ON APPLICATION OF NON-NEGATIVE MATRIX FACTORIZATION FOR AD HOC MICROPHONE ARRAY CALIBRATION FROM INCOMPLETE NOISY DISTANCES

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ABSTRACT

We propose to use non-negative matrix factorization (NMF) to estimate the unknown pairwise distances and reconstruct a distance matrix for microphone array position calibration. We develop new multiplicative update rules for NMF with incomplete input matrix that take into account the symmetry of the distance matrix. Additionally, we develop a convex matrix completion method which is related to an l_2-regularized symmetric NMF. Thorough experiments demonstrate that the proposed methods lead to substantial improvement over the state-of-the-art techniques in a wide range of signal-to-noise and unknown-distance ratios. The convex symmetric matrix completion method was found to be the most robust method with less computational cost.

Index Terms – Ad hoc array calibration, Non-negative matrix factorization, Euclidean Distance Matrix, Convex optimization

1. INTRODUCTION

Ad hoc microphone arrays are at the heart of ubiquitous sensing for future sound technologies. Application of microphones in an ad hoc setup enables high-quality acquisition of distant sound in a distributed and flexible infrastructure. The distant acquisition is widely used for source localization and separation [1–5], videoconferencing and distant speech recognition in multiparty environments [6–9]. A fundamental step to enable processing of an array of distant recordings is to account for the acoustic sampling effect via calibration of the microphone positions. Calibration is often achieved in two steps: estimation of the distances between the pairs of microphones and reconstruction of the array geometry given all the pairwise distances. In a scenario where the microphones are distributed randomly in a possibly large deployment area, the conventional techniques for distance estimation may fail to measure all the pairwise distances. This can happen due to device malfunctioning or architectural barriers. Furthermore, some of the microphones deployed far apart deteriorate in capturing the sound energy leading to a locality constraint in distance estimation. Hence, the main goal of this work in to develop a method for microphone array calibration using an incomplete set of pairwise distances which is robust against measurement noise.

The classic multi-dimensional scaling (MDS) method is a common technique to reconstruct the microphone array geometry [10]. This method applies a double centering transformation to subtract the row and column means of the distance matrix. Then a low-rank projection is applied to extract the relative microphone positions. If some of the pairwise distances are missing, an extension of this method called MDS-MAP can be used. This method approximates the unknown distances by the shortest path defined as the minimum sum of the distance measures of the constituent edges of the microphones. Recent advances incorporate the properties of the Euclidean distance matrix through definition of the appropriate cost functions via the algebraic s-stress method [11] and formulate efficient optimization schemes for finding the geometry via semidefinite programming (SDP) [12]. State-of-the-art microphone array calibration methods are often very sensitive to unreliably-measured or missing distances. There is very little work to address the problem of unknown pairwise distances. In [13], a joint source localization and microphone calibration algorithm is proposed using the bilinear optimization approach. This algorithm is able to handle missing distances, but it requires a minimum of five microphones and thirteen sound source events.

In this paper, we rely on the characteristics of Euclidean distance matrix (EDM) to estimate the unknown pairwise distances. The matrix consisting of the squared pairwise distances has very low rank (explained in Section 2.1). Candès et al. [14] showed that a small random fraction of the entries are sufficient to reconstruct a low-rank matrix *exactly*. Drineas et al. [15] exploited the low rank property to complete the distance matrix, but the full set of EDM properties are not incorporated. In our earlier work [16], we extended their approach by developing a EDM matrix completion algorithm within the framework proposed by Keshavan et al. [17]. In this paper, we study the application of non-negative matrix factorization (NMF) in low-rank matrix completion for distance matrix reconstruction and calibration. Although NMF has been previously used in various applications within the speech community, such as speech enhancement [18] and source separation [19], the current work is the first to address the microphone array calibration. We derive the procedure for EDM matrix factorization taking into account the symmetric constraint in multiplicative rules [20]. We further elaborate on convex relaxation of our objective using an energy constraint on the latent factors. We extend the optimization via alternative direction of multiplier method (ADMM) [21] and develop the convex optimization procedure for reconstruction of symmetric matrices. The matrix reconstruction is followed by EDM projection to find the microphone coordinates. We analyze the performance of each method in various noisy conditions at different ratios of missing distances. The importance of incorporating the EDM properties and the connectivity of the array with respect to the number of microphones are explicitly quantified.

The rest of the paper is organized as follows: In Section 2, we define the calibration problem from a subset of the pairwise distances. The procedure for reconstructing a low-rank matrix via NMF with missing entries is elaborated in Section 3 where the multiplicative update rules for symmetric NMF and the convex relaxation are explained along with the EDM projection. The experiments are presented in Section 4 and the conclusions are drawn in Section 5.

2. DISTANCE-BASED CALIBRATION

The problem of distance-based microphone array calibration is to find the Cartesian coordinates of the microphones, $\boldsymbol{x}_i \in \mathbb{R}^\kappa, i \in \{1, \ldots, N\}$ based on a subset of observed pairwise distances, $d_{ij} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ where $\|.\|$ stands for the Euclidean norm. This problem is formalized as the following:

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}}{\arg\min} \sum_{(i,j)\in E} \left(\|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2^2 - d_{ij}^2\right)^2 ; \tag{1}$$

$\boldsymbol{X} \in \mathbb{R}^{N\times\kappa}$ denotes the position matrix formed as $[\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N]^{\mathrm{T}}$ where $.^{\mathrm{T}}$ is the transpose operator and E denotes the set of known pairwise distances[1]. The algorithmic approaches to reach the objective of (1) requires all the pairwise distances. The goal of this paper is to enable microphone array calibration from a small subset of pairwise distances, i.e. $|E| \ll N^2$ where $|\cdot|$ denotes the cardinality of a set. The key idea to handle this problem is to exploit the properties of the squared distance matrix to recover the unknown distances prior to position estimation.

2.1. The Low-rank Squared Distance Matrix

Consider a matrix $\boldsymbol{M} \in \mathbb{R}^{N\times N}$ consisting of the squared pairwise distances between N microphones defined as $\boldsymbol{M} = \left[d_{ij}^2\right]$, The matrix $\boldsymbol{M}$ has rank at most $\eta = \kappa + 2$ [15]. Due to the rank deficiency, there is a strong dependency among the entries of the squared distance matrix and $O(\eta N)$ measurements suffice to recover the missing components [14].

In addition to the low-rank property, the squared distance matrix is a Euclidean distance matrix (EDM) which possesses additional structures. These structures should be incorporated to recover the matrix components. A common technique for low-rank matrix approximation relies on non-negative matrix factorization (NMF). In the following Section 2.2, we elaborate on the theory of NMF of partially-given matrices. Generalization of the proposed approach to include the EDM properties is explained in Section 3.

2.2. NMF Formulation with Missing Entries

The standard NMF is developed for matrices with known entries. In this section, we use NMF as a matrix completion method where a low-rank approximation of a distance matrix is obtained using only a subset of the entries, which are known. Let $\mathbf{M} \in \mathbb{R}^{N\times N}$ be the squared distance matrix of rank η. We define a matrix $\mathbf{M}_E$ consists of the observed squared pairwise distances, $d_{ij}^2, (i,j) \in E$ and zeros as the unknown components.

To apply NMF, a fidelity measures is required to obtain a cost function to quantify the goodness of the approximation. Various fidelity measures, such as Kullback-Leibler (KL) divergence and Euclidean distance, have been used in the literature, where one of these measures may be preferred depending on a given application. In this paper, we limit the discussions to KL divergence as it was observed to yield more accurate calibration performance in our experiments.

[1] Dropping the square power on distances as $(\|\boldsymbol{x}_i - \boldsymbol{x}_j\| - d_{ij})$ leads to the maximum likelihood estimation of the microphone coordinates, however achieving the exact solution is algorithmically more complex. Hence, the squared distance formulation (1) is often considered [22].

We define the following cost function to apply NMF with missing entries:

$$\hat{\boldsymbol{W}}, \hat{\boldsymbol{H}} = \underset{\boldsymbol{W},\boldsymbol{H}\geq 0}{\arg\min}\ \mathrm{KL}\left(\boldsymbol{M}_E \| \boldsymbol{W}\boldsymbol{H}\right), \tag{2}$$

where $\mathbf{W} \in \mathbb{R}^{N\times\eta}$, $\mathbf{H} \in \mathbb{R}^{\eta\times N}$ are both element-wise non-negative defined as $\boldsymbol{W}, \boldsymbol{H} \geq 0$, and $\mathrm{KL}\left(\mathbf{M}_E \| \mathbf{W}\mathbf{H}\right)$ denotes the generalized KL divergence:

$$\mathrm{KL}\left(\mathbf{M}_E \| \mathbf{W}\mathbf{H}\right) = \sum_{(i,j)\in E} \left(M_{ij} \log \frac{M_{ij}}{(WH)_{ij}} - M_{ij} + (WH)_{ij} \right).$$

After solving (2), the unknown entries of $\boldsymbol{M}_E$ are recovered as $\hat{\boldsymbol{M}} = \hat{\boldsymbol{W}}\hat{\boldsymbol{H}}$. Eq. (2) can be solved using an iterative gradient descent method with an adaptive learning rate [20]. Following this method, we can write:

$$\begin{aligned} W_{ia} &= W_{ia} - \gamma\left(W_{ia}\right) \frac{\partial \mathrm{KL}\left(\mathbf{M}_E \| \mathbf{W}\mathbf{H}\right)}{\partial W_{ia}} \\ H_{a\mu} &= H_{a\mu} - \gamma\left(H_{a\mu}\right) \frac{\partial \mathrm{KL}\left(\mathbf{M}_E \| \mathbf{W}\mathbf{H}\right)}{\partial H_{a\mu}}, \end{aligned} \tag{3}$$

where the learning rate γ is defined as:

$$\gamma\left(W_{ia}\right) = \frac{W_{ia}}{\sum\limits_{\mu} H_{a\mu}}, \quad \gamma\left(H_{a\mu}\right) = \frac{H_{a\mu}}{\sum\limits_{i} W_{ia}}, \quad \forall (i,j) \in E. \tag{4}$$

Substituting (4) in (3) leads to multiplicative update rules that guarantee the convergence to a local minimum of the cost function in (2) [20]. In Section 3, we elaborate on the procedure of recovering Euclidean distance matrices via non-negative matrix factorization and a closely related convex non-negative matrix completion approach.

3. EDM-NMF PROCEDURE

We start by revising the multiplicative update rules of the standard NMF for symmetric matrices in Section 3.1. Then, the convex formulation and full EDM projection are presented in Sections 3.2–3.3.

3.1. The Multiplicative Symmetric NMF Algorithm

The standard NMF algorithm recovers a low-rank matrix with components as close as possible to the known entries. However, the recovered matrix does not necessarily correspond to a Euclidean distance matrix; for example, EDMs are symmetric matrices and this additional structure is not included in the matrix factorization algorithm. Hence, we modify the NMF multiplicative update rules [20] to have, as output, matrices that are closer to EDMs. More specifically, the NMF objective is formulated with an additional symmetric constraint as follows:

$$\hat{\boldsymbol{W}}, \hat{\boldsymbol{H}} = \underset{\boldsymbol{W},\boldsymbol{H}\geq 0}{\arg\min}\ \mathrm{KL}\left(\boldsymbol{M}_E \| \boldsymbol{W}\boldsymbol{H}\right) + \mathrm{KL}\left(\boldsymbol{M}_E \| (\boldsymbol{W}\boldsymbol{H})^{\mathrm{T}}\right). \tag{5}$$

It may be noted that (5) is just one way to incorporate the symmetric property which was found to be very effective in our experiments (c.f. Fig. 2). Following the method presented in Section 2.2, we can derive multiplicative update rules to solve (5), which are given as:

$$\begin{aligned} W_{ia} &\longleftarrow W_{ia} \left[\frac{\sum_\mu H_{a\mu} (M_E)_{i\mu} \left(1/(WH)_{i\mu} + 1/(WH)_{\mu i}\right)}{2\sum_\nu H_{a\nu}} \right] \\ H_{a\mu} &\longleftarrow H_{a\mu} \left[\frac{\sum_i W_{ia} (M_E)_{i\mu} \left(1/(WH)_{i\mu} + 1/(WH)_{\mu i}\right)}{2\sum_k W_{ka}} \right]; \end{aligned} \tag{6}$$

Staring from non-negative initialization, convergence to non-negative factorization is guaranteed. The Eq. (6) is referred to as the multiplicative NMF or shortly *NMF-MUL* during the experimental evaluation presented in Section 4.

It is worth to mention that the above update rules can be easily modified to take into account the sparsity of the latent factors. One common method for this purpose is to regularize the NMF cost function with weighted l_1-norm (sum of the absolute values of the elements) of $\boldsymbol{H}$ or $\boldsymbol{W}$ using the basis pursuit framework [19]. As we will empirically observe in Section 4, the sparsity regularization is particularly helpful when the ratio of the missing entries is high.

3.2. Convex Relaxation and Optimization via ADMM

The non-convex problem stated in (5) can be relaxed as

$$\underset{\boldsymbol{WH}\geq 0}{\arg\min}\ \mathrm{KL}\left(\boldsymbol{M}_E \|\boldsymbol{WH}\right) + \mathrm{KL}\left(\boldsymbol{M}_E \|(\boldsymbol{WH})^{\mathrm{T}}\right) + \frac{\lambda}{2}\left(\|\boldsymbol{W}\|_{\mathrm{F}}^2 + \|\boldsymbol{H}\|_{\mathrm{F}}^2\right), \tag{7}$$

where λ is the regularization parameter; the additional energy-based penalty, i.e. the Frobenius norm $\|.\|_{\mathrm{F}}$ on the latent factors, has been motivated for stability reasons and empirically observed to improve the performance. Furthermore, the following relation holds between the nuclear norm $\|.\|_*$ of the matrix (defined as the sum of the absolute singular values) and the energy of the latent factors:

$$\|\boldsymbol{M}\|_* = \inf_{\boldsymbol{W},\boldsymbol{H}:\boldsymbol{WH}=\boldsymbol{M}} \frac{1}{2}\left(\|\boldsymbol{W}\|_{\mathrm{F}}^2 + \|\boldsymbol{H}\|_{\mathrm{F}}^2\right), \tag{8}$$

as long as η (column dimension of $\boldsymbol{W}$) $\geq$ rank$(\boldsymbol{M})$ [23]. Hence, the following theorem is a consequence:

Theorem 1. The non-convex symmetric NMF objective in (7) for any $\eta \geq$ rank$(\boldsymbol{M})$ can be relaxed to obtain the following convex optimization problem:

$$\underset{\boldsymbol{WH}\geq 0}{\text{minimize}}\ \mathrm{KL}\left(\boldsymbol{M}_E\|\boldsymbol{WH}\right) + \mathrm{KL}\left(\boldsymbol{M}_E\|(\boldsymbol{WH})^{\mathrm{T}}\right) + \lambda\|\boldsymbol{WH}\|_*. \tag{9}$$

The Theorem can be proved similar to [21]. The convex characterization guarantees that the global solution can be obtained in polynomial time. The non-negativity of the individual factors can be relaxed as the reconstruction is important to estimate the unknown distances for the microphone array calibration problem.

To achieve the minimization stated in (9), we adopt the framework of non-negative matrix completion (NMC) via alternating direction method of multipliers (ADMM) developed in [21]. Hence, the divergence and non-negativity constraint are decoupled from the nuclear norm penalty as

$$\underset{\boldsymbol{M}\geq 0}{\text{minimize}}\ \mathrm{KL}\left(\boldsymbol{M}_E\|\boldsymbol{M}\right) + \lambda\|\boldsymbol{Z}\|_* : \boldsymbol{M}\geq 0, \boldsymbol{M}=\boldsymbol{Z}, \boldsymbol{M}=\boldsymbol{Z}^{\mathrm{T}}. \tag{10}$$

The rest operates on the augmented Lagrangian to optimize over $\boldsymbol{M}$ and $\boldsymbol{Z}$ alternately. To account for the symmetric constraint, following the scheme elaborated in [21], a transformation through $\frac{\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}}{2}$ (element-wise averaging a matrix and its transpose) must be applied prior to updating the matrices $\boldsymbol{M}$ and $\boldsymbol{Z}$ at each iteration. This procedure is referred to as *NMC-ADMM* during the experimental evaluation presented in Section 4.

3.3. EDM Projection and Coordinate Estimation

In Sections 3.1 and 3.2, the symmetric characteristic of the EDMs are included in the optimization procedure. Hence, the algorithms are obtained to reconstruct a low-rank symmetric matrix with components best matching the observed distances. However, the recovered matrix does not necessarily correspond to a Euclidean distance matrix. To meet the full set of EDM properties, we propose to project the reconstructed matrix to the cone of Euclidean distance matrices, $\mathbb{EDM}^N$. To this end, we apply a two-step projection:

$$\mathcal{P}: \mathbb{R}^{N\times N} \longmapsto \mathbb{S}_h^N \longmapsto \mathbb{EDM}^N$$

to decrease the distance between the estimated matrix and the EDM cone. The $\mathbb{S}_h^N$ designates the space of symmetric, positive hollow matrices. The projection onto $\mathbb{S}_h^N$ is achieved by setting the diagonal values to zero followed by averaging the symmetric elements. The symmetric NMF and NMC algorithms yields a matrix very close to symmetry, however the projection into $\mathbb{S}_h^N$ guarantees this property. The ultimate projection to $\mathbb{EDM}^N$ ensures that the matrix satisfies the following properties [25]

$$\hat{\boldsymbol{M}} \in \mathbb{EDM}^N \Longleftrightarrow \begin{cases} -z^T\hat{\boldsymbol{M}}z \geq 0 \\ \mathbb{1}^T z = 0 \\ (\forall\|z\| = 1) \\ \hat{\boldsymbol{M}} \in \mathbb{S}_h^N \end{cases} \tag{11}$$

To achieve that, we search in the EDM cone using the following cost function

$$\mathcal{H}(\boldsymbol{X}) = \left\|\mathbb{1}_N\boldsymbol{\Lambda}^T + \boldsymbol{\Lambda}\mathbb{1}_N{}^T - 2\boldsymbol{X}\boldsymbol{X}^T - \hat{\boldsymbol{M}}\right\|_{\mathrm{F}}^2, \tag{12}$$

where $\boldsymbol{\Lambda} = (\boldsymbol{X}\circ\boldsymbol{X})\mathbb{1}_\kappa$ and $\circ$ denotes the Hadamard product; $\mathbb{1}$ stands for all ones vector. The minimum of $\mathcal{H}(\boldsymbol{X})$ with respect to $\boldsymbol{x}_i$ can be computed by equating the partial derivation of Eq. (12) to zero to obtain the new estimate of the coordinates $\hat{\boldsymbol{X}}$. The stopping criterion is satisfied when the new estimate differs from the old one by less than a threshold or the maximum number of iterations is reached.

4. EXPERIMENTAL ANALYSIS

4.1. Evaluation for Different Noise and Missing Ratios

The scenario consists of $N = 20$ microphones distributed at random locations within a cubic enclosure of unit dimensions. Different levels of random missing ratios equal to 18%, 25%, 38% and 48% are used at 5 different SNR levels including 5, 10, 15, 20, 30 dB. The value of SNR quantifies the level of noise on the known distances; if the noisy observed distance matrix is $\boldsymbol{D}+\boldsymbol{N}$ where $\boldsymbol{D} = [d_{ij}]$ and $\boldsymbol{N} = [n_{ij}]$ consists of the noise on individual distance measures; SNR is computed as $20\log\frac{\|\boldsymbol{D}\|_{\mathrm{F}}}{\|\boldsymbol{N}\|_{\mathrm{F}}}$.

It may be noted that the calibration algorithm extracts a possibly rotated or reflected coordinates of the microphones. Thus, we find the best match between the estimated geometry and the ground truth using the optimization procedure developed in [26] prior to quantifying the localization error.

The evaluation results in terms of microphone localization and distance estimation errors are illustrated in Fig. 1. We have compared our proposed methods, NMF-MUL (6) and NMC-ADMM (10) with three state-of-the-art techniques namely MDS-MAP, s-stress [10] and the NMF-based matrix completion algorithm (NMFC) proposed in [24]. Each number is obtained by averaging the results for 50 realizations at 100 distinct configurations. The error bars are smaller than the markers thus all the comparisons are significant. The authors of NMFC [24] recommend to use a higher rank for the latent factors $\boldsymbol{W}$ and $\boldsymbol{H}$ and it has been shown empirically to yield better results. Hence, following their insights and their implementation code, we set $\eta = 8$ to run NMFC and we observe that this rank overestimation improves the accuracy of their algorithm. For the proposed NMF-MUL and NMC-ADMM, however, the best result is obtained by setting $\eta = 5$ (the rank of $\boldsymbol{M}$).

As the results demonstrate, the convex approach after applying the symmetric constraint and EDM projection, yields the best results and it is also more stable in very noisy condition. Furthermore, we can observe that as the number of missing distances increases, the difference between performance of NMF-MUL (6) and NMC-ADMM (10) gets larger indicating the requirement of more observed distances for matrix reconstruction in the NMF framework.

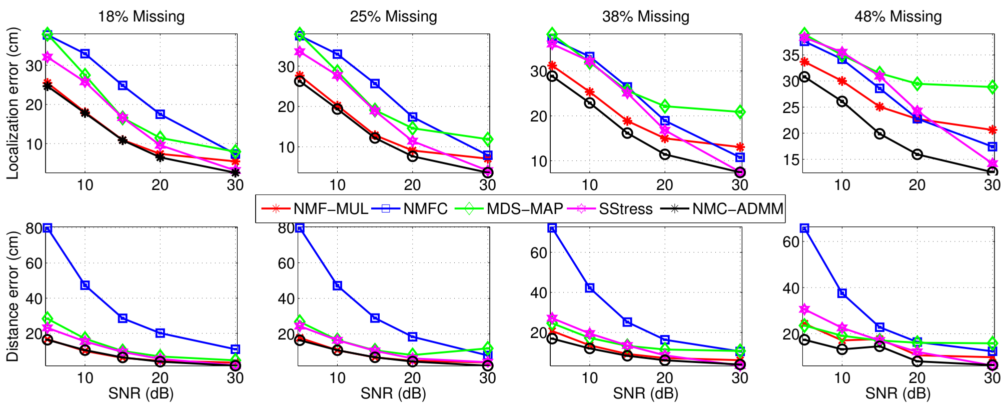

Fig. 1: Performance of the proposed NMF-MUL (6) and the convex NMC-ADMM (10) are compared with NMFC [24], MDS-MAP and S-stress [10] techniques in terms of the average error on microphone localization as well as pairwise distances estimation.

The average time required for NMC-ADMM is 5 times less than NMF-MUL. Hence, the convex approach speeds up the process while achieving higher accuracy. On a modern desktop computer, it takes less than 10 ms to run the NMC-ADMM algorithm.

To evaluate the denoising obtained through low-rank reconstruction, we run the calibration experiments at SNR = 10 dB when all the pairwise distances are known. The localization error using NMF-MUL, NMFC, MDS, S-stress and NMC-ADMM are $13.13, 20.30, 25.06, 21.67, 15.06$ cm respectively. We can see that NMF is able to denoise the distance matrix as the localization accuracy is 11.93 and 8.54 cm smaller than MDS and s-stress methods that directly use the observed noisy squared distances. In fact, the computed SNR after NMF reconstruction is 16.06 dB, thus NMF yields more than 6 dB denoising on the observed distances.

4.2. Importance of Incorporating EDM Properties

We quantify the gain explicitly obtained after constraining the optimization to yield a symmetric approximation, (5) and (9), as well as EDM projection (12). Fig. 2 illustrates the contribution of each additional structure to reduce the error of estimating the microphone positions. The trend in similar for both NMF-MUL and NMC-ADMM. We can see that the benefit of incorporating the

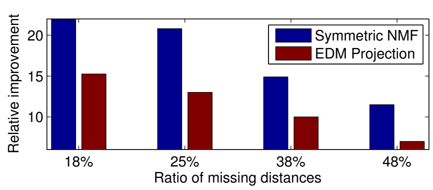

Fig. 2: Contribution of incorporating the symmetric and full EDM properties (excluding symmetric) in localization performance with respect to the standard NMF [20].

EDM properties is more illustrated when the ratio of missing distances is relatively small. Furthermore, the symmetric version of NMF achieves a huge gain with respect to the standard NMF and it is noticeable compared to the EDM projection.

In addition to the EDM properties, we observed that the sparsity constraint on the coefficient matrix is quite useful [19]; in fact, sparse NMF performs 11% better than NMF without sparsity at low-SNR regimes and highly incomplete distance matrix. The hyper-parameters for the sparse NMF [19] (NMF regularized with l_1 norm of $\boldsymbol{H}$ - defined as the sum of the absolute values of its components) are set to $\alpha_X^{[19]} = 0.1$ and $\alpha_{sX}^{[19]} = 1.001$.

4.3. Importance of the Number of Microphones

In this final experiment, we study the effect of the number of connected nodes for microphone array calibration. Different networks of size $N = \{10, 20, 30, 40, 50, 100\}$ are considered while the missing ratios are equal and the observed distances have 20 dB accuracy. The summary of localization error is listed in Table 1. We can see that as the number of microphones increases the calibration performance improves substantially.

Table 1: Localization error (cm) for different number of microphones and missing ratios at SNR=20 dB.

%Missing	Number of microphones					
	10	**20**	**30**	**40**	**50**	**100**
18%	11.64	6.52	4.94	4.01	3.62	2.59
25%	15.56	7.78	5.46	4.58	3.94	2.72
38%	20.05	11.11	7.25	5.68	4.80	3.19
48%	23.32	15.36	9.78	7.25	5.99	3.81

5. CONCLUSIONS

We studied the application of non-negative matrix factorization for calibration of ad hoc microphone arrays using a small subset of pairwise distances. We have shown that NMF enables estimation of the unknown distances exploiting the low-rank property of the squared distance matrix. The multiplicative update rules for factorization of symmetric matrices were derived. Furthermore, the convex relaxation of the reconstruction objective was elaborated and effective optimization was achieved via ADMM. The experiments demonstrated that the convex approach yields the best localization results. In addition, the symmetric constraint and EDM projection improved the calibration accuracy significantly. This study motivates derivation of the ADMM procedure to meet all the EDM properties. It further encourages us to revisit the formulation of NMF for other problems which admit convex objectives.

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