Modelling Lateral and Longitudinal Control of Human Drivers with Multiple Linear Regression Models

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ABSTRACT

In this paper, we describe results to *model lateral and longitudinal control behavior* of drivers with simple *linear multiple regression models*. This approach fits into the *Bayesian Programming* (BP) approach (Bessière, 2008) because the linear multiple regression model suggests an action selection strategy which is an alternative to the BP action selection strategies *draw* and *best.* Furthermore, the inference process provided by a linear multiple regression model is a kind of *short cut inference* compared to the inference approach used in Bayesian networks or Bayesian Programming.

Keywords: digital human modeling, driver modeling, lateral and longitudinal control, linear multiple regression model, Bayesian Programming

INTRODUCTION

Modeling driver behavior is essential for developing error-compensating assistance systems (Cacciabue, 2007). The Human Centered Design of Partial Autonomous Driver Assistance Systems (PADAS) requires Digital Human Models (DHMs) of human control strategies for simulating traffic scenarios (Möbus et al., 2009).

There are a number of control-theoretical driver models (Jürgensohn, 2007; Weir and Chao, 2007) available. Salvucci and Gray (2004) and Salvucci (2007) proposed an integrated model (S&G-model), which has been implemented as a production system within the wide-spread cognitive architecture ACT-R (Anderson et al., 2004). However, only lateral control in this integrated model has been achieved by using a control model, which uses visual signals as input for steering actions. Longitudinal control is missing.

Using linear multiple regression models, we estimated the optimal coefficients as well as parameters for the *lateral* control model from single human test drives. Additionally, the steering model has been reformulated in order to achieve a better fit with human data. The same techniques were applied to *longitudinal* control, i.e. acceleration and deceleration.

This approach fits into the Bayesian Programming (BP) approach (Bessière et al., 2008). The multiple regression model *E(Action | Percepts)* uses the conditional probability distribution *P(Action | Percepts).* This is a *form* in the BP framework. The linear multiple regression model suggests the action selection strategy *selection of expected conditional action* which is an alternative to the BP action selection strategies *draw* and *best.*

The regression models are learnt from multivariate time series of driving episodes generated by a single driver. The variables of the time series describe phenomena and processes of perception, cognition, and action control of drivers according to the S&G-Model. The real-time control of virtual vehicles is achieved by inferring the appropriate actions under the evidence of sensory percepts. This is a slightly different but more efficient action selection strategy than those used in the BP framework. Here the action selection strategies are $draw(P(Action \mid Percepts))$ or $best(P(Action \mid Percepts))$. According to the *draw* strategy the concrete action is randomly selected from the conditional probability distribution $P(Action \mid Percepts)$, while under the *best* strategy the concrete action with the highest probability or density is selected from the conditional probability distribution $P(Action \mid Percepts)$. This differs from our approach, where the selected action is the *conditional expected value* $E(Action \mid Percepts)$ under the constraint of a linear model.

THE TWO-POINT VISUAL CONTROL MODEL OF STEERING

In the Two-Point Visual Control Model (S&G-Model; Salvucci and Gray, 2004), steering actions are controlled by points which are obtained from the road. It is based on the experiments of Land and Horwood (Land and Horwood, 1995; Land, 1998), where participants were shown only small visual segments of the road. This resulted in the hypothesis that the quality of driving improved with the horizontal angle of *two* visual *segments*. This hypothesis was adapted by the S&G-Model and now states that these visual signals are in fact *two points*: The Near Point (N) is defined by its distance d_N to the vehicle. The Far Point's location (F) is dependent on the situation (Figure 1.1): On straight road strips, it is defined by the *escape* *point*, while on bent strips it is defined by the *tangential point*. A third situation is defined by a leading vehicle, but shall not be of further interest in this work.

The respective angles between F and N and the car's longitudinal axis, θ_F and θ_N, constitute the errors for two parallel-connected controllers. Thus, the steering angle φ is computed by a Proportional (P) controller for F and a Proportional-Integral (PI) controller for N in the original S&G-Model (Salvucci, 2004):

$$\varphi = k_N\theta_N + k_F\theta_F + k_I\int\theta_N dt$$

Thus, the coefficients k_N, k_F, and k_I are unknown. In the work of Salvucci and Gray, the distance between N and the car's location is given as 6.2m. However, it should be noted that all original experiments were conducted at the relatively low constant speed of 60.84 km/h. Thus, no longitudinal control was needed in their experimental setting.

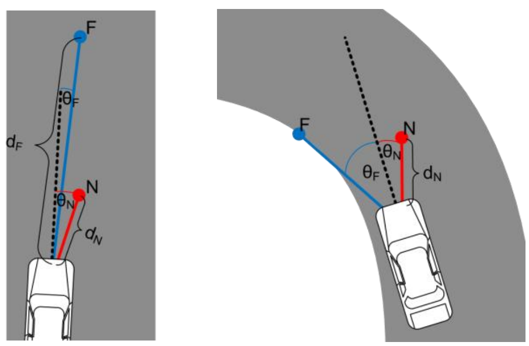

FIGURE 1.1. Straight (left) and Bend (right) situations in the S&G Model

This leads to the question whether the parameters are dependent on the vehicle speed. In order to achieve this, test drives can be used to identify the coefficients as well as the N distance parameter. A less coercive model should be able to cope with high and variable velocities as well.

REIMPLEMENTATION OF THE S&G-MODEL

We already implemented the S&G-Model before (Möbus et al., 2007). We had some doubts whether the autonomous control of a vehicle is dependent on foveal control as is hypothesized by Salvucci and Gray (2004). There is some evidence that ambient vision is sufficient for real-time control in routine driving situations (Horrey et al., 2006). A Bayesian model for longitudinal and lateral control which rests on the assumption of ambient vision has been presented by Möbus and Eilers (2009).

The current implementation uses the open-source TORCS[1] racing simulator as simulation environment, which has been augmented with a development environment for all kinds of control-theory based driver models (Lenk, 2008). In this implementation, the selection rules for the situation-dependent Far Point calculation had to be redefined, as the original ACT-R model could not be reused. The front-vehicle situation has not been considered, thus leaving the escape and tangential point situations. These were distinguished by introducing another parameter, the distance d_F (Figure 1.1) to the Far Point. Thus, if the road strip in the given distance ahead is curved, the tangential point is calculated, otherwise the escape point serves as F.

ESTIMATING LATERAL CONTROL

A simulated test drive has been conducted. The track chosen for this drive features many different types of curves with varying radii. The drive lasted around 14 minutes, during which a multivariate time series with 95522 episodes of data had been sampled. These included the car state, such as position, velocity, acceleration, and orientation, as well as the steering, braking, and accelerating actions of the driver. The human driver had been instructed to drive fast, but careful enough to stay on the road at all times, although cutting curves was permitted. Average speed was 30.65039 m/s (around 110 km/h) with a standard deviation of 5.475582 m/s. This may be a significantly higher average speed than any one encountered in day-to-day traffic, however we felt the model should work under extreme conditions.

REGRESSION FOR THE S&G MODEL

The locations of the control points N and F are hypothetical constructs. We had to infer the distances as a second set of parameters. In order to find the optimal distance parameters, we conducted a grid search over a set of regressions on the same test data with varying distances d_N and d_F. The controller's coefficients were estimated for any distance between $10m$ and $40m$ for d_N and $10m$ to $80m$ for d_F using a multiple linear regression model.

$$y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

The dependent variable y corresponds to the steering angle φ, while the independent variables are x_1 (θ_N), x_2 (θ_F), and x_3 ($\int \theta_N \, dt$). The parameters β_1, β_2, and β_3 are estimated. The model does not include an intercept term. The determination coefficient R^2 of the regressions is very high in general (Figure 2.1). Subsequently, the parameter estimations are used as controller coefficients k_N, k_F, and k_I, respectively. The d_N^{opt}, d_F^{opt}-tuple with the highest coefficient of determination R^2 determines two optimal angles θ_N^{opt} and θ_F^{opt}, which best explain

[1] http://torcs.sourceforge.net/ (last retrieved: 2/25/2010)

the actions of the human driver. Thus, the controller calculates the conditional expected value $E\left(\varphi \middle| \theta_N^{opt}, \theta_F^{opt}\right)$ using the sum of the products of the estimated coefficients with their respective angles.

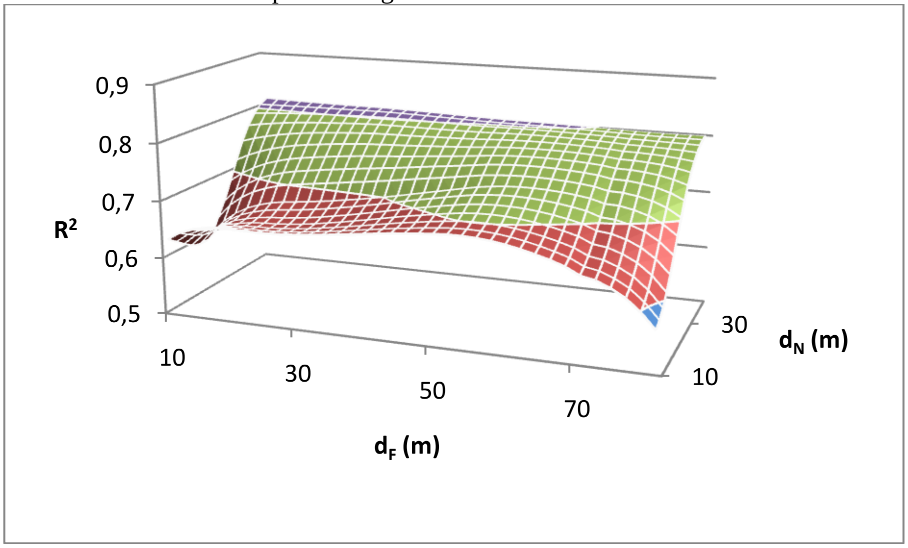

FIGURE 2.1. Coefficient of determination R^2 for multiple regressions on varying distances d_N and d_F using the original unmodified 3-parameter S&G lateral control model.

EXTENDING THE S&G MODEL

We adapted the S&G-Model by introducing a segmentation of the road into *bent* and *straight* strips. This approach effects in a doubling of the number of controller coefficients. Thus, k_{NS}, k_{FS}, and k_{IS} guide the model on straights, while k_{NB}, k_{FB}, and k_{IB} perform the same function on bends.

$$\varphi = \begin{cases} k_{NS}\theta_N + k_{FS}\theta_F + k_{IS}\int \theta_N dt, & \text{on straight segments} \\ k_{NB}\theta_N + k_{FB}\theta_F + k_{IB}\int \theta_N dt, & \text{on bent segments} \end{cases}$$

Accordingly, the data matrix for the predictors in the regression features six columns, which are either filled with the actual values for the angles in one condition or set to zero when the opposite condition applies. The type of Far Point calculation serves as discriminator in order to determine whether the straight or bent strip condition is in place. If the tangential point is calculated, the columns in the data matrix corresponding to θ_{*B} are filled with the actual values, or vice versa if the escape point is calculated.

The results of the regressions (Figure 2.2) generally show a higher coefficient of determination for all reviewed distances d_N and d_F. The maximum R^2 =*0.809* can be found at $d_N^{opt} = 40$ and $d_F^{opt} = 10$. These extreme values may seem surprising at first, but the estimated controller coefficients (Table 2.1) perform well. The resulting behavior is similar to that of the actual test drive, so that while the cutting corner behavior is reproduced, the car is kept stable in the middle of the road.

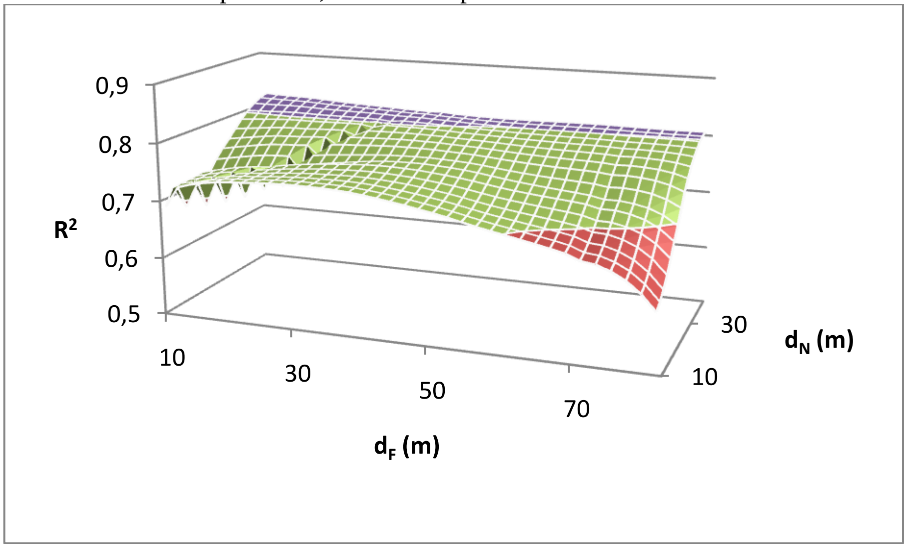

FIGURE 2.2. Coefficient of determination R^2 for multiple regressions on varying distances d_N and d_F using the modified lateral control model with segmentation.

Due to the fact that d_F^{opt} is actually lower than d_N^{opt}, the name characterization as a “Far Point” might be disputed. However, this is only the case for the straight condition, since the distance of the tangential point in the bent condition is usually higher than the distance to the Near Point. This could be an indicator that a single-point model could be preferable on straight roads.

Table 2.1 Estimated controller coefficients for $d_N^{opt} = 40$ and $d_F^{opt} = 10$.

k_{NS}	k_{FS}	k_{IS}	k_{NB}	k_{FB}	k_{IB}
0.166	0.0038	6.16×10^{-5}	0.1295	0.0461	-0.0002

ESTIMATING LONGITUDINAL CONTROL

In order to achieve longitudinal control emitting acceleration and deceleration actions, a naïve control model has been chosen first. Thus, braking and accelerating actions u are numeric values on the same axis with opposite signs. A PID controller adjusts this value using the difference between the actual velocity v and a set-point velocity v_d (Coller, 2007).

$$u = -\left(c_P(v - v_d) + c_I \int (v - v_d)dt + c_D \frac{d(v - v_d)}{dt}\right)$$

However, while the actual velocity v is known from the experiment data, the set-point velocity v_d might be considered an internal state of the driver, thus not being observable. However, it might be approximated by a heuristic using two parameters: a braking rate b and a maximum velocity v_{top}.

For each road segment s, a maximum velocity v_s^{max} may be determined. For a bent road segment, it is defined by the radius r_s, and a friction constant f_s.

$$v_s^{max} = \begin{cases} \min\left(\sqrt{f_s \cdot G \cdot r_s}, v_{top}\right) & \text{on bends} \\ v_{top} & \text{on straights} \end{cases}$$

All road segments within a velocity-dependent look-ahead distance $d_l = v^2/(2 \cdot b)$ are examined for their maximum velocity. Thus, the set-point velocity is approximated by $v_d = \min(v_s^{max} \mid \forall s \; with \; d_s < d_l)$.

With this approximation of v_d, it is possible to conduct yet another grid search over the parameters b and v^{top}, using multiple linear regressions to estimate c_P, c_I, and c_D (Figure 3.1). Once again, the coefficient of determination is used to select the optimal parameters b^{opt} and v_{top}^{opt}, which effect an optimal set-point speed v_d^{opt}. Thus, the controller output is the conditional expected value $E\left(u \middle| v_d^{opt}\right)$.

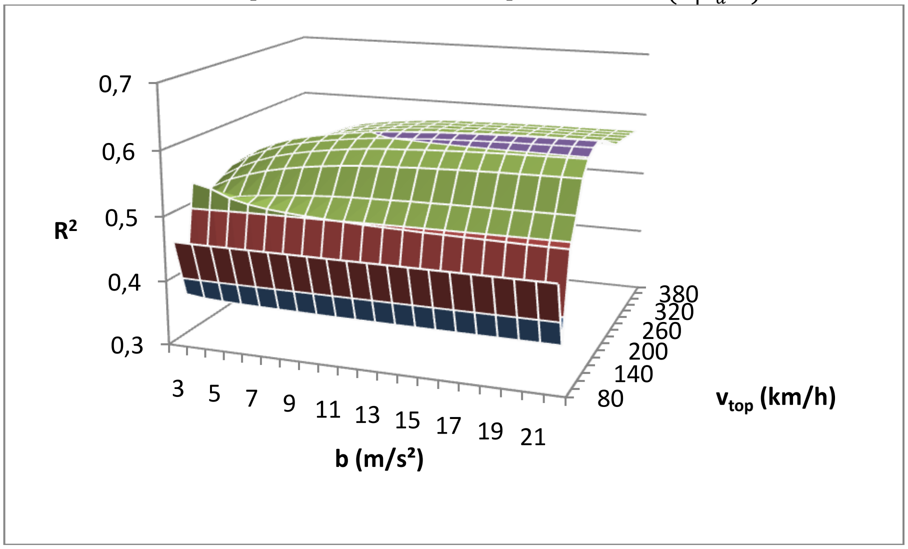

FIGURE 3.1. Coefficient of determination R^2 for multiple regressions on varying distances b and v_{top} using the longitudinal control model.

Again, the values $b^{opt} = 22$ and $v_{top}^{opt} = 220$ may seem extreme, but provide the best fit with human data with $R^2 = 0.612$. The estimated coefficients for this configuration (Table 3.1) provide adequate acceleration and deceleration during the model run. It should be noted that the longitudinal controller ignores the current gear state altogether, even though the fuel pedal state depends on it. Thus, the estimation for the controller would have an even better fit with human data if it were adapted to accommodate this variable.

Table 3.1 Estimated longitudinal controller coefficients for $b^{opt} = 22$ and $v_{top}^{opt} = 220$

c_P	c_I	c_D
0.0069	2.59×10⁻⁶	-5.35×10⁻⁵

DEPENDENCY OF LONGITUDINAL CONTROL ON VISUAL PERCEPTS

Clearly, the above controller is not an entirely plausible model of a human driver, since the set-point speed v_d^{opt} cannot be readily established from human data. However, it may be derived from the absolute values of the visual percepts θ_N^{opt} and θ_F^{opt}. Thus, a nested controller may be embedded in the above longitudinal controller, which estimates the expected value $E\left(v_d^{opt} \middle| \theta_N^{opt}, \theta_F^{opt}\right)$. Again, discrimination between straight and bent segments takes place. A constant k is needed to provide a positive value when both angles converge to zero.

$$v_d^{opt} = \begin{cases} c_{NS}\left|\theta_N^{opt}\right| + c_{FS}\left|\theta_F^{opt}\right| + k & \text{on straights} \\ c_{NB}\left|\theta_N^{opt}\right| + c_{FB}\left|\theta_F^{opt}\right| + k & \text{on bends} \end{cases}$$

Using a single regression, values for the coefficients may be estimated (Table 3.2). The coefficient of determination for this regression is $R^2 = 0.5716$. In a way, both regressions for the longitudinal controller effect a role change for v and v_d^{opt}. As the velocity v is provided by human data, it can be considered the set-point velocity, while its difference to the model-provided v_d^{opt} is minimized by the nested regressions.

Table 3.2 Estimated controller coefficients v_d^{opt}.

c_{NS}	c_{FS}	c_{NB}	c_{FB}	k
-43.1718	-12.7844	-105.2268	-9.0415	60.6528

MODEL RUN

If the integrated model is run on the same track as the test drive using the estimated coefficients (Tables 2.1, 3.2, and 3.3) the model performance shows similar behavior for lateral as well as longitudinal control. The achieved average velocity of $29.98395\ m/s$ is slightly lower than in the original human data and the model does not achieve the same high velocities as the original human driver (Figure 3.2) with a standard deviation of $5.123619\ m/s$. Nevertheless, it performs well on the road. Generally spoken, the model actions tend to be more temperate than those of the human driver.

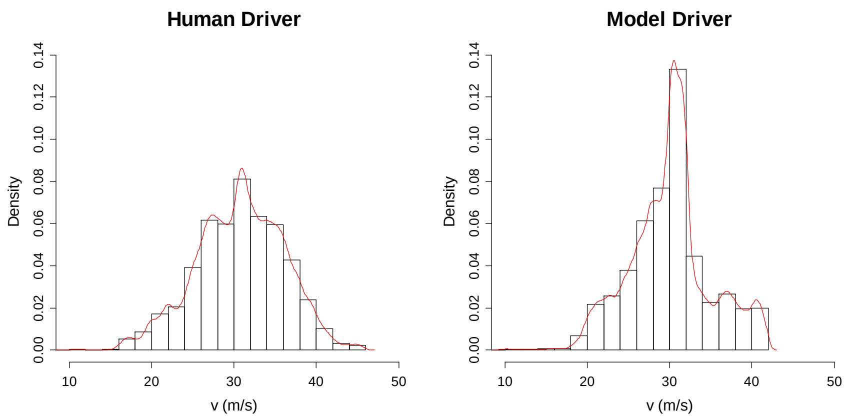

FIGURE 3.2. Density of achieved velocities by Human Driver (left) and integrated model (right) while driving.

CONCLUSION

We reformulate the S&G Model of steering as a linear multiple regression model and extend this model with a second linear multiple regression model for the purpose of longitudinal control. The inference process provided by a linear multiple regression model is a kind of *short cut inference* compared to the inference approach used in Bayesian networks or Bayesian Programming. From this perspective the percept-based inference or selection of actions on the basis of linear multiple regression models is suited for reactive agents and seems to be more efficient than the usual inference process of the BP approach. But there is an efficiency-flexibility trade-off. Linear models are more restrictive than the flexible Bayesian Programs. They only allow one direction of inference.

The proceedings introduced in this work are reproducible for other human test data, even though the concrete estimations may vary. Further study is required to determine the relationships between lateral and longitudinal control.

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