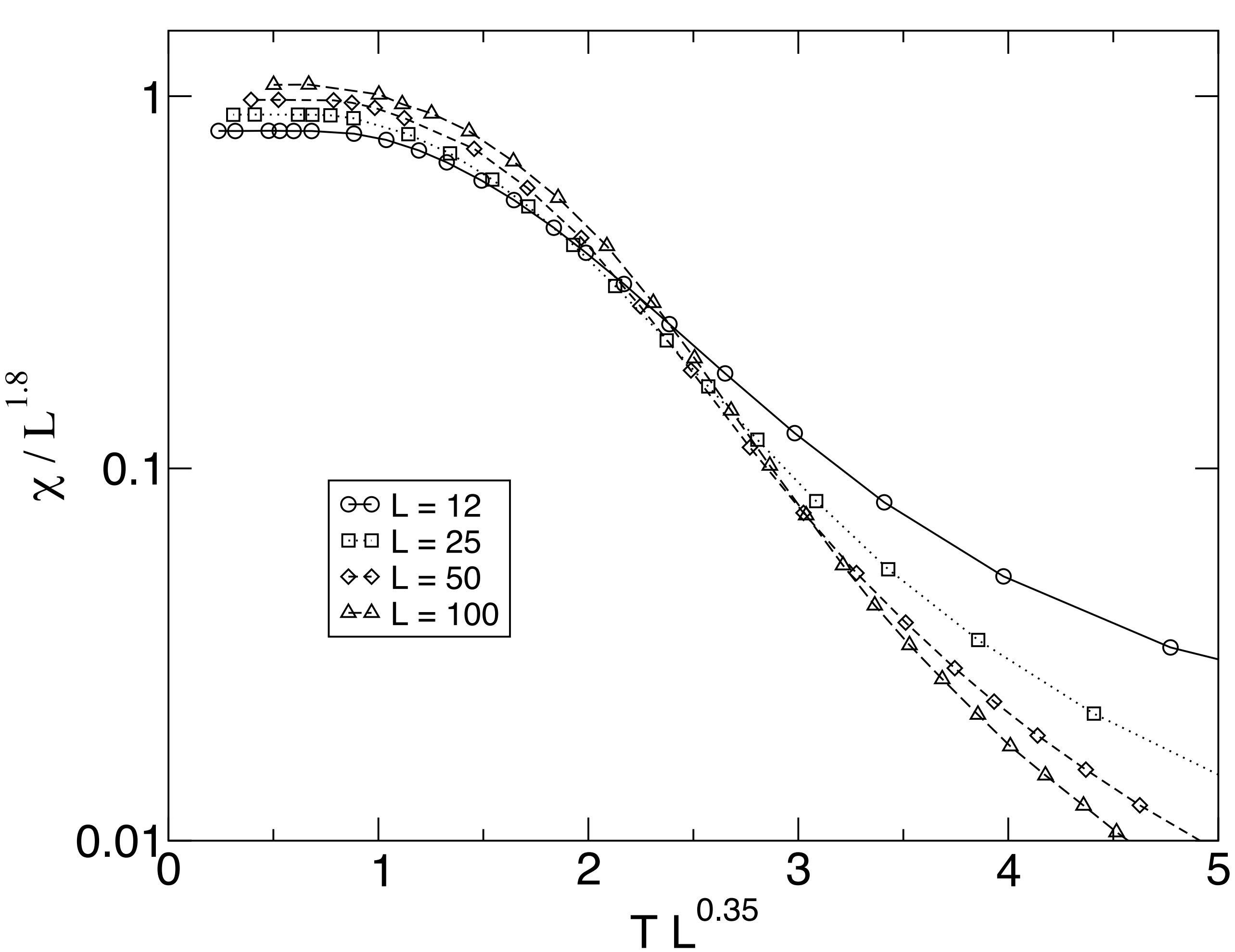
g is 0 is the paramagnetic phase and 1 in the spin glass phase

g is independent of L at T = Tc => the g curves intersect at T = Tc

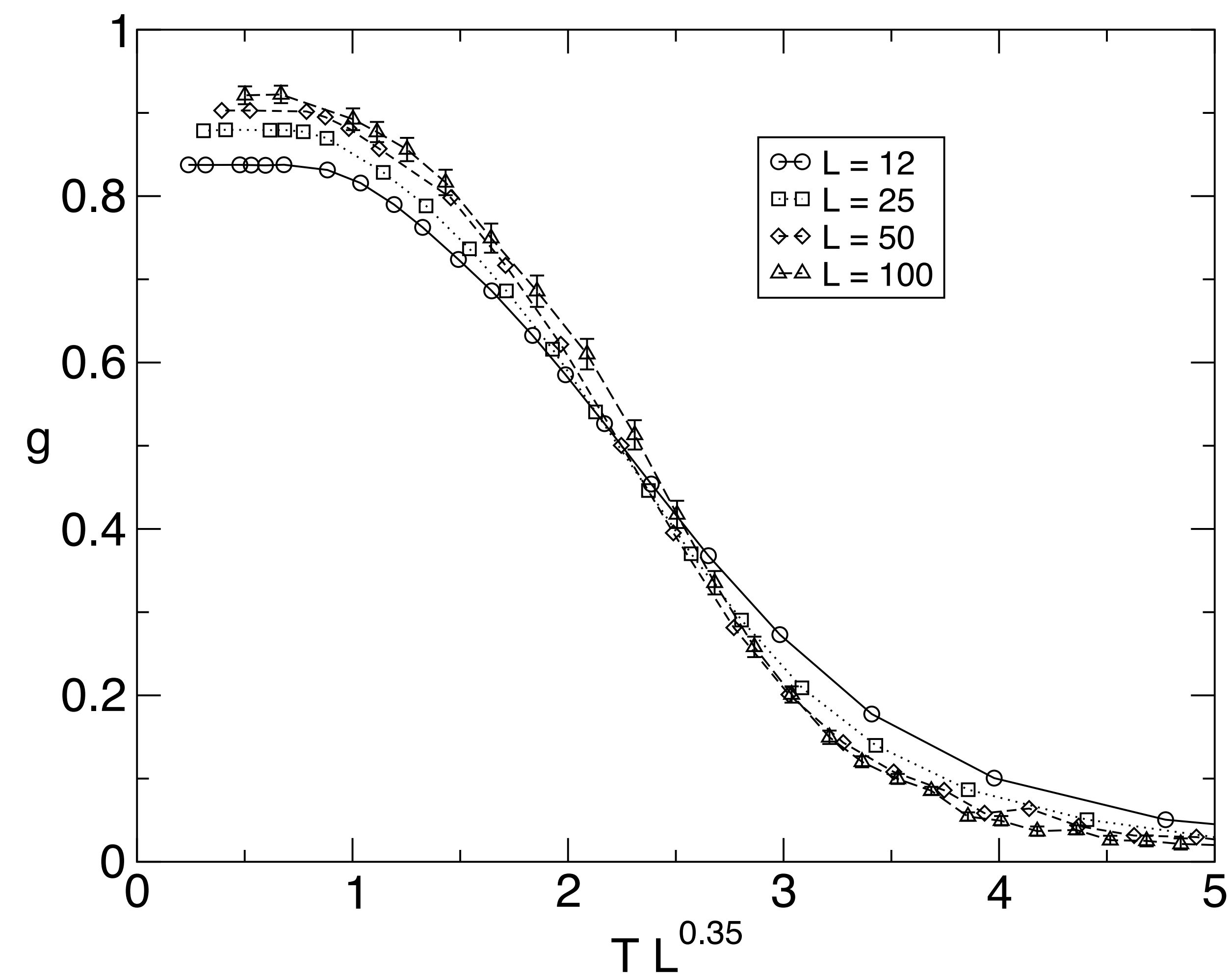

Tc = 0
g
β
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
L = 12
L = 25
L = 50
L = 100

$\xi \sim (T - T_c)^{-\nu}$? No !

$$\chi \sim L^{2-\eta}\tilde{\chi}(TL^{1/\nu})$$

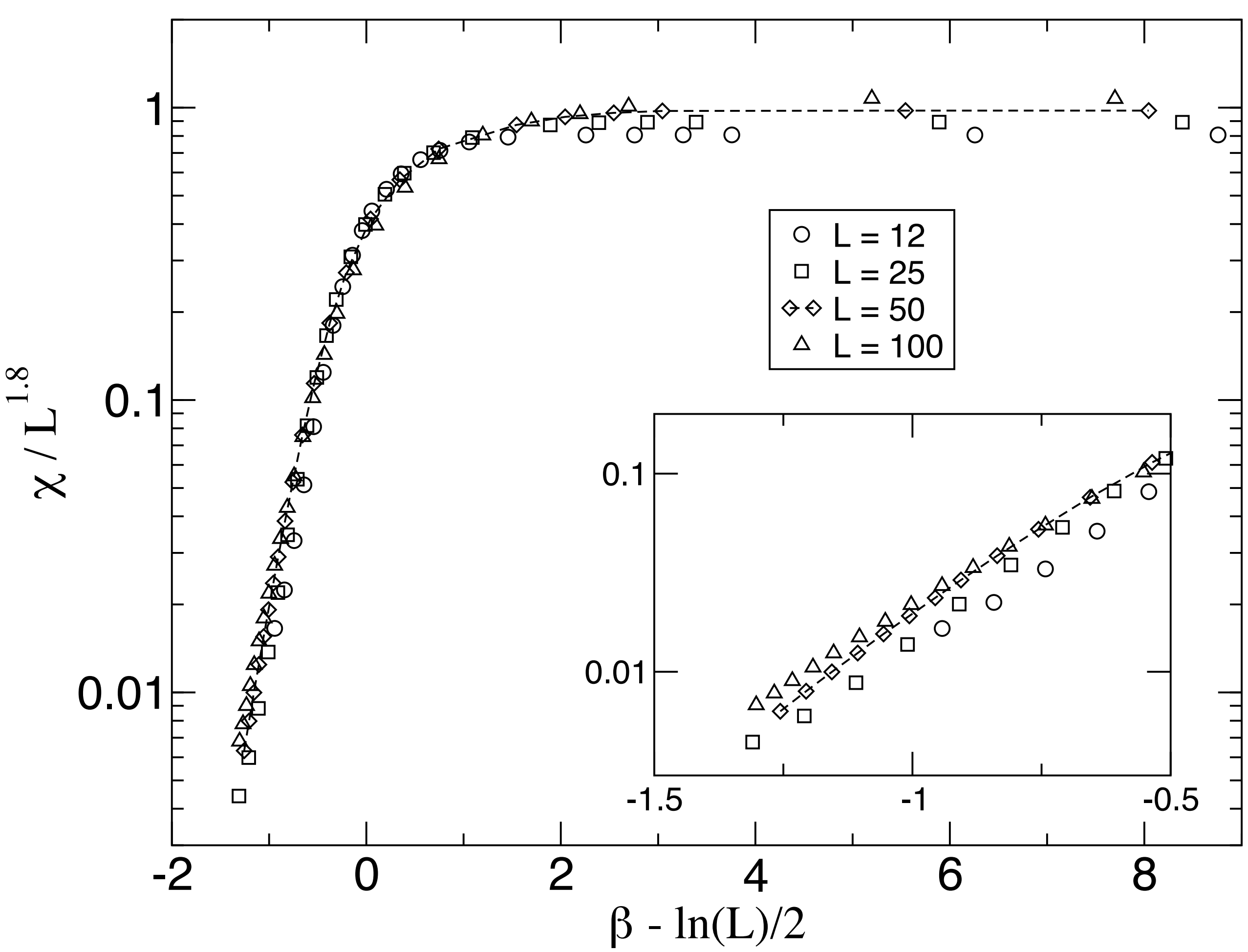


$$g \sim \tilde{g}(TL^{1/\nu})$$

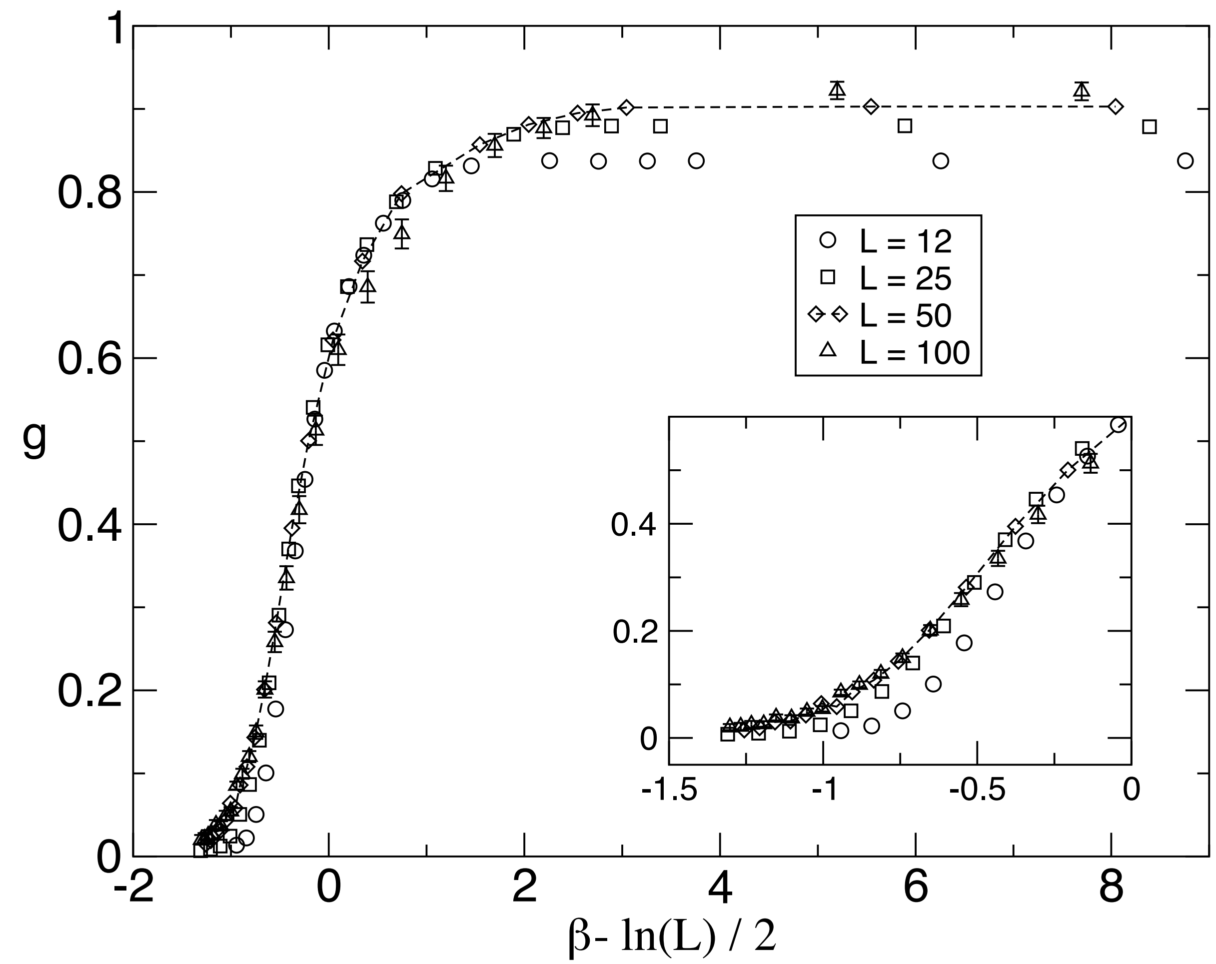


$\xi \sim e^{2\beta}$? Yes !

$$\chi \sim L^{2-\eta}\tilde{\chi}(2\beta - \ln L)$$



$$g \sim \tilde{g}(2\beta - \ln L)$$



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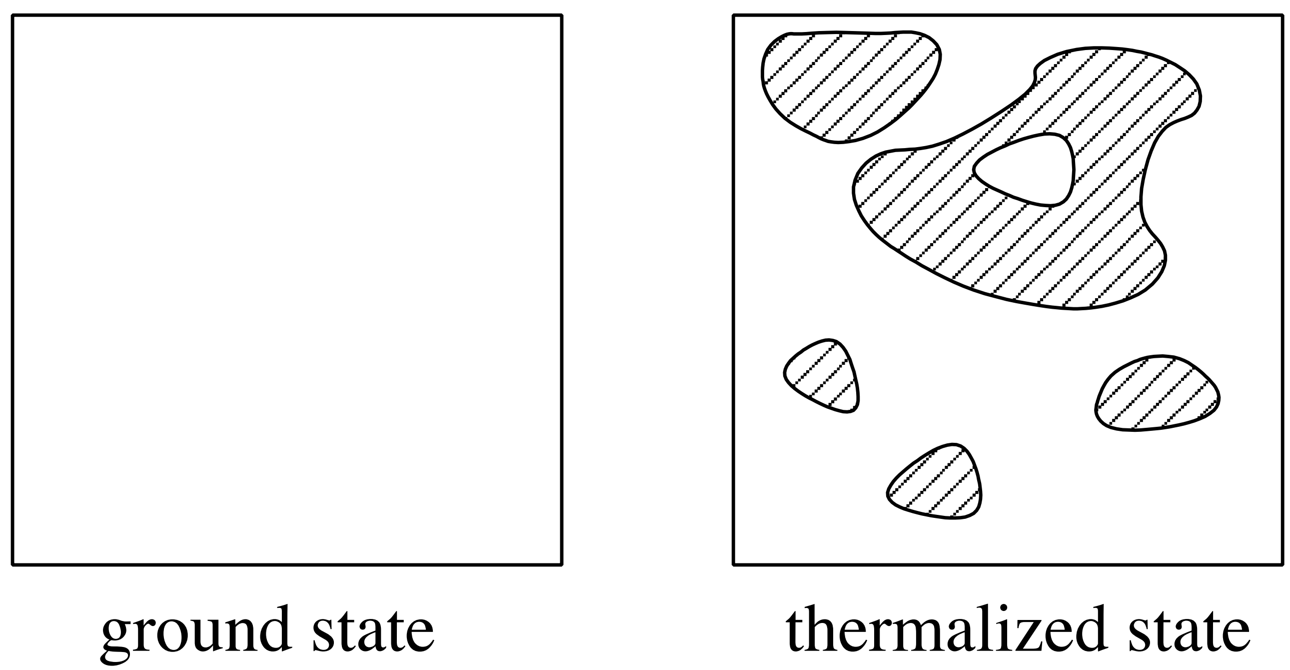
Finding low energy excitations

We can compute the ground state

We have low temperature equilibrated configurations

Let’s compare them !



Each connected cluster boundary defines an elementary excitation

Using excitations as MC moves

During equilibration, build the list of all excitations under a given energy

Choose an excitation at random in the list and flip it with Metropolis probability

Efficiency increases as the temperature decreases !

Used with the cluster algorithm it allows to go to extremely low temperatures: $\beta = 50$ for $L = 100$

Does not work with $\pm J$ interactions: too many 0 energy excitations

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Conclusions and outlook

- Algorithms
  - A cluster algorithm for 2-d spin glasses
  - Use of ground state to find excitations
  - Use of excitations as Monte Carlo moves
- Results for ±J model
  - $T_c = 0$
  - $\xi \sim e^{2\beta}$
- Work in progress on the Gaussian model with A. Hartmann
  - Critical exponents
  - Low energy excitations

The End