Scaling behavior of domain walls at the $T=0$ ferromagnet to spin-glass transition

O. Melchert and A.K. Hartmann
Institut für Physik, Universität Oldenburg, Germany

CARL VON OSSIETZKY universität OLDENBURG

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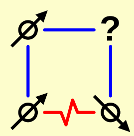
Ising spin glasses (ISGs)

Disordered model systems, governed by Edwards-Anderson Hamiltonian for Ising spins $\sigma_i = \pm 1$:

$$H(\sigma) = -\sum_{\langle i,j \rangle} J_{ij}\sigma_i\sigma_j$$

Frustration:



Quenched disorder:

$J_{ij} > 0$: (blue line)
$J_{ij} < 0$: (red zigzag)

Here: "Gaussian-like" distributed bonds

$$P(J) = (1-\rho)\; e^{-J^2/2}/\sqrt{2\pi} \;+\; \rho\;\delta(J-1)$$

$\rho=0$: SG with Gaussian disorder
$\rho=1$: Ferromagnet

Ground states (GSs)

Spin configuration with minimal energy.

Always:
Global spin flip connects GS pairs.
Gaussian disorder $\longrightarrow$ unique GS pair.

Calculation of GSs:

In 2d with periodic boundary conditions (BCs) in one direction: solvable in polynomial time through mapping to minimum weight perfect matching problem [1].

Domain walls (DWs)

Defined using two spin configurations:

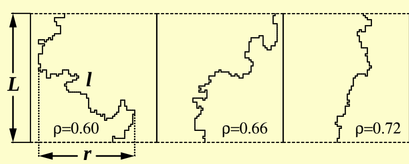
$\sigma^{(1)}$: GS for periodic BCs
$\sigma^{(2)}$: GS for anti periodic BCs

Comparison of spin configurations:
DW separates regions of agreeing/disagreeing spin orientations.

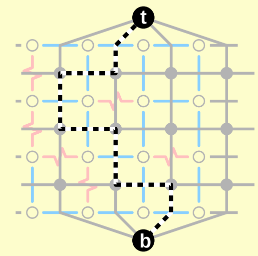
DW energy: $\delta E = E^{(1)} - E^{(2)}$

Here: Determine DWs as shortest paths on dual of spin lattice [2]. $\longrightarrow$

Sample DWs for system size $L=64$:



DW is shortest (t,b)–path on dual graph:



Here: undirected and negative edge weights $\rightarrow$ more complicated than usual shortest path problems.

Schramm-Loewner evolutions

Schramm–Loewner evolutions (SLEs) [5]:

Stochastic differential equation driven by $1d$ Brownian motion.

Describe continuum limit of random curves for $2d$ systems, applies e.g. to percolation, loop-erased random walks.

Geometric properties of SLEs $\longrightarrow$ statistics of critical interfaces.

Results [3]

Previous results at $\rho=0$:

Excitation energy of DWs [4]:

$$\Delta E = \langle|\delta E|\rangle \;\sim\; L^{\theta_1},\; \theta_1 = -0.287(4)$$
$$\sigma(\delta E) = \sqrt{\langle \delta E^2\rangle - \langle \delta E\rangle^2} \sim L^{\theta_2}$$

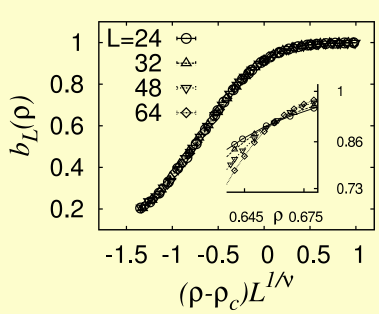
Scaling behavior of DWs [2]:

$$\langle \ell \rangle \;\sim L^{d_f},\; d_f = 1.274(2)$$
$$\langle r \rangle \;\sim L^{d_r},\; d_r = 1.008(11)$$

DWs can be described by SLEs [6], possibility to relate exponents via

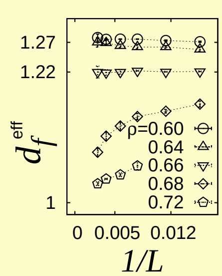
$$d_f = 1 + 3/[4(3+\theta)]$$

Does SLE scaling relation also hold for values $\rho > 0$?



Data collapse yields values:

$\rho_c = 0.660(1)$
$\nu = 1.49(7)$
$\beta = 0.097(6)$



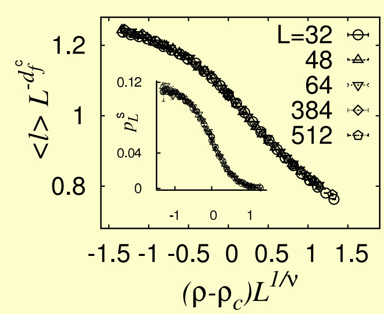
Scaling of magnetization m_L:

$$m_L = \sum_i \sigma_i / L^2$$
$$m_L(\rho) = L^{-\beta/\nu}\tilde{m}[(\rho-\rho_c)L^{1/\nu}]$$

and binder cumulant b_L :

$$b_L = (3 - \langle m_L\rangle^4 / \langle m_L^2\rangle^2)/2$$
$$b_L(\rho) = \tilde{b}[(\rho-\rho_c)L^{1/\nu}]$$

ρ	d_f	d_r	θ_1	θ_2
0.60	1.275(1)	1.003(3)	-0.28(1)	-0.28(2)
0.64	1.275(2)	1.012(4)	-0.28(1)	-0.28(4)
0.66	1.222(1)	1.002(2)	0.17(2)	0.16(1)
0.68	1.05(2)	0.74(3)	0.97(4)	0.35(3)
0.72	1.022(1)	0.698(6)	1.052(3)	0.27(2)



DW length yields data collapse under the scaling assumption:

$$\langle \ell \rangle \sim L^{-d_f^c}\tilde{\ell}[(\rho-\rho_c)L^{1/\nu}],$$
$$d_f^c = 1.222(1)$$

How does this relate to the values of d_f?

Probability that DW roughness is $O(L)$, curves intersect at ρ_c:

$$p_L^s(\rho) \sim \tilde{p}[(\rho-\rho_c)L^{1/\nu}]$$

Spin glass phase up to ρ close to ρ_c:

Scaling behavior of DW energy and DW length consistent with scaling relation derived from SLE processes.

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