COMMON PART ESTIMATION OF ACOUSTIC FEEDBACK PATHS IN HEARING AIDS OPTIMIZING MAXIMUM STABLE GAIN

Henning Schepker and Simon Doclo

Signal Processing Group, Dept. Medical Physics and Acoustics,
and Cluster of Excellence Hearing4All, University of Oldenburg, Germany
Email: `henning.schepker@uni-oldenburg.de`

ABSTRACT

The computational complexity and convergence speed of adaptive feedback cancellation algorithms depend on the number of adaptive parameters used to model the acoustic feedback path. To reduce the number of adaptive parameters it has been proposed to decompose the acoustic feedback path as the convolution of a (time-invariant) common part and a (time-varying) variable part. Typically the problem of estimating all the required coefficients has been formulated as a least-squares optimization problem. In contrast, in this paper we propose to formulate the estimation problem as a minmax optimization problem and show how this is associated with the maximum stable gain of a hearing aid. Experimental results using measured acoustic feedback paths from a two-microphone behind-the-ear hearing aid show that the proposed minmax optimization outperforms the least-squares optimization in terms of maximum stable gain. Furthermore, the robustness of proposed common part decomposition for different feedback paths is evaluated.

***Index Terms*—** acoustic feedback cancellation, common part modeling, invariant part extraction, minmax optimization, hearing aids

1. INTRODUCTION

The number of hearing impaired persons supplied with open-fitting hearing aids has been steadily increasing over the last years. Although largely alleviating problems related to the occlusion effect, open-fitting hearings aids are especially susceptible to acoustic feedback, often perceived as whistling or howling. This demands for robust and fast-adapting feedback cancellation algorithms.

While different strategies can be used to reduce the acoustic feedback (see e.g. [1, 2]), adaptive feedback cancellation (AFC) is one of the most promising approaches. In AFC an adaptive filter is used to estimate the impulse response (IR) of the acoustic feedback path, theoretically allowing for perfect cancellation of the feedback signal [1]. In general, the convergence speed and the computational complexity of the adaptive filter is determined by the number of adaptive parameters [3, 4]. In order to reduce the number of adaptive parameters and hence reduce the complexity and improve the convergence speed it has been proposed [5, 6, 7, 8] to model the acoustic feedback path as the convolution of two filters: a time-invariant *common part* and a time-varying *variable part*. While the time-invariant *common part* models parts that are common in several acoustic feedback paths, e.g., transducer characteristics and individual ear canal characteristics, the time-varying *variable part* enables to track fast changes, e.g., in the presence of a handheld telephone.

This work was supported in part by the Research Unit FOR 1732 "Individualized Hearing Acoustics" and the Cluster of Excellence 1077 "Hearing4All", funded by the German Research Foundation (DFG).

For modeling the common part different filters have been proposed, i.e., an all-zero filter [6], an all-pole filter [9] and the general pole-zero filter [7]. For practical reasons the variable part is commonly modeled using an all-zero filter to enable easy and stable adaptation using standard adaptive filtering techniques.

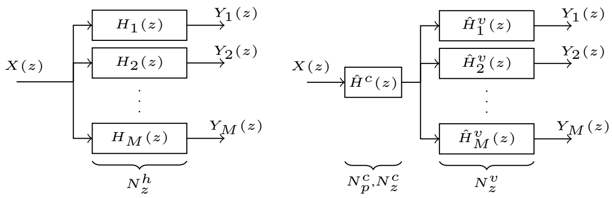
Assuming that the IRs of at least two acoustic feedback paths are available (e.g., using measurements), the common and variable parts are usually estimated by minimizing a least-squares (LS) cost function [6, 7, 8, 9]. This corresponds to optimizing the misalignment between the true and estimated IRs, which is often used to quantify the performance of adaptive filters. However, although these approaches allow for good performance in terms of the misalignment, the closed-loop transfer function of the hearing aid may already be unstable for smaller gains. Therefore, in this paper we propose to directly optimize the maximum stable gain (MSG) [10], i.e., the maximum applicable gain that leads to a stable closed-loop response of the hearing aid. After reviewing the commonly used technical measures to assess the performance of AFC algorithms in Section 3, in Section 4 we show how the problem of maximizing the MSG can be formulated as a minmax optimization problem. The resulting nonlinear minmax cost function is then minimized by an alternating optimization procedure. Experimental results using measured acoustic feedback paths from a two-microphone behind-the-ear hearing aid indicate that: 1) using the proposed minmax optimization procedure to estimate the common pole-zero filter of acoustic feedback paths yields an increase in MSG compared to existing LS techniques, and 2) a reduction of the number of variable part parameters can be achieved even for unknown feedback paths.

2. PROBLEM FORMULATION

Consider a single-input-multiple-output (SIMO) system with M outputs as depicted in Figure 1a. Such a SIMO system arises, e.g., in a single-loudspeaker multiple-microphone setup in a multi-microphone hearing aid. The m-th output signal $Y_m(z)$ is related to the input signal $X(z)$ by the m-th acoustic transfer function (ATF) $H_m(z)$ as $Y_m(z) = H_m(z)X(z)$. We assume that the true ATFs $H_m(z)$ are causal all-zero filters of finite order N_z^h each. To reduce the number of coefficients required to model all ATFs, the following approximation (depicted in Figure 1b) is introduced:

$$\boxed{\begin{bmatrix} H_1(z) \\ \vdots \\ H_M(z) \end{bmatrix} \approx \begin{bmatrix} \hat{H}_1(z) \\ \vdots \\ \hat{H}_M(z) \end{bmatrix} = \hat{H}^c(z) \begin{bmatrix} \hat{H}_1^v(z) \\ \vdots \\ \hat{H}_M^v(z) \end{bmatrix},} \tag{1}$$

where $H^c(z)$ denotes the microphone-independent *common part* and $H_m^v(z)$ the microphone-dependent *variable parts*. Assuming the *common part* can be modeled as a pole-zero filter with N_p^c poles



(a) SIMO system (b) Approximation of SIMO system

Fig. 1. System models.

and N_z^c zeros and each of the M *variable parts* can be modeled as an all-zero filter with N_z^v zeros, their transfer functions are defined as

$$\hat{H}^c(z) = \frac{B^c(z)}{A^c(z)} = \frac{\sum_{l=0}^{N_z^c} b^c[l]z^{-l}}{1 + \sum_{l=1}^{N_p^c} a^c[l]z^{-l}}, \tag{2}$$

$$\hat{H}_m^v(z) = B_m^v(z) = \sum_{l=0}^{N_z^v} b_m^v[l]z^{-l}, \tag{3}$$

where $a^c[j]$, $b^c[j]$ and $b_m^v[j]$ denote the coefficients of the polynomials associated with the common poles, common zeros and variable zeros, respectively. Note that $a^c[0] = 1$, i.e., $A^c(z)$ is assumed to be a monic polynomial. The coefficients in vector notation are defined as

$$\mathbf{h}_m = [\ h_m[0] \quad h_m[1] \quad \dots \quad h_m[N_z^h]\ ]^T, \tag{4}$$

$$\mathbf{a}^c = [\ a^c[1] \quad a^c[2] \quad \dots \quad a^c[N_p^c]\ ]^T, \tag{5}$$

$$\mathbf{b}^c = [\ b^c[0] \quad b^c[1] \quad \dots \quad b^c[N_z^c]\ ]^T, \tag{6}$$

$$\mathbf{b}_m^v = [\ b_m^v[0] \quad b_m^v[1] \quad \dots \quad b_m^v[N_z^v]\ ]^T, \tag{7}$$

where $[\cdot]^T$ denotes transpose operation. We also define the concatenation of the variable part coefficient vectors $\mathbf{b}_m^v$ as

$$\mathbf{b}^v = [\ (\mathbf{b}_1^v)^T \quad (\mathbf{b}_2^v)^T \quad \dots \quad (\mathbf{b}_M^v)^T\ ]^T. \tag{8}$$

3. TECHNICAL MEASURES OF FEEDBACK CANCELLATION PERFORMANCE

To characterize the performance of AFC algorithms two measures are commonly used [1, 6]: the normalized misalignment ϵ and the MSG $\mathcal{M}$. The normalized misalignment ϵ_m in the m-th microphone indicates the performance of the estimated feedback paths in terms of its Euclidean distance to the true feedback paths, i.e.,

$$\epsilon_m = \frac{\|\mathbf{h}_m - \hat{\mathbf{h}}_m\|_2^2}{\|\mathbf{h}_m\|_2^2}, \tag{9}$$

where $\hat{\mathbf{h}}_m$ is the estimated IR.

The MSG $\mathcal{M}$ indicates the gain that can be applied in a hearing aid until instability of the closed-loop system and hence howling or whistling occur. Assuming a broadband hearing aid gain, the MSG $\mathcal{M}_m$ of the m-th feedback path can be computed as [10]

$$\boxed{\mathcal{M}_m = 20\log_{10} \frac{1}{\max_{0\le\Omega\le\pi} |H_m(e^{j\Omega}) - \hat{H}_m(e^{j\Omega})|},} \tag{10}$$

where Ω denotes normalized frequency. Note that the system is actually only unstable if also the phase at the frequency of the maximum absolute difference is a multiple of 2π [11] and hence (10) provides the worst-case assumption.

4. OPTIMIZING THE MAXIMUM STABLE GAIN

In [6, 7, 8, 9] LS procedures minimizing the misalignment in (9) have been presented to estimate the coefficients vectors $\mathbf{a}^c$, $\mathbf{b}^c$, $\mathbf{b}^v$ of the common and variable parts. In contrast in this section we propose to estimate the coefficient vectors $\mathbf{a}^c$, $\mathbf{b}^c$, $\mathbf{b}^v$ of the common and variable parts in order to maximize the MSG in (10). This leads to a novel optimization procedure for the approximate SIMO system depicted in Figure 1b. Maximizing $\mathcal{M}_m$ in (10) corresponds to minimizing the maximum absolute difference of the frequency response of the true and estimated ATFs. Thus maximizing $\mathcal{M}_m$ for all M IRs of the considered approximate SIMO system can be formulated as a minmax optimization problem, where we aim to minimize

$$\boxed{J_{MM}(\mathbf{a}^c, \mathbf{b}^c, \mathbf{b}^v) = \max_{\substack{0\le\Omega\le\pi \\ 1\le m\le M}} \left|\tilde{E}_m(e^{j\Omega})\right|} \tag{11}$$

with the so-called output-error $\tilde{E}_m(e^{j\Omega})$ defined as

$$\tilde{E}_m(e^{j\Omega}) = H_m(e^{j\Omega}) - \hat{H}_m(e^{j\Omega}) \tag{12}$$

$$= H_m(e^{j\Omega}) - \frac{B^c(e^{j\Omega})}{A^c(e^{j\Omega})} B_m^v(e^{j\Omega}). \tag{13}$$

For conciseness we will omit the variable $e^{j\Omega}$ in the remainder of this paper. Note that the cost function in (11) aims at maximizing the minimum $\mathcal{M}_m$ for the considered set of M IRs, which is a reasonable assumption since this will presumably dominate the MSG in, e.g., multi-microphone hearing aids. The output-error $\tilde{E}_m$ is non-linear in A^c, B^c and B_m^v and thus minimization of (11) is not straight forward. To ease the optimization we rewrite the output-error as

$$\tilde{E}_m = \frac{1}{A^c} E_m = \frac{1}{A^c}\left(A^c H_m - B^c B_m^v\right), \tag{14}$$

where E_m is the so-called equation-error which is non-linear in only B^c and B_m^v. This formulation suggests the following iterative optimization procedure to approximate (11) where at iteration i we aim to minimize

$$\boxed{J_{WM}(\mathbf{a}_i^c, \mathbf{b}_i^c, \mathbf{b}_i^v) = \max_{\substack{0\le\Omega\le\pi \\ 1\le m\le M}} \frac{1}{|A_{i-1}^c|} |E_{m,i}|} \tag{15}$$

where the equation-error $E_{m,i}$ is weighted by the inverse frequency-response of A_{i-1}^c from the previous iteration. Thus at convergence ideally $A_i^c \approx A_{i-1}^c$ and hence $(A_{i-1}^c)^{-1} E_{m,i} \approx \tilde{E}_m$ approximating the desired output-error minimization. This approach is similar to the well-known Steiglitz-McBride method [12] for LS identification of single-input-single-output (SISO) systems, which was also successfully applied to LS estimation of the approximate SIMO system in [8]. It should be noted that the Steiglitz-McBride method [12] as well as the approach in [8] use iterative LS estimation whereas minimizing (15) yields an iterative minmax optimization problem. A similar iterative optimization procedure was presented in [13] in the context of SISO digital filter design.

The cost function in (15) is non-linear in B^c and B_m^v and hence yields a non-linear optimization problem. However, to minimize (15) the problem can be split into two separate convex subproblems. Hence, we employ a two-step alternating optimization procedure similar to the two-step alternating optimization procedure proposed to minimize the LS equation-error in [7] or the the weighted LS equation-error in [8].

Step 1: We assume $\mathbf{a}^c$ and $\mathbf{b}^c$ fixed to be fixed to values from the previous iteration and estimate the coefficient vector $\mathbf{b}^v$ that minimizes

$$\boxed{J_{WM}(\mathbf{b}_i^v) = \max_{\substack{0\le\Omega\le\pi \\ 1\le m\le M}} \frac{1}{|A_{i-1}^c|}|E_{m,i}^v|} \tag{16}$$

with

$$E_{m,i}^v = A_{i-1}^c H_m - B_{i-1}^c B_{m,i}^v. \tag{17}$$

This equation-error can be written in the time-domain as [7]

$$\mathbf{e}_{m,i}^v = \tilde{\mathbf{h}}_m + \tilde{\mathbf{H}}_m \mathbf{a}_{i-1}^c - \tilde{\mathbf{B}}_{i-1}^c \mathbf{b}_{m,i}^v \tag{18}$$

where $\tilde{\mathbf{h}}_m$ is the $(\tilde{N}_z^h + N_p^c + 1)$-dimensional zero-padded version of $\mathbf{h}_m$ with $\tilde{N}_z^h = \max\{N_z^h, N_z^c + N_z^v\} + 1$, $\tilde{\mathbf{H}}_m$ is the $(\tilde{N}_z^h + N_p^c + 1) \times N_p^c$-dimensional convolution matrix of $\tilde{\mathbf{h}}_m$ and $\tilde{\mathbf{B}}_{i-1}^c$ is the $(\tilde{N}_z^h + N_p^c + 1) \times (N_z^v + 1)$-dimensional convolution matrix of $\mathbf{b}_{i-1}^c$. The minimization of (16) can be reformulated as the following linear program (LP) [14]

$$\begin{aligned}
&\min_{t,\mathbf{b}_i^v} \quad t && \text{(19a)}\\
&\text{s.t.} \quad t \ge 0 && \text{(19b)}\\
&\qquad |\mathbf{c}^T(\Omega)\big(\frac{1}{A_{i-1}^c(q^{-1})}\mathbf{e}_{m,i}^v\big)| \le t \quad \forall m, \Omega && \text{(19c)}\\
&\qquad |\mathbf{s}^T(\Omega)\big(\frac{1}{A_{i-1}^c(q^{-1})}\mathbf{e}_{m,i}^v\big)| \le t \quad \forall m, \Omega && \text{(19d)}
\end{aligned}$$

with q^{-1} the unit delay operator [1], i.e., $q^{-1}h_m[k] = h_m[k-1]$, k the sample index, hence essentially a filtering operation on the equation-error $\mathbf{e}_{m,i}^v$ is performed and

$$\mathbf{c}(\Omega) = [\ 1 \quad \cos\Omega \quad \dots \quad \cos(\tilde{N}_z^h + N_p^c)\Omega\ ]^T, \tag{20}$$

$$\mathbf{s}(\Omega) = [\ 0 \quad \sin\Omega \quad \dots \quad \sin(\tilde{N}_z^h + N_p^c)\Omega\ ]^T. \tag{21}$$

The expressions in (19c) and (19d) essentially compute the absolute values of the real and the imaginary part of the frequency response of the weighted equation-error in (16). This LP can then be efficiently solved using convex optimization methods, e.g., CVX [15, 16].

Step 2: We assume $\mathbf{b}^v$ fixed to be fixed to the value from the previous step and estimate the coefficient vectors $\mathbf{a}^c$ and $\mathbf{b}^c$ that minimize

$$\boxed{J_{WM}(\mathbf{a}_i^c, \mathbf{b}_i^c) = \max_{\substack{0\le\Omega\le\pi \\ 1\le m\le M}} \frac{1}{|A_{i-1}^c|}|E_{m,i}^c|} \tag{22}$$

with

$$E_{m,i}^c = A_i^c H_m - B_i^c B_{m,i}^v \tag{23}$$

Note that stability of the IRs estimated by minimizing (22) is not guaranteed. Hence, the location of the poles, i.e., the roots of $A_i^c(z)$, needs to be constrained. In general, stability of a causal pole-zero filter is guaranteed if all poles are located strictly inside the unit-circle. A sufficient condition for stability of $\frac{1}{A_i^c(z)}$ is that the real part of its frequency response is strictly positive [17], i.e.,

$$\mathfrak{Re}\{A_i^c\} > 0, \quad \forall\Omega. \tag{24}$$

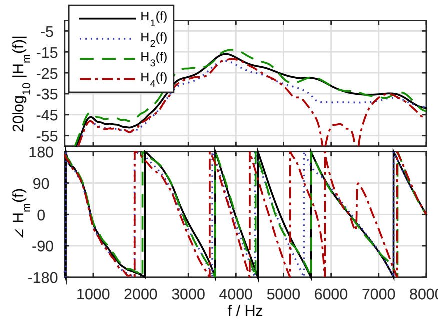


Fig. 2. Frequency response and phase responses of the IRs $m = 1, 2, 3, 4$ used in the experimental evaluation.

To control the strength of the constraint a small positive constant δ is typically introduced [17], i.e.,

$$\mathfrak{Re}\{A_i^c\} \ge \delta, \quad \forall\Omega. \tag{25}$$

In order to rewrite the cost function in (22) subject to the constraint in (25) as an LP we again first rewrite $E_{m,i}^c$ in the time-domain as [7]

$$\mathbf{e}_{m,i}^c = \tilde{\mathbf{h}}_m + \tilde{\mathbf{H}}_m \mathbf{a}_i^c - \tilde{\mathbf{B}}_{m,i}^v \mathbf{b}_i^c \tag{26}$$

with $\tilde{\mathbf{B}}_{m,i}^v$ the $(\tilde{N}_z^h + N_p^c + 1) \times N_z^c + 1$-dimensional convolution matrix of $\mathbf{b}_{m,i}^v$. This allows to reformulate the minimization of (22) subject to the stability constraint in (25) as the following LP

$$\begin{aligned}
&\min_{t,\mathbf{a}_i^c,\mathbf{b}_i^c} \quad t && \text{(27a)}\\
&\text{s.t.} \quad t \ge 0 && \text{(27b)}\\
&\qquad |\mathbf{c}^T(\Omega)\big(\frac{1}{A_{i-1}^c(q)}\mathbf{e}_{m,i}^c\big)| \le t \quad \forall m, \Omega && \text{(27c)}\\
&\qquad |\mathbf{s}^T(\Omega)\big(\frac{1}{A_{i-1}^c(q)}\mathbf{e}_{m,i}^c\big)| \le t \quad \forall m, \Omega && \text{(27d)}\\
&\qquad -\sum_{l=1}^{N_p^c} a^c[l]cos(l\Omega) \le 1 - \delta \quad \forall\Omega && \text{(27e)}
\end{aligned}$$

where in (27e) the stability constraint (25) has been reformulated to be applicable to linear programming.

The LPs in (19) and (27) are solved alternatingly until some convergence criterion is achieved or a maximum number of iterations is exceeded. Furthermore, the continuous variable Ω is discretized by using an equidistant grid of N frequencies Ω_n in the range $0 \le \Omega_n \le \pi$.

5. EXPERIMENTAL EVALUATION

In this section the proposed minmax estimation procedure maximizing the MSG is evaluated and compared to the LS estimation procedure of [8] minimizing the misalignment. Acoustic feedback paths were measured using a two-microphone behind-the-ear hearing aid with an open-fitting ear mold on a dummy head with adjustable ear canals [18]. The IRs were sampled at $f_s = 16$ kHz and truncated to order $N_z^h = 99$. The frequency and phase responses of four acoustic feedback paths used in the evaluation are depicted in Figure 2. Two IRs ($m = 1, 2$) were measured without obstruction and two IRs ($m = 3, 4$) were measured with a telephone receiver in close distance.

The performance was evaluated in terms of the MSG defined in (10). Evaluations were made for the following set of parameters: $N_p^c, N_z^c \in \{0, 4, 8, \dots, 24\}$, $N_z^v \in \{6, 12, \dots, 48\}$. In the

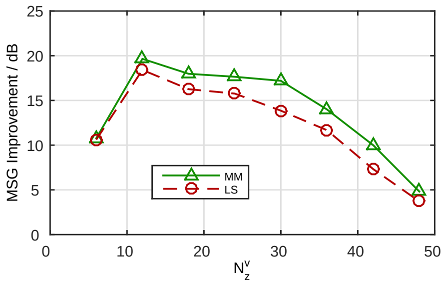


Fig. 3. MSG improvements of the proposed minmax optimization (MM) and LS optimization of [8] using $N^c = 8$ compared to using $N^c = 0$.

following N^c denotes the number of common part parameters, i.e., $N^c = N_p^c + N_z^c$. Note that different combinations of N_p^c and N_z^c can lead to the same value of N^c. We used $N = 1025$ discrete frequencies and $\delta = 10^{-6}$ to control the stability margin in (27e). Since it was experimentally found that a good initialization for the proposed minmax optimization procedure is crucial, we used the solution obtained from the LS approach proposed in [8] to initialize the coefficient vectors $\mathbf{a}^c$ and $\mathbf{b}^c$.

5.1. Improved maximum stable gain

Figure 3 shows exemplary average MSG improvements for IRs $m = 1, 2$ when using $N^c = 8$ compared to using $N^c = 0$, i.e., not using any common part information, for the proposed minmax (MM) optimization and the LS optimization of [8]. The average has been computed for a given combination of common part parameters N_p^c and N_z^c across both IRs. Note that three different choices of N_p^c and N_z^c from the considered parameter set lead to $N^c = 8$ where only the combination with the maximum improvement is depicted in Figure 3. The results indicate that using common part knowledge improves the MSG for both the proposed minmax optimization and the LS optimization. In general these large improvements are expected due to the fact that by including common part information the total number of available parameters is increased. In addition improvements decrease towards larger N_z^v since most the energy of the IRs is located within the first 50 samples. Nevertheless, the proposed minmax optimization clearly outperforms the LS optimization of [8] in terms of MSG.

To compare the two optimization procedures for different choices of N^c Figure 4 depicts the MSG improvement of the minmax optimization compared to the LS optimization for different choices of N^c. Here the average improvement is computed across the different combinations of N_p^c and N_z^c leading to the same N^c and both IRs ($m = 1, 2$). Furthermore the error bars indicate the minimum and maximum improvement. Again the minmax optimization procedure outperforms LS optimization procedure in terms of MSG with average improvements as large as 4dB. Improvements appear to be rather constant in the range of 2-3 dB across different values of N_z^v and tend to increase for larger values of N^c and N_z^v.

5.2. Robustness to unknown feedback paths

Modeling of the acoustic feedback path as the combination of a common part and variable part(s) is motivated by the goal to reduce the number of adaptive parameters, i.e., the number of parameters N_z^v used to model the variable part, to reduce complexity and increase convergence speed.

Here we thus investigate the robustness of the common part estimate to unknown feedback paths, i.e., feedback paths that were not included during optimization, in terms of the minimum number of variable part coefficients N_z^v required to achieve a desired MSG. Compared to the IRs depicted in Figure 2 four additional IRs were

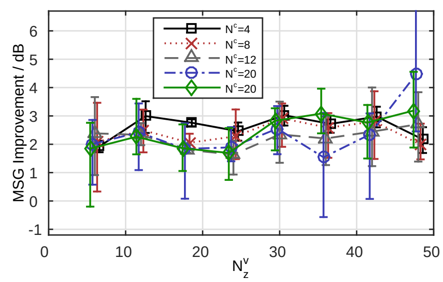


Fig. 4. MSG improvements of the proposed minmax optimization (MM) compared to the LS optimization proposed in [8]. Symbols indicate mean improvements across different combinations of N_p^c and N_z^c for a fixed N^c, while error bars indicate the minimum and maximum improvements across all possible combinations.

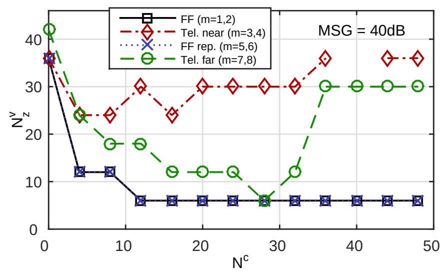


Fig. 5. Minimum number of variable part parameters N_z^v as a function of N^c given a desired MSG of 40 dB for different acoustic feedback paths.

included in the evaluation. Two IRs ($m = 5, 6$) were measured after repositioning of the hearing aid without any obstruction (similar to IRs $m = 1, 2$), while two IRs ($m = 7, 8$) where measured with a telephone receiver at a distance of approximately 24cm. First the common part $H^c(z)$ is estimated with the proposed minmax optimization procedure using only IRs $m = 1, 2$. Keeping the common part fixed the variable parts for IRs $m = 3, 4$, $m = 5, 6$ and $m = 7, 8$ are then estimated using (16).

Figure 5 depicts the minimum number of variable part parameters N_z^v required to achieve a desired MSG of 40 dB as a function of the number of common part parameters. Note that $N^c = 0$ corresponds to using no common part information. For IRs $m = 1, 2$ best performance is achieved as could be expected as those were included during optimization. Similar results are obtained for IRs $m = 5, 6$ indicating that the common part is robust to minor changes of the hearing aids position. However, in the presence of a telephone receiver the performance degrades depending on the positioning. Nevertheless, for low N^c a reduction of the variable part parameters is still possible for both telephone positions. These results indicate that even for changing acoustic conditions the proposed common part decomposition can achieve a reduction for the variable part parameters given a desired MSG.

6. CONCLUSION

In this paper the problem of estimating a common pole-zero filter from a set of measured acoustic feedback paths was formulated as a minmax optimization problem aiming to maximize the MSG. The resulting non-linear cost function was minimized using alternating optimization techniques. Experimental results using measured acoustic feedback paths indicate that the proposed estimation procedure leads to a higher MSG compared to existing LS optimization procedures minimizing the misalignment. Furthermore, for a desired MSG the proposed minmax optimization is able to reduce the number of the variable part parameters even for unknown feedback paths.

7. REFERENCES

[1] A. Spriet, S. Doclo, M. Moonen, and J. Wouters, "Feedback Control in Hearing Aids," in *Springer Handbook of Speech Processing*, pp. 979–999. Springer-Verlag, Berlin, Germany, 2008.

[2] T. van Waterschoot and M. Moonen, "Fifty Years of Acoustic Feedback Control: State of the Art and Future Challenges," *Proc. IEEE*, vol. 99, no. 2, pp. 288–327, Feb. 2011.

[3] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, 3rd edition, 1996.

[4] A. H. Sayed, *Fundamentals of Adaptive Filtering*, John Wiley & Sons, 2003.

[5] J. M. Kates, "Feedback Cancellation Apparatus and Methods," *US Patent, 6,072,884*, 2000.

[6] G. Ma, F. Gran, F. Jacobsen, and F. Agerkvist, "Extracting the invariant model from the feedback paths of digital hearing aids.," *J. Acoust. Soc. Am.*, vol. 130, no. 1, pp. 350–63, July 2011.

[7] H. Schepker and S. Doclo, "Modeling the common part of acoustic feedback paths in hearing aids using a pole-zero model," in *Proc. Int. Conf. Acoust. Speech Signal Process. (ICASSP), Florence, Italy*, May 2014, pp. 3693–3697.

[8] H. Schepker and S. Doclo, "Estimation of the common part of acoustic feedback paths in hearing aids using iterative quadratic programming," in *Proc. Int. Workshop Acoust. Signal Enhancement (IWAENC), Antibes - Juan Les Pins, France*, Sept. 2014.

[9] Y. Haneda, S. Makino, and Y. Kaneda, "Common acoustical pole and zero modeling of room transfer functions," *IEEE Speech Audio Process.*, vol. 2, no. 2, pp. 320–328, 1994.

[10] J. M. Kates, "Room reverberation effects in hearing aid feedback cancellation," *J. Acoust. Soc. Am.*, vol. 109, no. 1, pp. 367–378, Jan. 2001.

[11] H. Nyquist, "Regeneration theory," *Bell System Technical Journal*, vol. 11, no. 1, pp. 126–147, 1932.

[12] K. Steiglitz and L. McBride, "A technique for the identification of linear systems," *IEEE Trans. Autom. Control*, vol. 10, no. 4, pp. 461–464, 1965.

[13] W.S. Lu and A. Antoniou, "Design of digital filters and filter banks by optimization: A state of the art review," in *Proc. European Signal Proc. Conf., Tampere, Finland*, 2000.

[14] S. P. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge university press, 2004.

[15] Michael Grant and Stephen Boyd, "Graph implementations for nonsmooth convex programs," in *Recent Advances in Learning and Control*, V. Blondel, S. Boyd, and H. Kimura, Eds., Lecture Notes in Control and Information Sciences, pp. 95–110. Springer-Verlag Limited, 2008.

[16] Michael Grant and Stephen Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[17] A. Chottera and G.A. Jullien, "A linear programming approach to recursive digital filter design with linear phase," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 29, no. 3, pp. 139–149, Mar 1982.

[18] M. Hiipakka, M. Tikander, and M. Karjalainen, "Modeling the External Ear Acoustics for Insert Headphone Usage," *J. Audio Eng. Soc.*, vol. 58, no. 4, pp. 269–281, Apr. 2010.