Negative-weight percolation

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Outline

- Introduction
- Percolation problem
- Results
- Summary

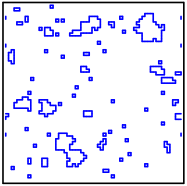
Model

- $L \times L$ lattice, fully periodic boundary conditions
- Undirected edges, weight (cost) distribution:

$$P(\omega) = \rho\,(2\pi)^{-1/2}\exp(-\omega^2/2) \;+\; (1-\rho)\,\delta(\omega-1)$$

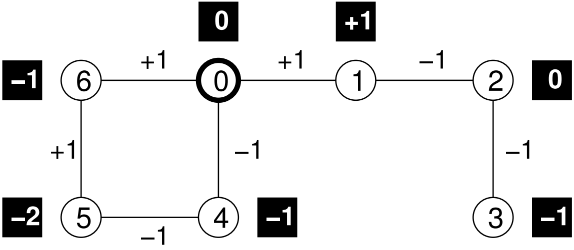
- Allows for loops $\mathcal{L}$ with negative weight $\omega_{\mathcal{L}}$
- Agent on lattice edges: pay/receive resources
- Configuration $\mathcal{C}$ of loops, with

$$E \equiv \sum_{\mathcal{L}\in\mathcal{C}} \omega_{\mathcal{L}} \stackrel{!}{=} \min$$



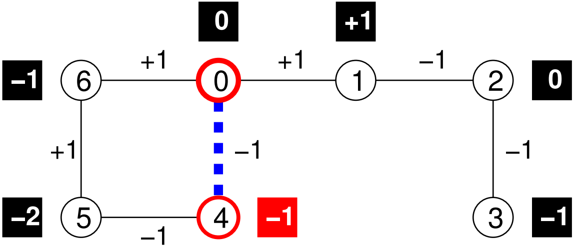
- Obtain $\mathcal{C}$ through mapping to minimum weight perfect matching problem

Minimal distances



- $d(i) = \min_{j \in N(i)}(d(j) + \omega(i,j))$ not fulfilled

Minimal distances



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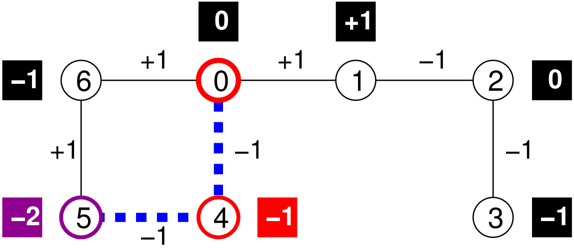

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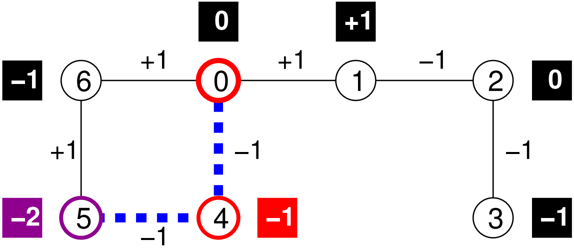
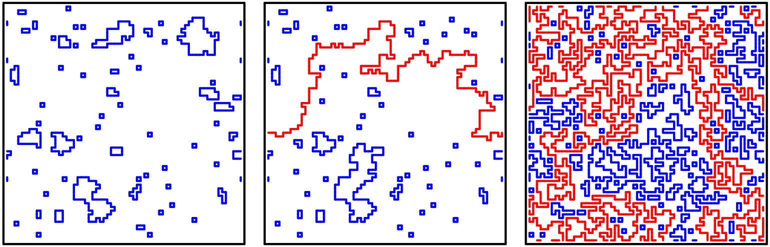


- $d(i) = \min_{j \in N(i)}(d(j) + \omega(i,j))$ not fulfilled
- Standard minimum-weight path algorithms, e.g. Dijkstra, Bellman-Ford, Floyd-Warshall, don't work

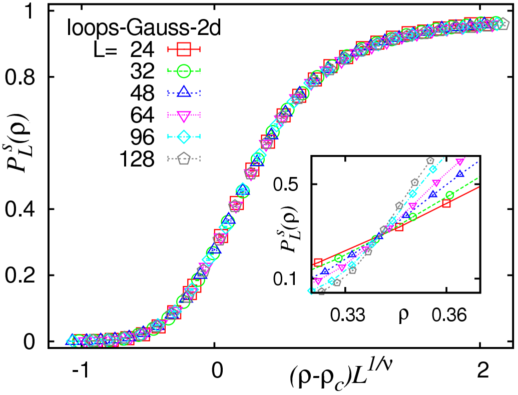
Loop percolation



($L = 64$ at $\rho = 0.335,\ 0.340,\ 0.750$)

- Observe system spanning loops above critical ρ
- Disorder induced, geometric transition
- Characterize loops using observables from percolation theory (finite-size scaling (FSS) analysis)

Percolation probability



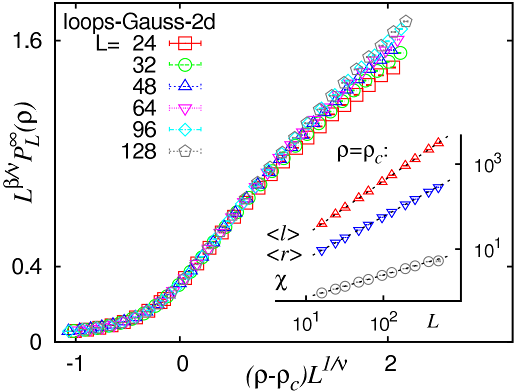
Percolation probability exhibits FSS:

$P_L^s \sim f[(\rho-\rho_c)L^{1/\nu}]$

$\rho_c = 0.340(1)$
$\nu = 1.49(7)$
(rand. perc.: $\nu = 1.33$)
$S = 0.91$

- S = "quality" of the scaling assumption
- Similar scaling for mean number of spanning loops

Percolation strength



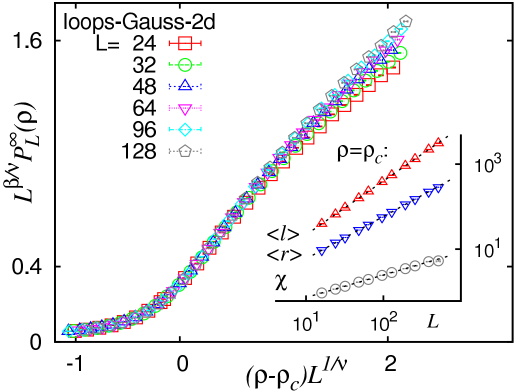
Exhibits FSS:

$$P_L^\infty \sim L^{-\beta/\nu} f[(\rho-\rho_c)L^{1/\nu}]$$

$\beta = 1.07(6)$
$S = 1.16$

- Probability $P_L^\infty \equiv \langle \ell \rangle / L^d$ that edge belongs to percolating loop, finite-size suszeptibility $\chi \equiv L^{-d}(\langle \ell^2 \rangle - \langle \ell \rangle^2)$

Percolation strength



At ρ_c ($L_{max} = 512$):

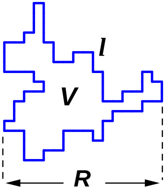
loop length $\langle \ell \rangle \sim L^{d_f}$,
roughness $\langle r \rangle \sim L^{d_r}$,
suszept. $\chi \sim L^{\gamma/\nu}$

$d_f = 1.266(2)$
$d_r = 1.001(4)$
$\gamma = 0.77(7)$

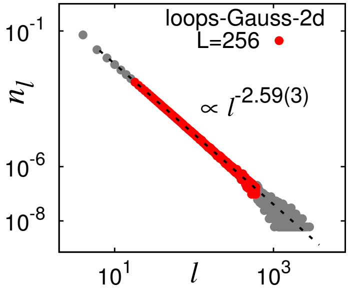
- Probability $P_L^\infty \equiv \langle \ell \rangle / L^d$ that edge belongs to percolating loop, finite-size suszeptibility $\chi \equiv L^{-d}(\langle \ell^2 \rangle - \langle \ell \rangle^2)$
- Scaling relations $d_f = d - \beta/\nu$ and $\gamma + 2\beta = d\nu$ are fulfilled

Non-percolating loops

- Scaling properties of the small loops:
  Consistent with percolating loops
  $\langle v \rangle \sim R^2$ (loop spanning lenght R)
  $\langle \omega \rangle \sim \ell$



- Distribution n_ℓ of the loop lengths ℓ at ρ_c for $L = 256$

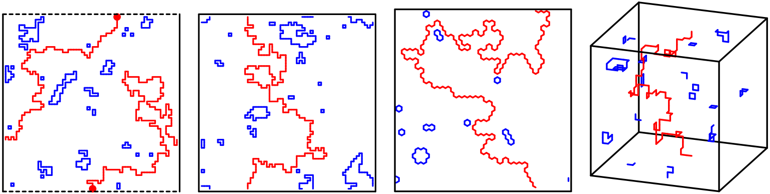


Expected FSS:

$$n_\ell \sim \ell^{-\tau}$$

$\tau = 2.59(3)$
Consistent with
$\tau = 1 + d/d_f$

More results



Type	ρ_c	ν	β	γ	τ	d_f
P$\pm$J 2d sq	0.1032(5)	1.43(6)	1.03(3)	0.76(5)	2.51(4)	1.268(1)
L$\pm$J 2d sq	0.1028(3)	1.49(9)	1.09(8)	0.75(8)	2.58(6)	1.260(2)
L$\pm$J 2d hex	0.1583(6)	1.47(9)	1.07(9)	0.76(8)	2.59(2)	1.264(3)
L-Gl 2d sq	0.340(1)	1.49(7)	1.07(6)	0.77(7)	2.59(3)	1.266(2)
L$\pm$J 3d cu	0.0286(1)	1.02(3)	1.80(8)	–	3.5(3)	1.30(1)

- Exponents seem to be universal in 2*d*
- Random bond Ising model at $T=0$:
  $\rho_c = 0.103(1)$, $\nu = 1.55(1)$, $\beta = 0.9(1)$
  [Amoruso & Hartmann, PRB 2004]

Summary

- Negative-weight percolation of loops
- Distinct from random bond/site percolation
- $2d$: critical exponents close to RBIM
- More details: arXiv:0711.4069

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- Thank you!