Networks and Complexity

Networks are everywhere. Examples include the internet on which you are reading this text, the power grid that delivers electricity to your home, the food webs which form the backbones of ecosystems, the social networks which allows opinions, ideas and diseases to spread among humans and the networks of biochemical reactions that sustain all life on earth.

In this course we will understand how network thinking can be used to make sense of the many complex processes around us. Along the way we will be drawing on ideas from Physics, Mathematics, Computer Science, Ecology and Sociology.

The lectures will revolve around **real world examples** that pose specific challenges. These range from finding the shortest path to a destination to analyzing the stability of complex ecosystems. We will then discover **broadly applicable methods** to overcome these challenges and in every case we will be able to apply the methods to **small examples with just pen and paper**.

The course will equip you with a set of tools that you can use to understand complex real world systems. We will build up an understanding why these tools work and which lines of thinking could have led to their discovery. In this way we will learn how to think about complexity to develop new tools and overcome new challenges.



Joining Instructions

Lectures will take place 18:15 - 19:45 CEST on Mondays and Thursdays from Apr 21 till Jul 21.

Students from Oldenburg and Neighboring Universities please join using the lecture number

5.15.755

The lecture is taught completely in English and will be open to international guests. If you want to join as a guest please contact

thilo2gross@gmail.com

Schedule and Topics

- Apr 21 Kruskal's greedy algorithm The language of networks, brute force solutions, greedy algorithms, counting networks, spanning trees, some combinatorics, and how to build an optimal power grid.
- Apr 25 The shortest darkest path Walks, paths, components, directed networks, Dijkstra's algorithm and how to branch-and-bound into Mordor.
- Apr 28 **Bridgewalk** Eulerian circuits, trial-and-error, Hierholzer's algorithm, building bridges, Chinese Postmen and how to patrol a building.
- May 02 SUMMARY NETWORK ALGORITHMS
- May 05 The small world Six degrees of separation, Milgram's letters, Bacon numbers, diameter, tree-like approximation, clustering coefficient, the strength of weak ties, and the secret power of wrong assumptions.
- May 09 **Friends of friends** The magic of heterogeneity, degree distribution, excess degree, Kronecker deltas, how to become a network scientist and the friendship paradox.
- May 12 Random graphs ER random graphs, the Binomial Formula, ensembles, independence, scaling, motifs, symmetry, phase transitions, and a surprising result on triangles.
- May 16 A curse of cows Configuration model, giant components, network fragmentation and a deadly disease.
- May 19 Massively parallel maths Generatingfunctionology, more components, series solved, excess degree done elegantly, and bags full of gold.
- May 23 **Attacks on networks** Infrastructure robustness, cutting links, random damage, targeted attacks, viral spreading and how to plan vaccination campaigns.
- May 30 NETWORK STRUCTURE SUMMARY
- Jun 02 **Growth of the Internet** Laplace's demon, mass action laws, integration, surprising explosions, scale free networks, and a proof that the internet is still young.
- Jun 09 A secret of bees Dynamical systems. Ancestral trees, separationsansatz, operator splitting, golden ratios, eigendecomposition, and a secret of bees.
- Jun 13 The last great hunters Ecological dynamics, stability analysis as a modeling tools, more differential equations, and the reason why tigers are good for nature.
- Jun 16 **The critical brain** Some neuroscience, bifurcation theory, types of bifurcations, and the edge of chaos.
- Jun 20 Rumors and other epidemics Epidemic spreading, rumors, decision making, mean-field and moment expansion, conservation laws, and a model of social distancing.

Jun 23 - **More is different** - Coarse graining, predator-prey dynamics, slow-fast systems, manifolds, separatrices, and the emergence of simplicity in complex systems.

Jun 27 - NETWORK DYNAMICS SUMMARY

- Jun 30 The most important node Spectral methods, network centrality, some localization, implicit function theorem, impact and sensitivity and how to find the most important fish in a lake.
- Jul 04 Complexity-Stability debate May's random matrix model, stability, Ghershgorin's theorem, motifs, and the mysterious stability of large ecological systems.
- Jul 07 **Measuring the vibe of a city** Vibrations, Laplacians, hearing the shape of drums, diffusion maps and an application to urban data analytics.
- Jul 11 **Meta-ecosystems** Diffusion in space and on networks, Kronecker products, master stability functions, Turing-bifucation, pattern-forming instabilities, and how to design highways for foxes.
- Jul 14 **No current without heat** Global stability, master equations, Lyapunov functions, different entropies, Kirchhoff's theorem and a derivation of thermodynamic laws.
- Jul 18 One Ring to Rule them All? Another look at Dijkstra, the beauty of rings, exotic algebras, making connections, tying up loose ends, planning construction projects with tropical semiring.

Jul 21 - FINAL SUMMARY

Promises

- Everything we do can be applied to the real world. We will see examples and you invest some effort you will leave this course with many powerful tools that can be applied to a wide range of topics.
- 2. All methods can be applied to small examples with **pen and paper** or at most a handheld calculator. No programming skills are required (but if you are a good programmer this course will give you some ideas).
- 3. We take an **interdisciplinary** perspective, using methods and insights from different disciplines including physics, math, computer science, ecology, sociology and of course network and data science. By making links between these disciplines we will get a much deeper understanding of the respective methods.
- There is a strong focus on how to think about complexity. Instead of just presenting or deriving methods I want to explain how you would have needed to think about the system to discover the solution yourself.
- 5. There will be a lot of **exercises with worked solutions**, so there will be a lot of opportunities to practice what we learned.

Frequently Asked Questions

Is this going to be very difficult?

I don't know. You tell me, is it difficult to drive a car? I'd say it isn't if you know how to do it, otherwise it's crazy difficult. Is it difficult to build a house? Juggle with five balls? Develop a computer game? It's always the same answer, isn't it? I hope that the content of this course will be difficult the first time you see it but easy at the end of the term. I will try my best to make the transition as smooth as possible.

What's the nature of the difficulty?

Things can be difficult for different reasons. Some things are difficult because they are badly explained or because the lecturer does not understand them very well in the first place. I will do my best to avoid these scenarios. There will still be difficult bits and this is because we will be training our brains to think in new ways. So the difficulty that we want to encounter is the difficulty of new concepts: Learning to think in high-dimensional spaces, and generally speaking handling complexity with your mind.

Will there be a lot of math?

Yes, but I will explain all the math that we need along the way. So, there are no prerequisites except high-school mathematics (e.g. manipulating equations, derivatives, knowing what a vector is). In particular I will try to explain the math more deeply than it is normally explained. While most textbooks and lectures just show how to solve mathematical problems I will try to explain how to think about problems to discover the solution. In my experience this makes it easier and leads to a deeper understanding of the maths.

Will this be a lot of work?

Yes it will, but it will be worthwhile. My promise to you is that I will work hard to make the learning curve as smooth as possible. However that alone will not make things 'easy' or 'intuitive'. Ultimately you can only make things easy for yourself, by practicing them. Think of juggling: a good theoretical explanation of what to do can help you get started, but ultimately the only way in which you can make juggling easy is to practice (a lot). To help you with this I will provide a lot of exercises with worked solutions. Still to keep up you need to invest some time in this course.

I am a PLACEHOLDER. Can I do this course?

Yes, I have given this course at 4 different universities and in 4 different departments. I remember students from the following departments taking the course: Mathematics, Physics, Biology, Social Sciences, Hydrology, Media Studies, Engineering (of various kinds), Ecology, Public health, Psychology, Neuroscience, etc. I am aware of about 10 or so students who subsequently used methods from the course in research publication during their PhDs.

No Prerequisites, really?

Yes, we will only need high-school level math. Of course if you have already done a lot of math or physics you will have an easier time with some of the topics. However, even if you know a lot of these things already, much will still be new. For example if you are a Computer Scientist much of what I say about algorithms will be familiar, but will also start to see algorithms in a completely different light that is not normally taught in Computer Science.