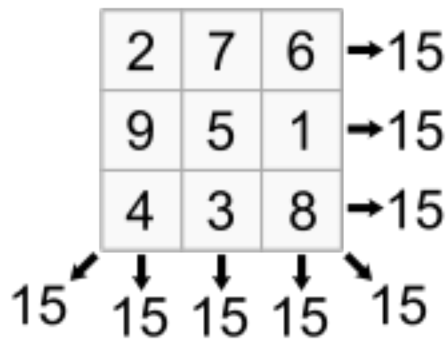
Magic squares and water retention

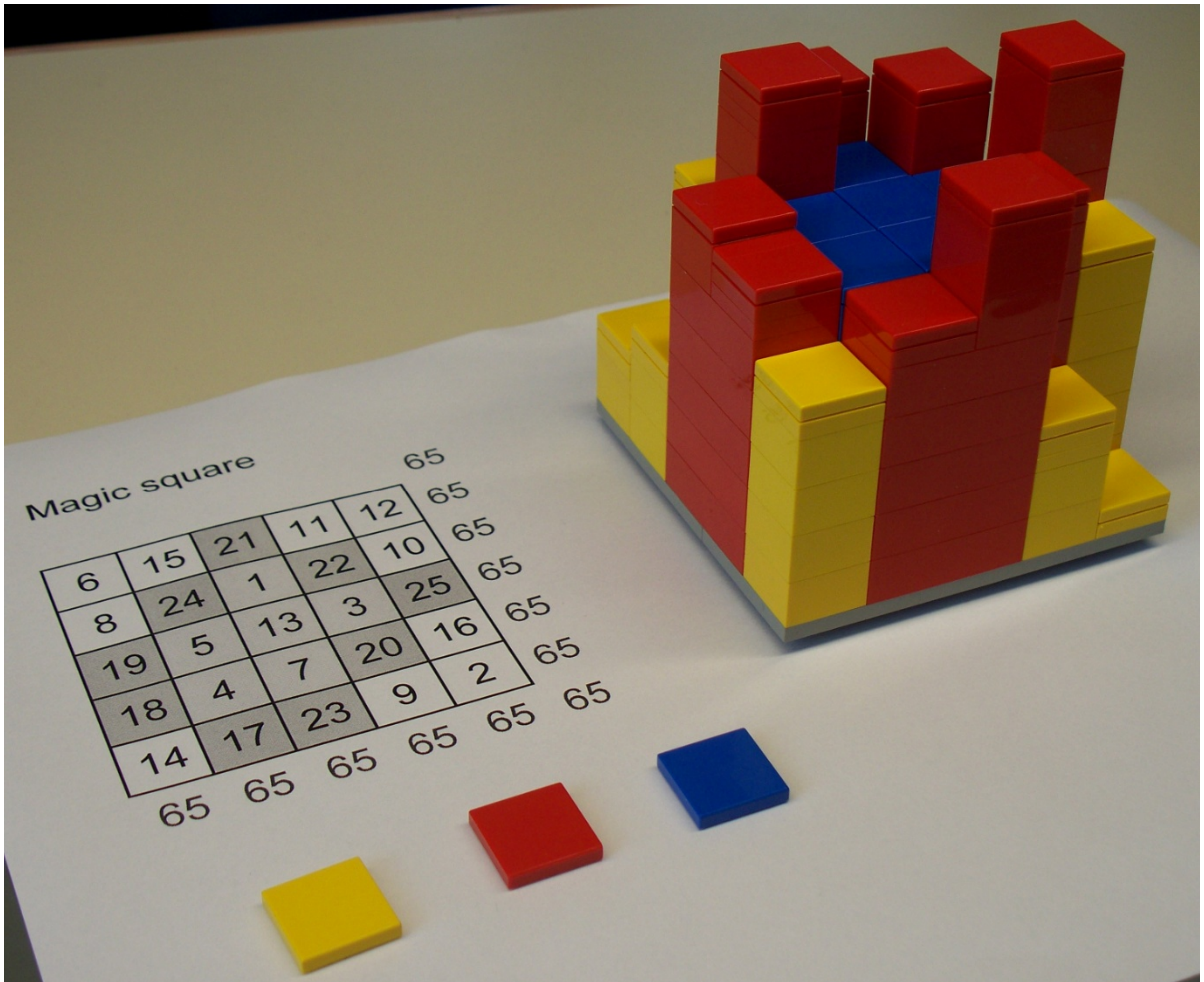
In a magic square, all rows, columns and diagonals add to the same value, and all numbers 1, 2, …, L^2 occur just once.



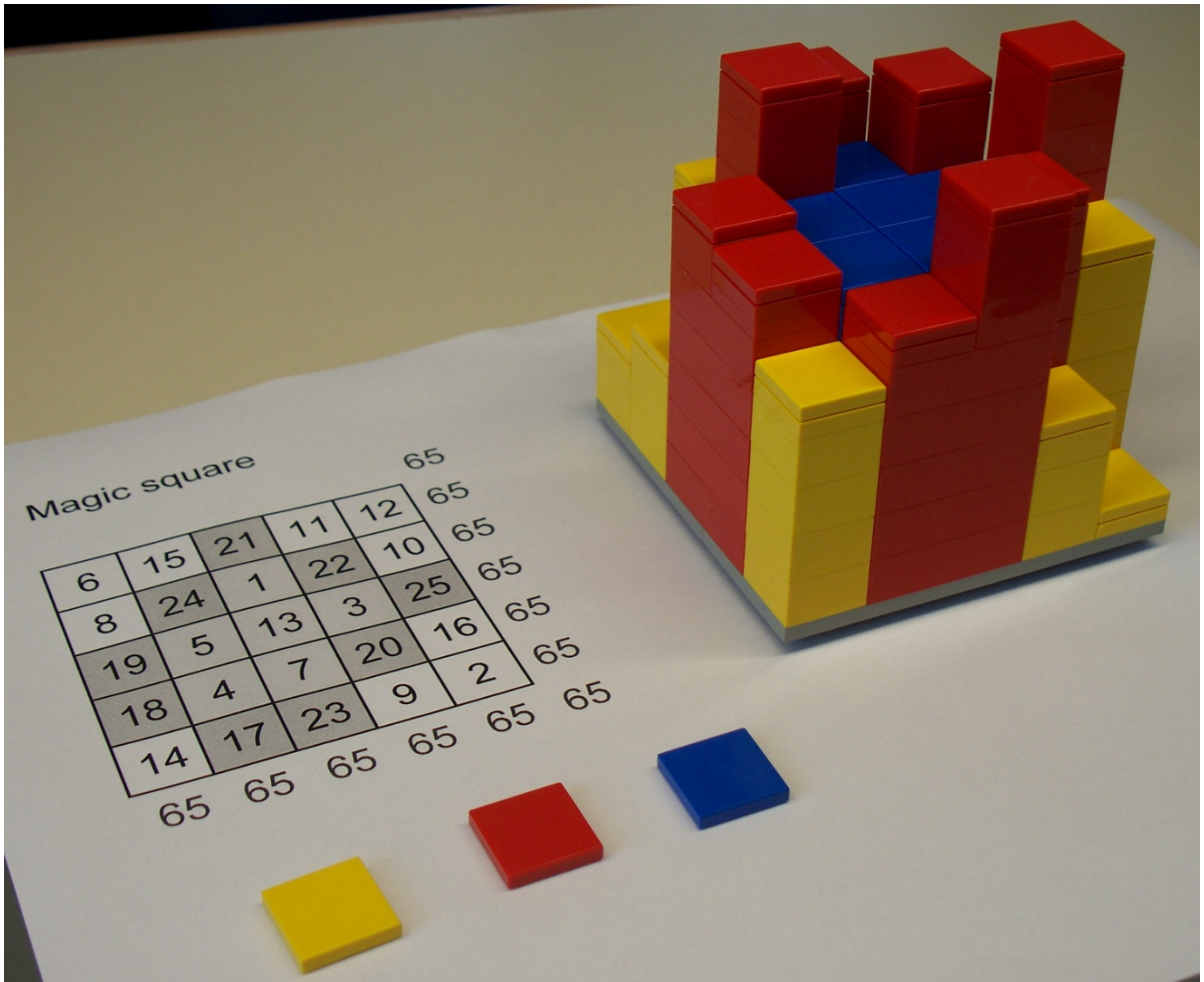
Magic squares have been studied since at least 650 BC in China. (The above magic square is the Lo Shu Square), and continue to be objects of intense study in recreational mathematics today.

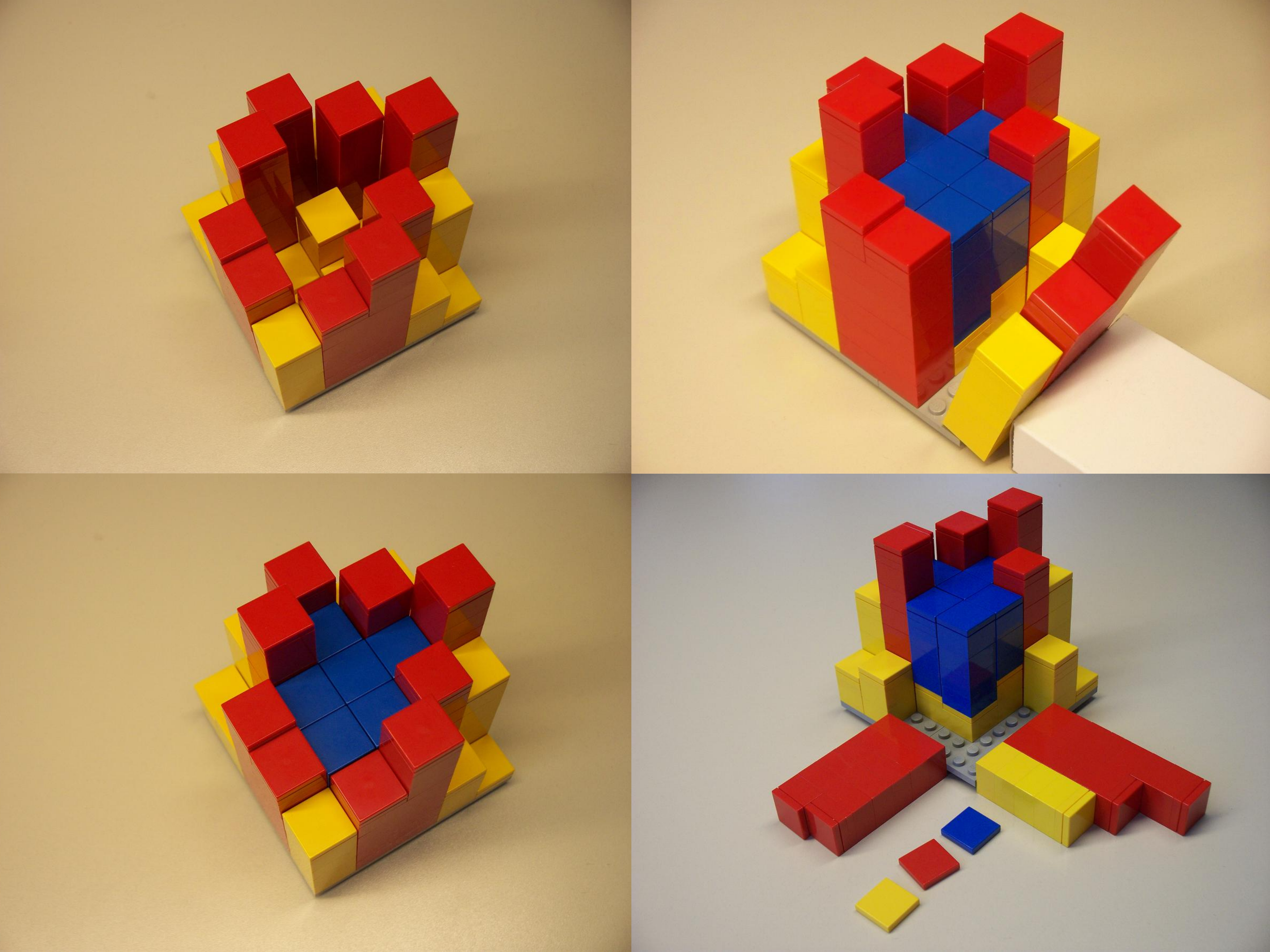
Chris Knecht proposed studying water retention on magic squares, and it became a problem in an international computation competition, with Walter Trump as one of the winners.

Magic Square work drove the discovery of water retention algorithms. This photo by Walter Trump shows the order 5 magic square that retains the most water.



Magic Square work drove the discovery of water retention algorithms. This photo by Walter Trump shows the order 5 magic square that retains the most water.





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Al Zimmermann's Programming Contests

Contests > About

About

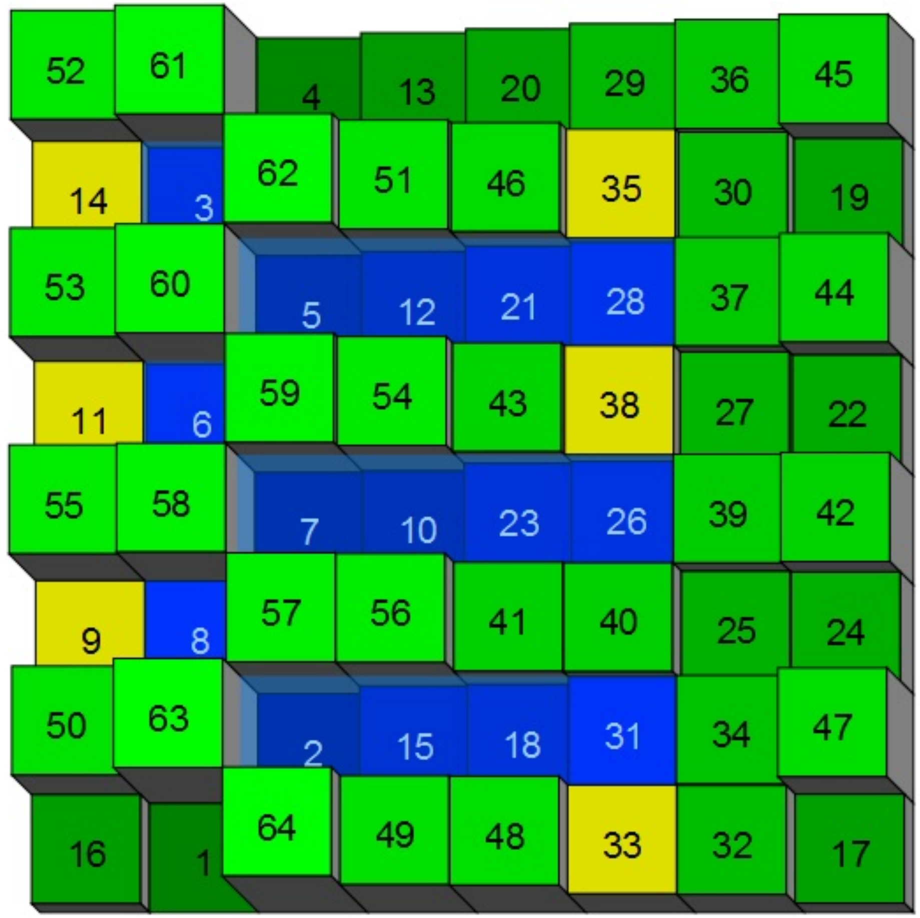
Zimmermann's contest in March 2010 was to find the maximum water retention for magic squares orders 4 – 28. The contest demonstrates that magic squares of any order can be produced and manipulated to explore their physical properties.

http://www.knechtmagicsquare.paulscomputing.com/

1	42	73	56	83	61	79	80	27	3	505
49	74	17	92	20	98	8	12	82	53	505
72	4	99	16	23	22	93	50	81	45	505
59	87	19	33	40	41	48	100	7	71	505
85	14	28	29	54	65	43	32	91	64	505
60	89	35	38	66	47	39	34	11	86	505
70	9	97	62	37	36	30	21	88	55	505
51	78	46	94	24	25	18	96	5	68	505
52	77	15	10	95	26	90	13	69	58	505
6	31	76	75	63	84	57	67	44	2	505
505	505	505	505	505	505	505	505	505	505	

Water retention on Ben Franklin's magic square

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17



In a Franklin magic square, diagonals do not add to the value of the columns, but all bent diagonals (black sites here) add to the value of the columns!

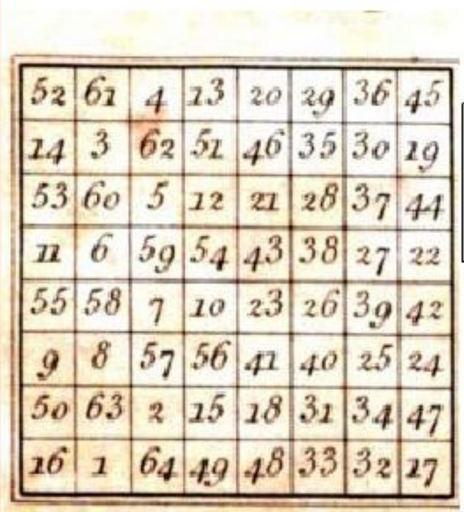
Albrecht Dürer's Magic Square

Dürer 1471 – 1528

- http://kk.haum-bs.de/?id=a-duerer-wb3-0122

It has been said that the first scientific paper written in the new world was a paper on magic squares by Ben Franklin. He was bored out of his mind sitting in the committees writing out the US constitution and doodled out three magic squares that are named after him to this day.

- http://www.magic-squares.net/square-update.htm



52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17

USA 2006: *Scott* 4022

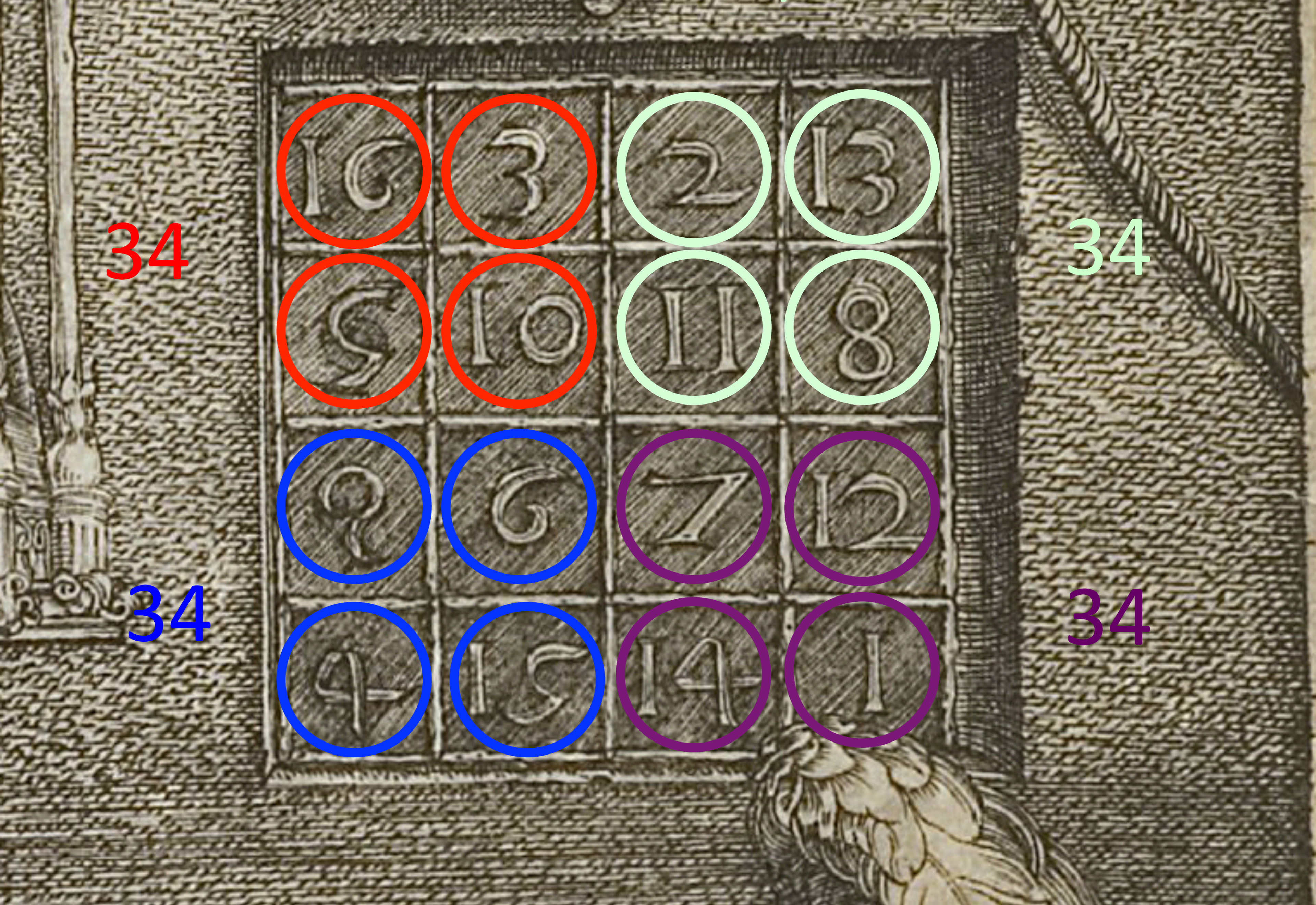
First US postage stamp!

Dürer’s Melancholia I

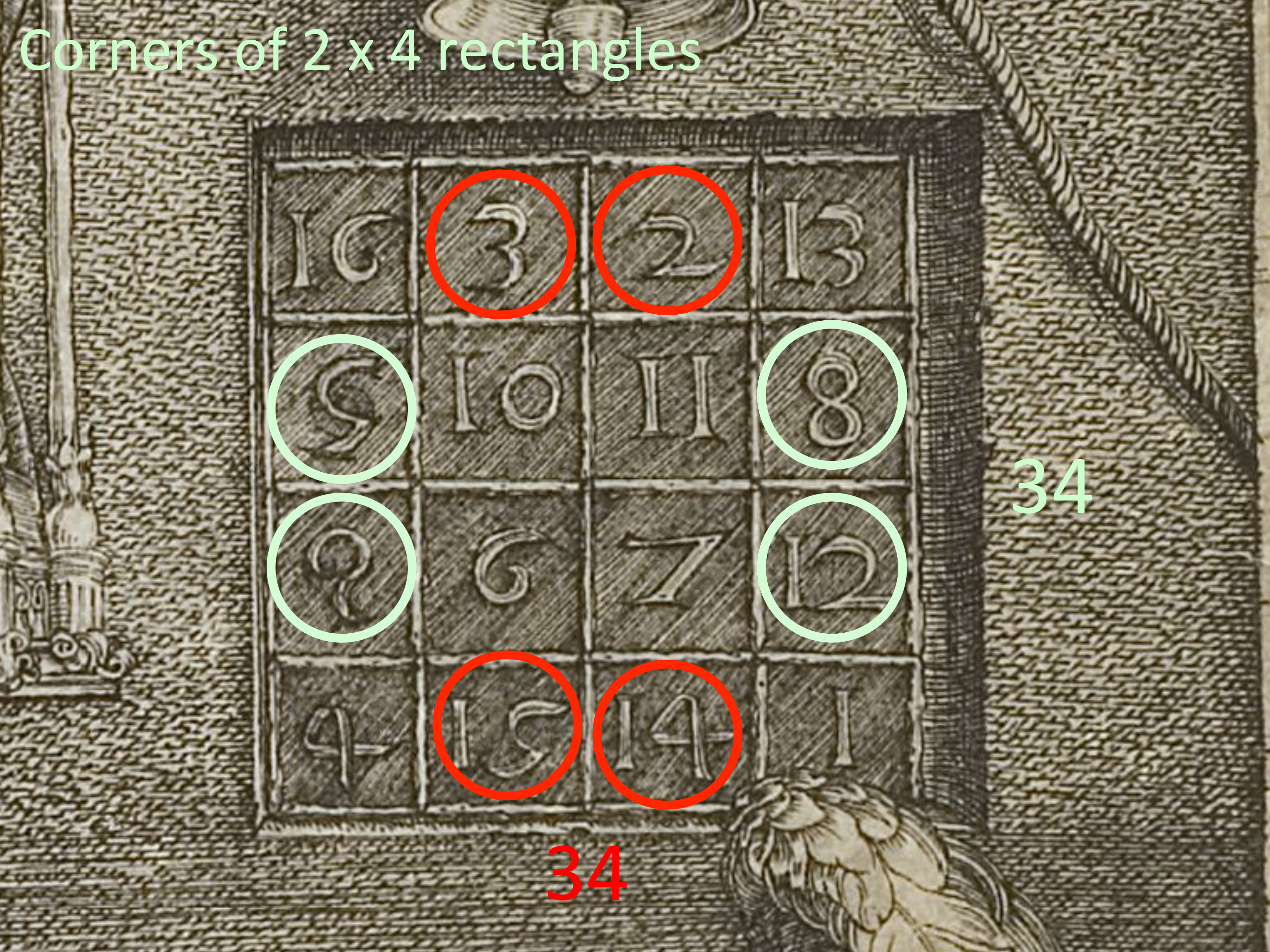
1514

Centrally symmetric pairs add to 17
("associative magic square")

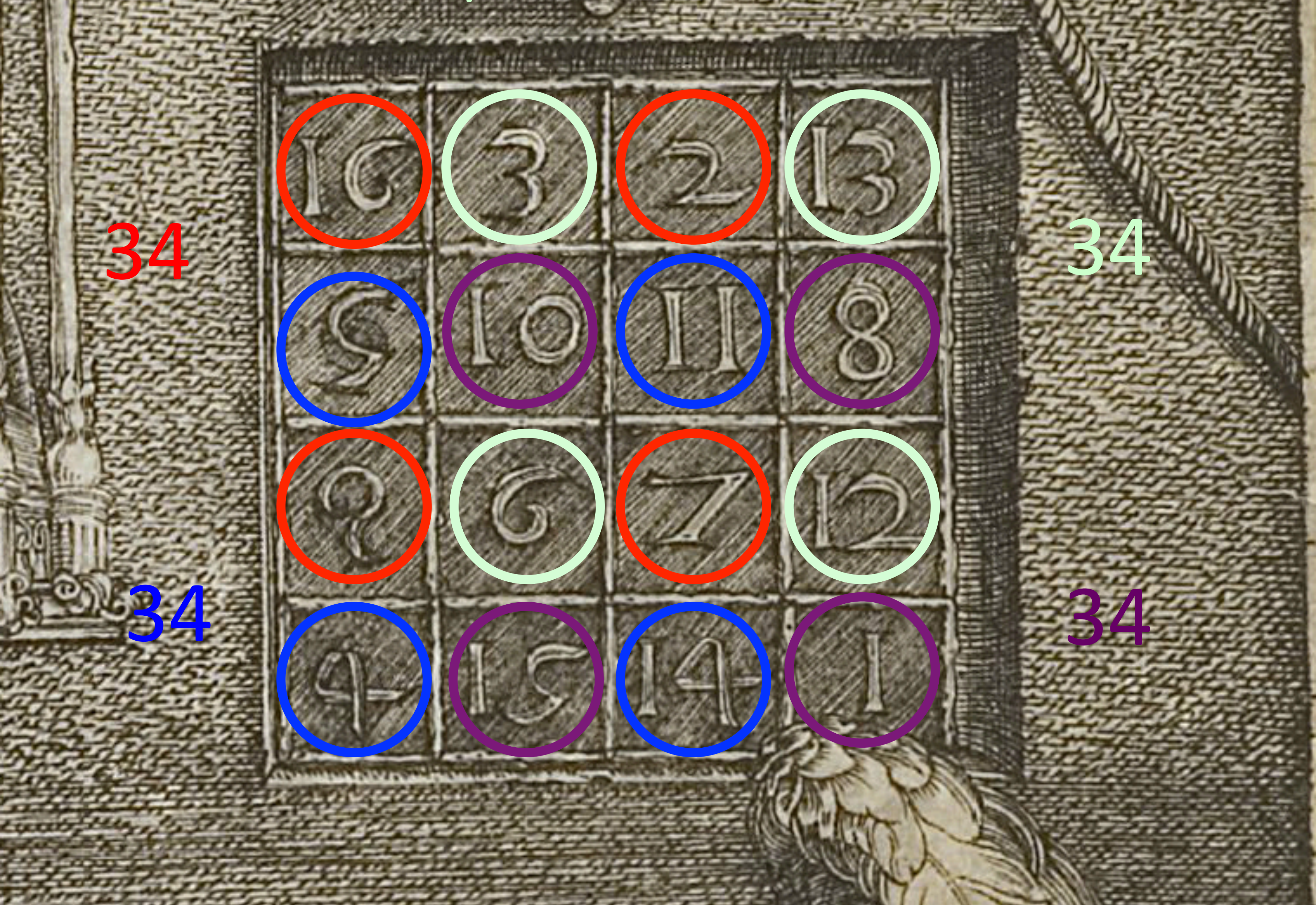
In addition, here all corner squares add to 34

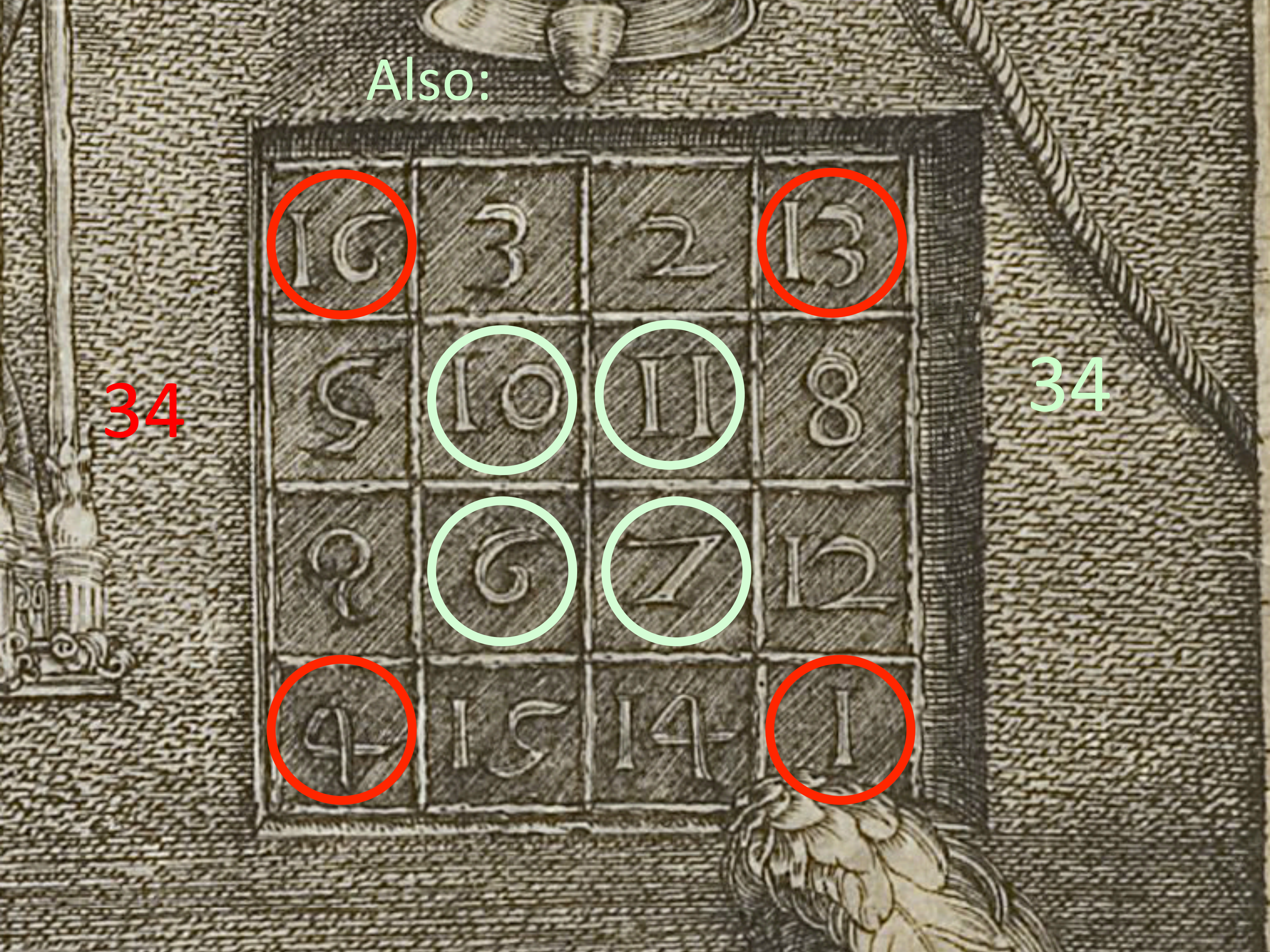


Corners of 2 x 4 rectangles

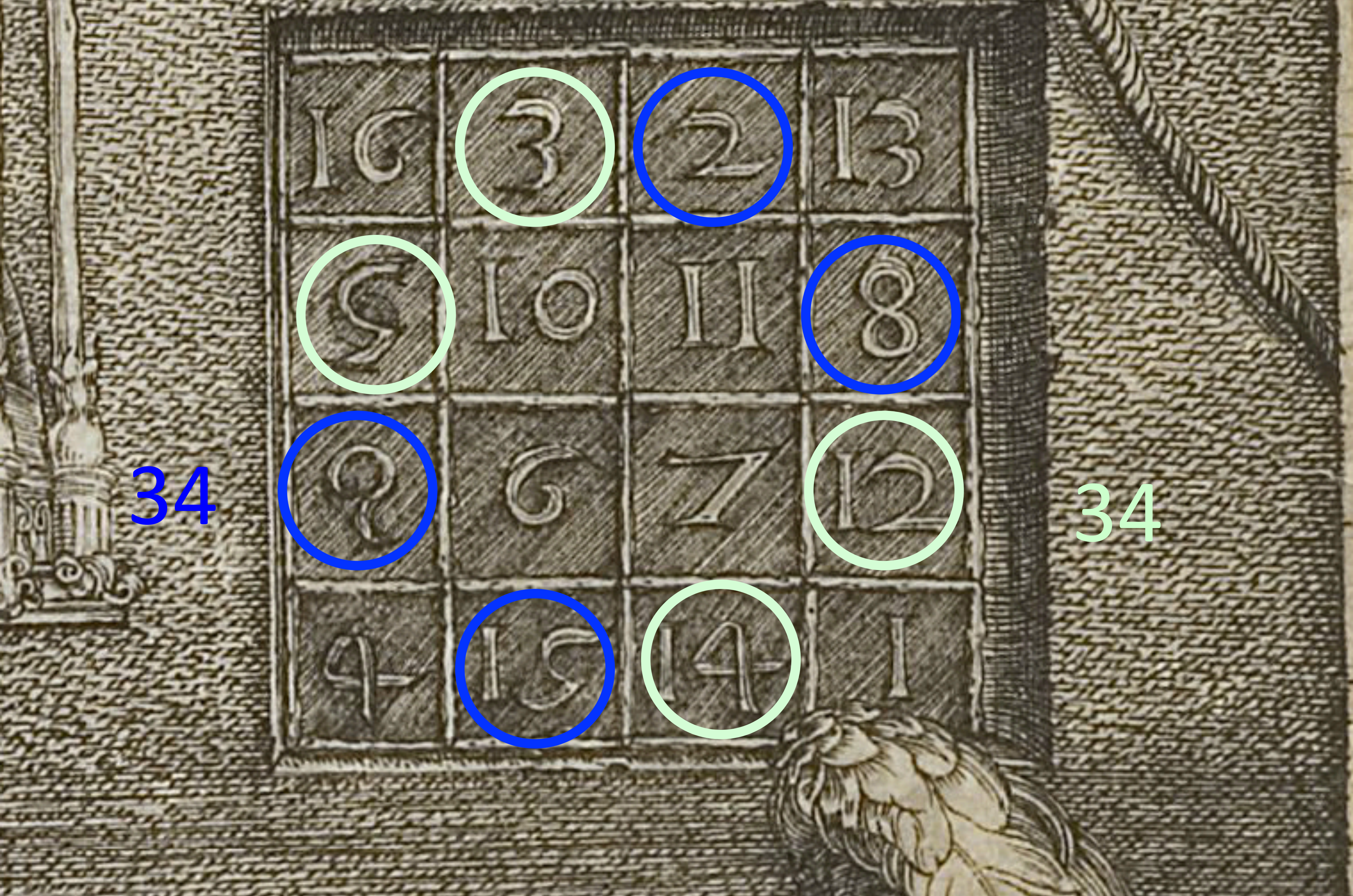


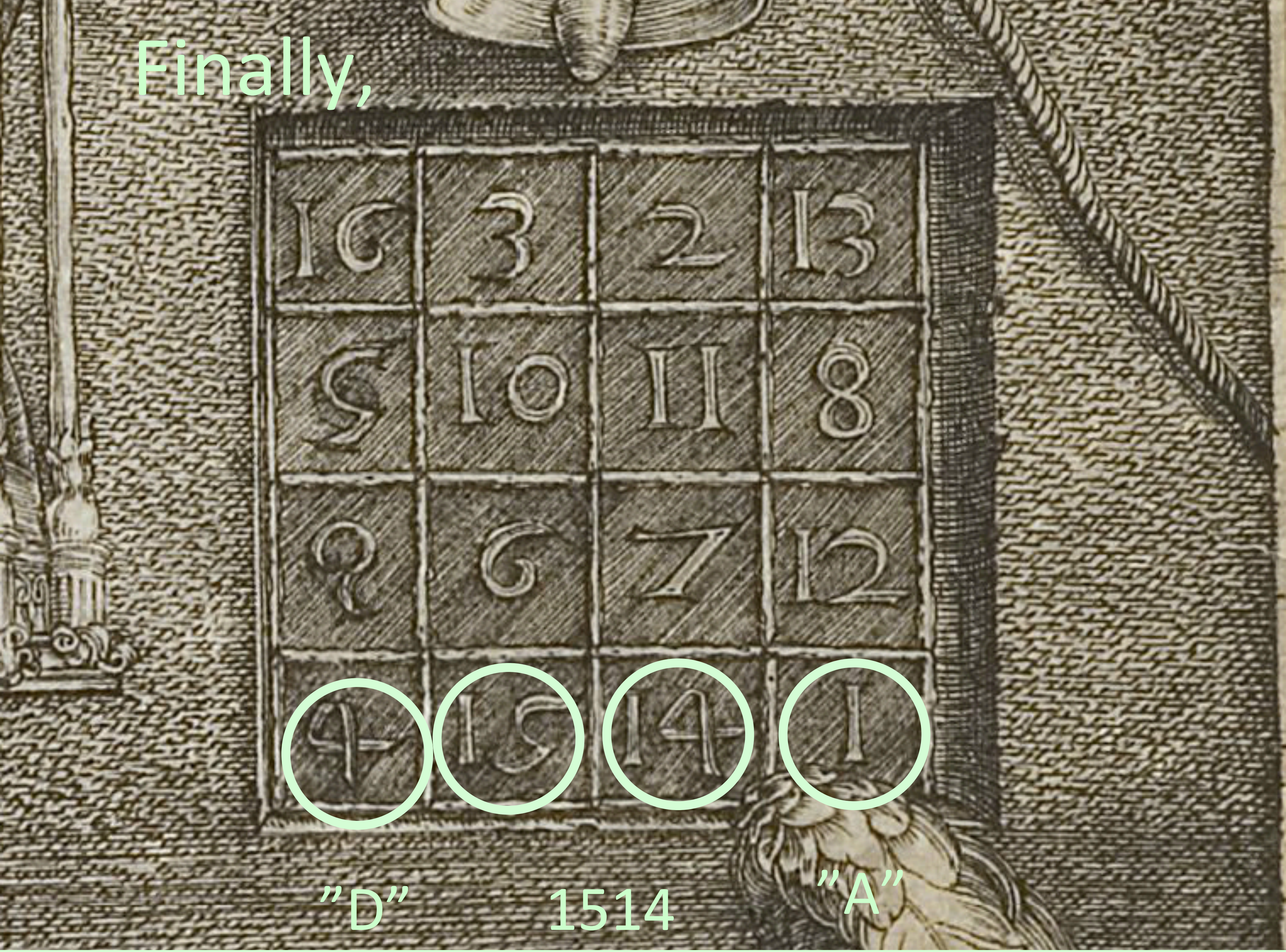
Corners of 3x3 squares:




Also:
16 3 2 13
5 10 11 8
9 6 7 12
4 15 14 1
34
34

Also:




Finally,
16 3 2 13
5 10 11 8
9 6 7 12
4 15 14 1
"D"
1514
"A"

Did Dürer Intentionally Show Only His Second-Best Magic Square?

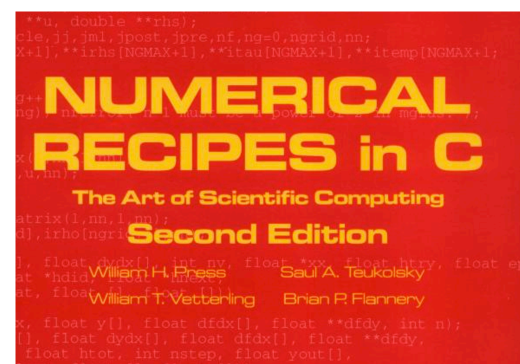
William H. Press
The University of Texas at Austin

December 25, 2009

William Press 2012
AAAS President
University of Texas, Austin

Interchanging two rows gives four additional cruciform sums…

“We want to call out to the angel, ‘Don’t worry! Be happy! Interchange the second and third rows!’ ”



http://www.nr.com/whp/notes/DurerSquare.pdf

Water retention on Dürer magic square:

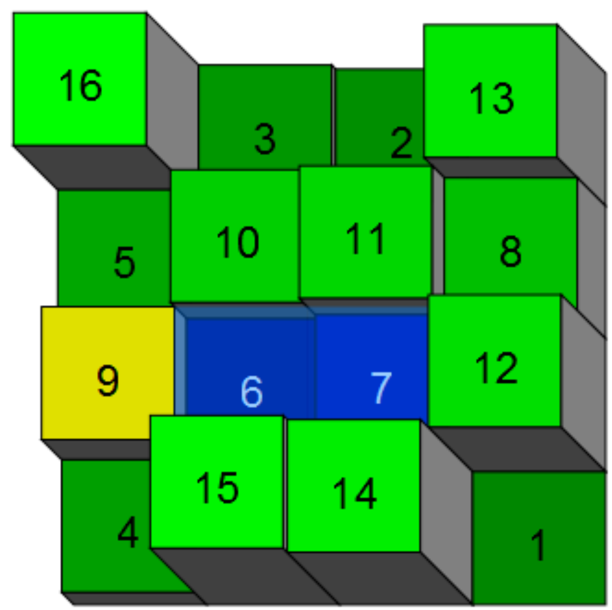
Order 4, cells retaining water 2 (12.5%), units retained 5

Cells

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Units

	3	2	



How many magic square are there?

Results of historical and computer enumerations and estimates.

Order	semi-magic (A)	normal (B)	associative (C)	pandiagonal (D)	ultramagic (E)
3	9	1	1	0	0
4	68 688	880	48	48	0
5	579 043 051 200	275 305 224	48 544	3 600	16
6	**9.4597** (13) $\cdot 10^{22}$	**1.775399** (42) $\cdot 10^{19}$	0	0	0
7	**4.2848** (17) $\cdot 10^{38}$	**3.79809** (50) $\cdot 10^{34}$	**1.125151** (51) $\cdot 10^{18}$	**1.21** (12) $\cdot 10^{17}$	20 190 684
8	**1.0806** (12) $\cdot 10^{59}$	**5.2225** (18) $\cdot 10^{54}$	**2.5228** (14) $\cdot 10^{27}$	**C8 + ?**	**4.677** (17) $\cdot 10^{15}$
9	**2.9008** (22) $\cdot 10^{84}$	**7.8448** (38) $\cdot 10^{79}$	**7.28** (15) $\cdot 10^{40}$	**81·E9 + ?**	**1.363** (21) $\cdot 10^{24}$
10	**1.4626** (16) $\cdot 10^{115}$	**2.4149** (12) $\cdot 10^{110}$	0	0	0

Variants of a square by means of rotations and reflections are not counted.
Statistical notation: **1.2345** (25) $\cdot 10^{9}$ means that the number is not known precisely but is in the interval **(1.2345 ±0.0025)** $\cdot 10^{9}$ with a probability of **99%**.
Ultramagic squares are associative (centrally symmetrical) and pandiagonal.

Source: Walter Trump, http://www.trump.de/magic-squares/howmany.html

Water-filled magic square created for the 500th anniversary of Dürer's Melancholia

100	132	187	6	192	75	9	80	119	74	4	166	144	91
118	168	63	101	62	31	188	34	131	30	161	35	143	114
110	122	59	78	58	151	155	22	145	23	173	26	146	111
116	182	67	149	66	41	189	39	181	37	38	40	140	94
24	115	54	90	134	28	183	21	193	156	138	25	126	92
97	112	56	81	55	27	163	79	184	98	154	19	158	96
3	113	142	82	137	129	176	86	87	88	89	99	147	1
5	95	130	83	171	106	84	85	105	107	123	125	152	8
150	20	71	179	70	53	48	186	47	164	43	194	46	108
121	167	61	104	60	191	32	169	33	128	29	139	36	109
120	42	69	159	68	190	49	148	50	174	44	51	45	170
178	18	180	103	57	133	16	117	14	157	177	124	12	93
135	17	65	162	64	52	15	153	13	136	196	195	11	165
102	76	175	2	185	172	72	160	77	7	10	141	73	127

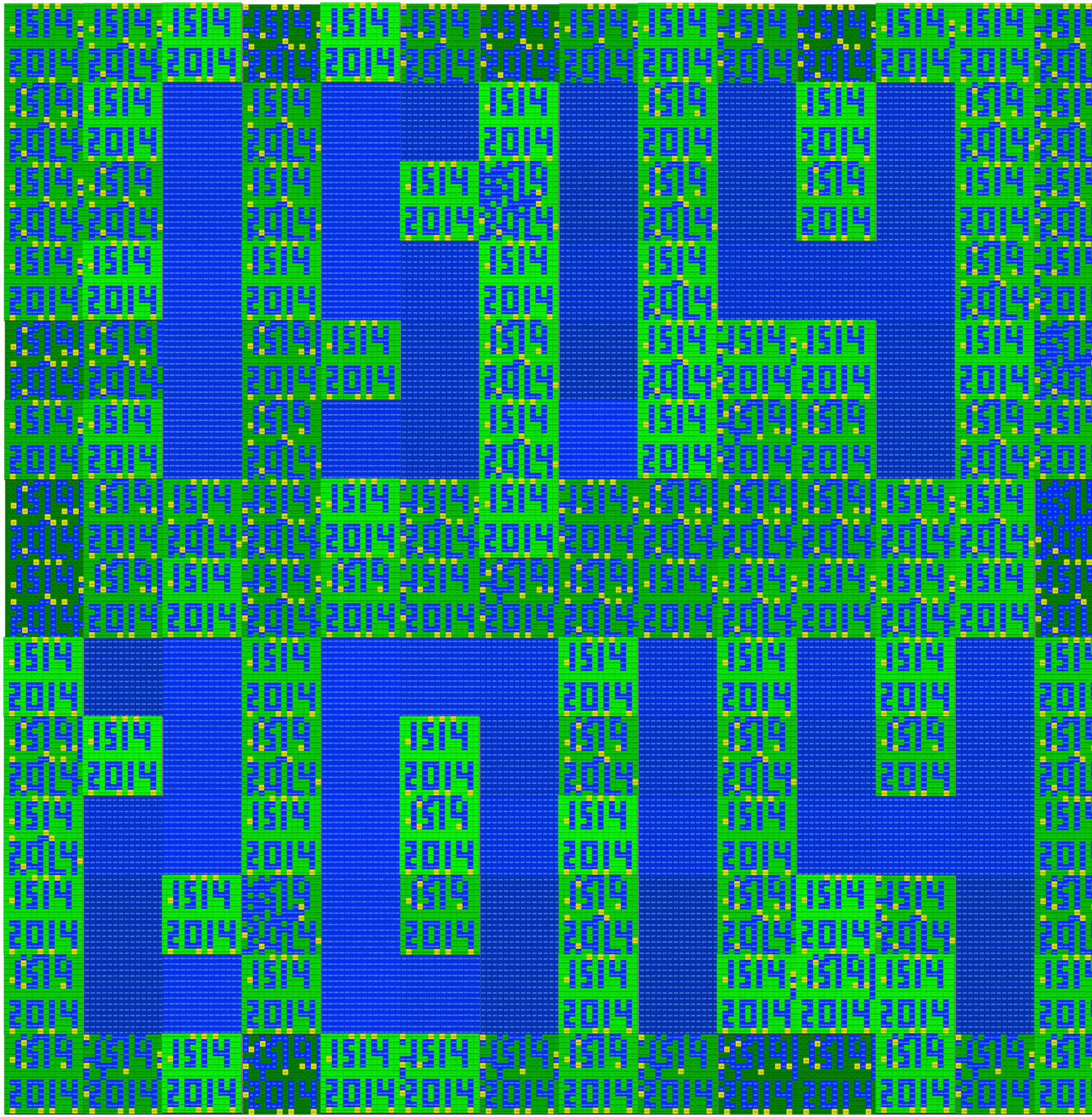
Blue = sites retaining water.

Durer's birth/ death: 1471-1528

Total retention = 2014 units

This is a 196x196 magic square made by convolving the smaller 14x14 magic squares, showing the water retention!

Harry White, Sept. 5, 2012



Retention of first 14 x 14 square

Order 14, cells retaining water 61 (31.1%), units retained 2014

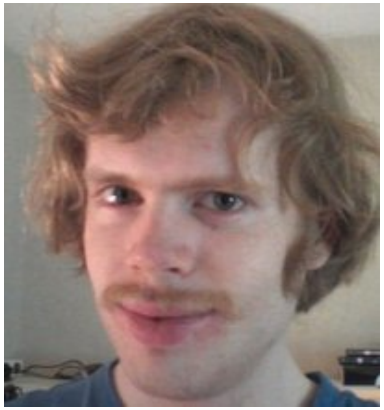
Cells

19504	19523	19587	19410	19598	19479	19413	19484	19558	19478	19408	19544	19554	19495
19497	19572	19467	19500	19466	19435	19596	19438	19555	19434	19586	19439	19547	19503
19505	19509	19463	19516	19462	19577	19566	19426	19533	19427	19562	19430	19548	19511
19518	19591	19471	19553	19470	19445	19593	19443	19571	19441	19442	19444	19540	19513
19428	19482	19458	19499	19538	19432	19569	19425	19585	19560	19567	19429	19565	19498
19494	19557	19460	19485	19459	19431	19574	19483	19588	19512	19525	19423	19527	19517
19407	19508	19514	19486	19578	19510	19580	19490	19491	19492	19493	19530	19551	19405
19409	19501	19559	19487	19542	19515	19488	19489	19496	19519	19521	19541	19556	19412
19581	19424	19475	19545	19474	19457	19452	19582	19451	19570	19447	19575	19450	19552
19526	19599	19465	19529	19464	19595	19436	19520	19437	19532	19433	19537	19440	19522
19561	19446	19473	19563	19472	19594	19453	19597	19454	19546	19448	19455	19449	19524
19564	19422	19584	19507	19461	19536	19420	19543	19418	19534	19600	19528	19416	19502
19535	19421	19469	19549	19468	19456	19419	19576	19417	19579	19589	19592	19415	19550
19506	19480	19590	19406	19583	19573	19476	19539	19481	19411	19414	19568	19477	19531

Units

		15		13	44		46		44		39		
		19		17			58		51		48		
		11		9	34		41		37	36	34		
		24			47		59				49		
		22		20	48		1				55		
	56	5		2	19	24		30		30		27	
		15		12		40		44		44		37	
	34	7		4		23		27		29	22	28	
	58			15		56		63				61	
	59	11		8	20	57		64				62	

- Diagrams created by Harry White and Craig Knecht.
- A constraint-based solver for water retention was also written by Johan Öfverstedt, Uppsala University (Bachelor's thesis under Pierre Flener).



- http://www.ted.com/profiles/963652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